Parametric Invariance of the Relativistic Coordinate Pulsar Time Scales

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> *The accuracy of modern astronomical observations so high that in order to understand what we see and how you can, you must use meaningful relativistic models of the observable processes. Effect of relativity can't be reduced in this case to small corrections of the Newtonian model. On the contrary, the whole concept of astronomical reference systems and astronomical observations should be revised and adapted to the framework of the theory of relativity.* (S.Klioner, 1995)

Problems. Objectives

Physical processes observed in different coordinate reference systems are different in time. On the measured coordinate time we can't confirm nor their identity, nor the accuracy of the measured values: «*It is a common mistake to believe that intervals of proper time $\Delta\tau_1$ and $\Delta\tau_2$ measured by different observers can be "uniquely" and "naturally" compared to each other. The only way to do so in General Relativity is to define a 4-dimensional relativistic reference system having coordinate time t, establish a relativistic procedure of coordinate synchronization of clock with respect to t, and convert the intervals of proper time $\Delta\tau_1$ and $\Delta\tau_2$ of each observer into corresponding intervals of coordinate time Δt_1 and Δt_2. These two intervals of coordinate time can indeed be compared directly*». (Klioner, Capitaine, Folkner, Guinot, Huang, Kopeikin, Pitjeva, Seidelmann and Soffel, 2009).

Because of the uncertainty of the observed intervals of pulsar time which are determined by the physical conditions that are known very approximately, it can not be compared pulsar and atomic time scales to reveal the instabilities in AT: *"If physical phenomena (intrinsic irregularities of the pulsar rotation, propagation, ephemerides of the solar system, etc…) introduced no error to the prediction of the arrival times of its pulses, the pulsar would realize an ideal pulsar time. If, in addition, the measurement noise was also negligible, the only source of error in AT-PT would be AT itself. In this ideal case, however, we still have to perform the adjustment of the pulsar parameters because the position, proper motion, period of rotation and its derivative are unknown. If we had an exact knowledge of their values, PT would be perfect and AT-PT would reveal exactly the instabilities in AT"* (G. Petit, P. Tavella, 1996).

Pulsar time as Time of Arrivals (TOAs) obtained by fitting of the physical parameters, doesn't permit to interpret the post-fit residuals as difference of the pulsar and atomic time for their mutual synchronization: *"The problem is that the timing measurements of pulsars are obtained only after a global fit has provided the necessary physical parameters, as these are not known apriori with sufficient accuracy. <…>For this reason it is not possible to interpret the post-fit residuals as pure differences of pulsar time and atomic time"* (B.Guinot, G.Petit, 1991).

Our approach, in general, is to find analytical relation of the pulsar time intervals and the physical parameters so that the numerical values of these parameters should be determined and best matched with measured values of the observed intervals. Analytical relations and numerical values should be extended to both, the barycentric and topocentric reference system. From fitting can be excluded any parameters that can't be obtained directly from observations.

Parametric Model of the Pulsar Time

Analytical form of the pulsar time intervals is reduced to Maclaurin power series:

$$f(x)=\sum_{n=0}^{\infty} f^{(n)}(0)\frac{x^n}{n!}=f(0)+f'(0)x+\frac{f''(0)x^2}{2!}+\ldots+\frac{f^{n}(0)x^n}{n!}+R_n, \quad (1)$$

$\lim_{n\to\infty} R_n = 0$ – condition of convergence of the series.

Observed events of pulsar radiation is expressed as a function of pulse number N in the frequency domain:

$$N=\upsilon_0 t+\frac{1}{2}\dot{\upsilon}_0 t^2+\frac{1}{6}\ddot{\upsilon}_0 t^3.$$

As frequency and period together with their derivatives are interconnected:

$$\upsilon=\frac{1}{P_0}\ [\mathrm{c}^{-1}]; \qquad \dot{\upsilon}=-\frac{\dot{P}}{P_0^2}\ [\mathrm{c}^{-2}]; \qquad \ddot{\upsilon}=-\frac{\ddot{P}}{P_0^2}+2\frac{\dot{P}^2}{P_0^3}[\mathrm{c}^{-3}],$$

the Maclaurin series of the observed pulsar events are identical in frequency and time domain:

$f^{(n)}(0)$	Rotation parameters of PSR	
$f'(0)$	υ_0	$1/P_0$
$f''(0)$	$\dot{\upsilon}$	$-\dot{P}/P_0^2$
$f'''(0)$	$\ddot{\upsilon}$	$-\ddot{P}/P_0^2+2\dot{P}^2/P_0^3$

PT intervals, expressed by the rotation parameters of the pulsar in the time domain:

$$PT(P_0,\dot{P},\ddot{P})=P_0 N+\frac{1}{2}P_0\dot{P}N^2+\frac{1}{6}(P_0^2\ddot{P}-2P_0\dot{P}^2)N^3,\ N=1,2,3,\ldots \quad (2)$$

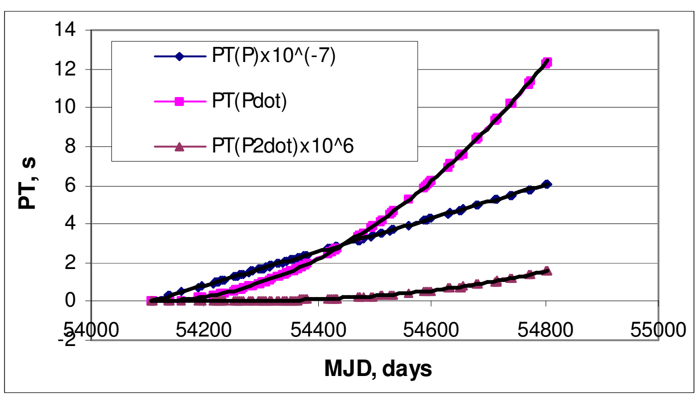


Fig.1. The parametric components of the *PT* intervals (2) of the PSR J1509+5531:

$PT(P_0)=P_0N$ at P_0 =0,739681922904 s (MJD 49904.0, [7])

$PT(\dot{P})=0{,}5P_0\,\dot{P}N^2$ at $\dot{P}$ = 4,99821·10^{-15} s·s^{-1} [8]

$PT(\ddot{P})=\frac{1}{6}(P_0{}^2\ddot{P}-2P_0\dot{P}^2)N^3;$ at $\ddot{P}$=3,0669·10^{-29} s^{-1} (it satisfy the convergence of (2), together with P_0, $\dot{P}$).

Consistency of the observed rotation parameters on the age span

The equation of the observed intervals of PT in accordance with (2) is:

$$PT_i = (1+\alpha_i)(P_0^* N + \frac{1}{2} P_0^* \dot{P} N^2 + \frac{1}{6}(P_0^{*2} \ddot{P} - 2P_0^* \dot{P}^2) N^3)_i \,. \qquad (3)$$

Here are: PTi is the numerical values of the observed intervals obtained from the planetary ephemeris;

P_0^*, $\dot{P}$, $\ddot{P}$ are the pulsar rotation parameters obtained by solving equation (3);

α_i is divergence of series (3) of the PTi approximated by the rotation parameters of pulsar.

By parametric approximation of the intervals PTi (3) counted from the initial observed pulsar event, the fixed rotation period and its derivatives on the initial epoch in accordance with the power series expansion Maclaurin (2), are defined.

It is obvious, for any choice of the epoch of initial event, the value of period will be different, taking into account the gap between epochs and the derivatives $\dot{P}$, $\ddot{P}$. The corresponding settings of rotation parameters also satisfy the convergence of the series expansion (3) for any extension in the vicinity specified by the variable $t = P_0^* N$:

$$P(t) = P_0^* + \dot{P}_0 \cdot t + \ddot{P} \cdot t^2; \;\; t = P_0^* N, \;\; 1 < N < \infty. \qquad (4)$$

Here are: $P(t) = P_0^* + \dot{P} \cdot t$; $\quad \dot{P} = \dot{P}_0 + \ddot{P} \cdot t$

Values of N_i, determined by the equation (3), unlike the calculated ratio (2), are not integer due to random variations in the pulse time of arrival (propagation, error of AT, ephemeris of the Solar system, fitting, etc.). Founded in accordance with the equation (3) the real values N_i are different from integer value by $\Delta N_i = \frac{\Delta\varphi(t)_i}{2\pi}$ determined by the observed pulse phase shift $\Delta\varphi(t)_i = \frac{2\pi}{P}\Delta t_i$ within the current period of rotation.

Real value $(N_i + \Delta N_i)$ includes himself in the solution of equation (3), in addition to the P_0^*, $\dot{P}$, $\ddot{P}$. It corresponds to the minimum of random variations of the divergence α_i and matches the phase of the observed event radiation determined by the stable rotation parameters of the pulsar at any real values of N_i.

Unmodeled variations of the observed intervals of the coherent pulsar radiation are limited a nanosecond range values, although the scattering of the time of pulse arrival can be up to several milliseconds.

Parametric model of PT firmly detects the second derivative of the period by the cubic component $PT(\ddot{P})$ in the observed intervals, the value of which is only about 1-2 mks within a 2-year observation (Figure 1). From observations PSR J1509 +5531 on the radio telescope LPA was found the numerical value $\ddot{P}$= 3,0669·10^{-29} s^{-1}, which is agreed with the period P_0^* and its derivative $\dot{P}$ in accordance with (4).

For comparison, at the specified in the ATNF Pulsar Catalogue [8] value $\ddot{\upsilon}$ for PSR J1509 +5531 corresponds to $\ddot{P}=-2{,}82\cdot10^{-26}$ s^{-1}. This value of $\ddot{P}$ shows anomalous discrepancy of the second derivative due to the unmodeled variations of pulse TOAs and residual deviations. This fact have been noted by Hobbs, Lyne and Kramer in the fundamental investigation [9]: *«The observed values $\ddot{\upsilon}$ for the majority of pulsars are not caused by magnetic dipole radiation or by any other systematic loss of rotational energy, but are dominated by the amount of timing noise present in the residuals and the data span»*.

Our observations has shown that for some pulsars, as B0809+74, B1919+21, the contribution of the second derivative during about 2 yr. observations, is negligible, it does not reach even the nanosecond threshold of detection. For these pulsars the right side of the equation (3) is limited by two components of the power series expansion only: $PT = P_0 N + 0{,}5 P_0 \dot{P} N^2$.

Parametric invariance of the PT intervals in the coordinate systems

According to the principle of relativity, which has formulated Poincare (1906), all physical processes occurring in any inertial system under the same conditions, are identical and correspond to the metric of four-dimensional space-time defined by the invariant interval

$$(d\sigma)^2 = c^2 (dT)^2 - (dX)^2 - (dY)^2 - (dZ)^2 . \qquad (5)$$

Spatial coordinates and time in the invariant (5) are related by direct and inverse Lorentz transformations that define common local time T for any points in three-dimensional space:

Direct: $T' = \gamma(T - \frac{v}{c^2} X)$, $X' = \gamma(X - vT)$;

Inverse: $T = \gamma(\tau + \frac{v}{c^2} X')$, $X = \gamma(X' + v\tau)$;

Here are: $\gamma = \frac{1}{\sqrt{1 - v^2 / c^2}}$; $\frac{1}{\gamma} T' = \tau$; T' – changed local time of the T.

Lorentz transformations overcome effects of different conditions of observation in the coordinate systems due to movement, current position of the observer, signal propagation time, thus leads physical processes to common conditions of observations.

By developing and generalizing the principle of relativity of Poincare, A. Logunov (1987) extended it without any changes physical entity to non-inertial reference systems as well, by showing that the interval (5) is invariant in respect any coordinate system [5].

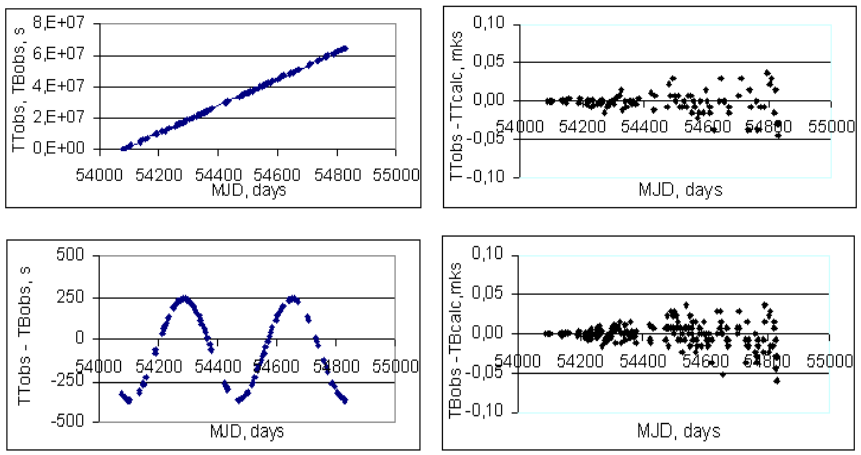
Equation of the PT intervals (3) transformed into the accelerated topocentric and inertial barycentric coordinate systems, are respectively [6]:

$$TT_i = (1 + \alpha_i)(P_0^{\,*} N_T + 0{,}5 P_0^{\,*} \dot{P} N_T^2)_i \qquad (6)$$

$$TB_i = (1 + \alpha_i)(P_0^{\,*} N_B + 0{,}5 P_0^{\,*} \dot{P} N_B^2)_i \qquad (7)$$

Left parts of the equations (6), (7) are interpreted as observed topocentric *TTobs* and barycentric *TBobs* intervals. The right parts are the intervals *TTcalc* and *TBcalc* which are calculated according to the observed rotation parameters of pulsar obtained by approximation of the *TTobs* and *TBobs.*

On the example of the pulsar B0809 +74 Figure 2 shows the intervals *TTobs* and *TBobs* and their difference on the two-year observations 2006 – 2008 yrs. at the radio telescope LPA FIAN (Pushchino).



a) the observed intervals (*up*) and their difference (*down*)

b) differences of the observed and calculated intervals of *TT* (*up*) and *TB* (*down*)

Fig.2. Observed topocentric (*TT*) and baricentric (*TB*) intervals of the PSR B0809+74 (*left*), inconsistency of the intervals in the coordinate systems (*right*)

Monotonically growing intervals *TTobs* and *TBobs* have a cyclical changes of their difference (*left, up*) due to the orbital motion of the Earth around the Sun (*left, down*). At these intervals in accordance with equations (6) and (7) have been determined the values of the rotation period P^*_{TT} and P^*_{TB} on the epoch *MJD* of the observed pulse counted in local coordinate time scales:

P^*_{TT} = 1.29224151775083 *s* at MJD_{TT} = 54080.0098

P^*_{TB} = 1.29224151775088 *s* at MJD_{TB} = 54080.0137

Difference in the values of the observed rotation period in the coordinate systems corresponds to the difference of the epoch of pulse observed in the coordinate systems:

$$P^*_{TB} = P^*_{TT} + \dot{P}(MJD_{TB} - MJD_{TT}) \cdot 86400, \text{ s.} \qquad (8)$$

Here are: $\dot{P}$ = 1,676·10^{-16} s· s^{-1} [7]; *TTobs* – *TBobs* = – 332.96872 *s* (LPA, Fig. 2a)

Note that the value of period in the Cat. [7]: *P* = 1.292241446861(**…**) *s* at MJD = 49162.0(**…**) is consistent with the (8), but precision is insufficient for nanosecond accuracy and subnanosecond resolution of the measured intervals of pulsar time.

This is an evidence of the principle of relativity: the physical process of periodic radiation of pulsar observed in barycentric and topocentric coordinate systems under the same conditions, is the same. The numerical values of the observed rotation period are coincide in any coordinate systems at the same epoch of local time.

Fig.2b presents the differences *TTobs–TTcalc* and *TBobs–TBcalc* that show inconsistency of intervals expressed in the metric of General relativity (GR) based on the numerical ephemeris, and metric of Special relativity (SR) based on the parametric form of *PT* intervals, in both topocentric and barycentric coordinate systems.

The differences of observed and calculated intervals are located in the same range of values in both coordinate systems. Standard statistical evaluation of their small inconsistency is about of 20 ns within the two-year span. This inconsistency can be associated with the inaccuracy of coordinate transformations of the intervals from metric GR to metric SR and unmodeled variations of the atomic time scales using for measuring of TOAs.

Thus, the intervals of coordinate pulsar time, determined by the observed rotation parameters, are synchronized and can indeed be compared directly in the coordinate systems.

Conclusion

The identity of the pulsar time intervals obtained in numerical form by the planetary ephemeris and approximated in analytical form by the rotation parameters of the pulsar, confirm the equivalence of the metric GR and metric SR.

The rotation parameters of the pulsar obtained from the equations of the observed intervals, are the same in any coordinate system at coincide epoch of local coordinate time, irrespective of choice of the initial epoch and duration of observation.

Intervals of coordinate pulsar time, which are determined by the observed rotation parameters with inconsistency within 10^{-18}-10^{-19} for 40-year duration of observations, are the precise astronomical *4-dimensional relativistic reference* measure within the Solar system that are 2-3 orders exceeds of the atomic clock standards.

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