Lunar influence on equatorial atmospheric angular momentum and consequences for nutation

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• Lunar influence on atmosphere: on old subject
• Global diagnostic: equatorial atmospheric angular momentum function $\chi$ — a strong retrograde diurnal component (~10 mas) as large as the seasonal one, squeezed in a band from 20 to 30 hours.
• By demodulation (removing diurnal carrier):

$$\chi'_x + i\chi'_y \equiv \frac{\chi'}{CEAMF} = -\left(\chi_x + i\chi_y\right) e^{i\theta}$$

$EAMF$ referred to $Gxyz$ (ITRF)

$\chi$: Equatorial Angular Momentum Function (EAMF)

$\chi'$: Celestial Equatorial Angular Momentum Function ($CEAMF$)

$\theta = \Omega t + \varphi$: Earth’s rotation angle

it spreads over the frequency band of the precession-nutation from 2 days
Most prominent terms are thermal waves caused by sun heating: $+365 \text{ d (S1, 24 h)}$, $+182 \text{ d (P1, 24.07h)}$, $\infty \text{ (K1, 23.93h)}$, $-365 \text{ d (y1, 23.87h)}$ and it mostly perturbs annual nutation at the level of 100 $\mu$as.

- Below 100 d, sharp peaks at $13.66 \text{ d (O1, 25.8h)}$ and $13.63 \text{ d}$
- Broad band peak around $7 \text{ d (28 h)}$
- Is it related to the lunar tide O1?
“Lunar band” 2-30 days - First striking feature

Pressure term $\chi'p$ and wind term $\chi'w$ are almost proportional by contrast to seasonal band (S1); wind term variations are $\sim$2–5 times larger than the ones of the pressure term (NCEP).

High band pass Filter
$\rightarrow$ periods < 30 days

N.B.: 1 mas = 5 $10^{-9}$ rad
Interpretation – local torques are negligible with regard to the bulge torque

- At a given frequency $\sigma'$ proportionality $\chi'^{\text{w}} / \chi'^{\text{p}}$ explained theoretically from atmospheric angular momentum balance, as far as

\[
\begin{align*}
\text{local torque} & \quad \ll \quad \text{bulge torque} \\
\text{pressure on topography} & \quad \text{pressure – gravitation} \\
\text{– friction} & \quad \text{on ellipsoidal Earth}
\end{align*}
\]

- This is not the case for seasonal band in a non-rotating frame (Marcus et al, 2004).

- Expected ratio: $\frac{\chi'^{\text{w}}}{\chi'^{\text{p}}} = \frac{\Omega - \sigma'}{\sigma'} = \begin{cases} \sim 13 \text{ around } 13.6 \text{ d} \\ \sim 6 \text{ around } 7 \text{ d} \end{cases}$
“Lunar band” 2-30 days - Second striking feature

By contrast to S1 band (of thermal origin), almost equal contributions of northern (NH) and southern hemispheres (HS) to the wind term. This hints a global simultaneous cause, like gravitational tides.
Tidal waves O1 (13.66 d, tidal argument $\phi_1$) and side lobe (13.63 d, argument $\phi_2$) estimated over the period 1949 – 2013

\[
\chi^p_{(IB)} [\text{mas}] = (0.05 - i 0.02) e^{i(\phi_1 + \pi/2)} + (0.02 - i 0.00) e^{i(\phi_2 + \pi/2)} \\
\chi^p_{(NIB)} [\text{mas}] = (0.17 - i 0.06) e^{i(\phi_1 + \pi/2)} + (0.06 - i 0.01) e^{i(\phi_2 + \pi/2)} \\
\chi^w [\text{mas}] = (0.73 - i 0.04) e^{i(\phi_1 + \pi/2)} + (0.23 - i 0.01) e^{i(\phi_2 + \pi/2)}
\]

\[
\frac{\chi^{r^w}}{\chi^{r^p(IB)}} \sim 14 \quad ; \quad \text{expected ratio} \sim 13
\]

Pressure NIB term fits a simple equilibrium tidal model:

\[
\chi^p_{(NIB)} = -\frac{8\pi}{15} \frac{r_0^4}{(C - A) g} q_0 G_M \sin \varepsilon e^{i(\phi_1 + \pi/2)}
\]

$A, C$ Earth inertia moments, $q_0$ mean atmospheric density, $G_M=2.64$ m$^2$/s$^2$ Doodson constant for the Moon, $\varepsilon = 23.5^\circ$ (obliquity)
Broad band peak round 7 days

- More powerful than 13.6 d harmonics
- $\psi_1^1$ atmospheric resonance excited by lunar tides Q1 (6.86 d), $\sigma_1$ (7.05 d)

$$\chi_{P/W} = (m_c + im_s ) e^{i \phi Q_1}$$

Least square fit over 6 year sliding window

Expected ratio of $\frac{\chi^{\text{NW}}}{\chi^{(\text{NIB})}} = \frac{\Omega - \sigma_i}{\sigma_i} \approx 6$ around 7 d
Synthesis

• NIB pressure term O1 explained by tidal equilibrium model: clearly a lunar effect

• At 13.6 d expected ratio works for IB (over continents) pressure term; at 7 d expected ratio works for NIB (full) pressure term

• Proportionality wind / pressure term is consistent with 2 facts
  ➢ Lunar tesseral tidal pressure do not cause topographic torque
  ➢ Tidal wind only blows in the upper layer of the atmosphere and do not cause any friction torque
  ➢ Local atmospheric torque much smaller than the bulge torque, leading to proportionality of the pressure and wind term at a given period
Conclusion

Below 30 days Celestial Equatorial Atmospheric angular momentum is only significant for prograde band and is characterized by:

- a 13.6 day wave, resulting from the fortnightly lunar tide
- a broad band weekly oscillation, possibly resulting from lunar tidal effect amplified by the $\psi_1^1$ atmospheric resonance.

- Proportionality wind/pressure term at a given period & equal contributions of northern and southern hemispheres strongly supports the fact that the whole band is caused by the lunar tide

- Effect on nutation (Liouville equation in Celestial Frame, Brzezinski 1994): $\sim 5 \mu\text{as at 13.6 days }$ (not observable), up to $30 \mu\text{as at 7 days }$ (densification needed)