On the definition and use of the ecliptic in modern astronomy

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Introduction

- The ecliptic was a fundamental reference plane for astronomy (astrometry, solar system dynamics and measurements), from antiquity unto the realization of the FK5 reference system.

- The situation has changed considerably with the adoption of the International Celestial Reference system (ICRS) by the IAU since 1998 and the IAU resolutions on reference systems that were adopted between 2000 and 2009. These correspond to major improvements in concepts and realizations of astronomical reference systems, in the use of observational data and the accuracy of the models for the motions of the solar system objects and Earth's rotation.

- In that modern context, consistent with GR, the ecliptic is no more a fundamental plane. Although IAU 2006 Resolution B1 clarifies some aspects of the definition of the ecliptic, the concept of an ecliptic is not as clear as those of the ICRS, the intermediate equator, etc.. It is therefore necessary to review in which works such a concept is still required and whether a definition in the GR framework is needed.

1) The adoption of the ICRS and ICRF (IAU 1997 Resolution B2)

International Celestial Reference System (ICRS)*: the idealized **barycentric coordinate system** to which celestial positions are referred. It is kinematically non-rotating with respect to the ensemble of distant extragalactic objects. It has no intrinsic orientation but was aligned close to the mean equator and dynamical equinox of J2000.0 for continuity with previous fundamental reference systems. Its orientation is independent of epoch, ecliptic or equator and is realized by a list of adopted coordinates of extragalactic sources.

International Celestial Reference Frame (ICRF)*: a set of extragalactic objects whose adopted positions and uncertainties realize the ICRS axes and give the uncertainties of the axes. It is also the name of the radio catalog whose 212 defining sources is currently the most accurate realization of the ICRS. Successive revisions of the ICRF are intended to minimize rotation from its original orientation. Other realizations of the ICRS have specific names (e.g. Hipparcos Celestial Reference Frame).

ICRS and ICRF were adopted by the IAU since 1998 as the replacement of the FK5 system and the fundamental catalogue of stars FK5 (based on the determination of the ecliptic, the equator and the equinox)

*: *definitions from the IAU 2006 NFA Glossary, http://syrte.obspm.fr/iauWGnfa/*

2) The IAU 2000/2006 definitions and models

IAU 2000 Resolutions

Resolution B1.3

Definition of BCRS and GCRS

Resolution B1.6

IAU 2000 Precession-Nutation Model

Resolution B1.7

Definition of Celestial Intermediate Pole (CIP)

Resolution B1.8

Definition and use of CEO and TEO

IAU 2006 Resolutions

Resolution B1

Adoption of the P03 Precession and definition of the ecliptic

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Resolution B2

Harmonization of the names to CIO and TIO

3) The IAU 2009 Resolutions

Resolution	Aim
Adoption of the IAU 2009 System of astronomical constants	to adopt an improved system of astronomical constants consistent with the current measurement accuracy
Adoption of the 2d realization of the International Celestial Reference Frame	to improve the realization of the ICRF with densification of the frame and a more precise definition of the axes

The Barycentric and Geocentric celestial reference systems, BCRS and GCRS

IAU 2000 Resolution B1.3: Definition of BCRS and GCRS as coordinate systems in the framework of GR

- a) for Solar System (BCRS) which can be considered to be *a global coordinate system* e.g. to be used for planetary ephemerides

- b) for the Earth (GCRS) which can only be considered as a *local coordinate system* e.g. to be used for Earth rotation, precession-nutation of the equator

Transformation BCRS/GCRS: extension of the Lorentz transformation (PN approximation)

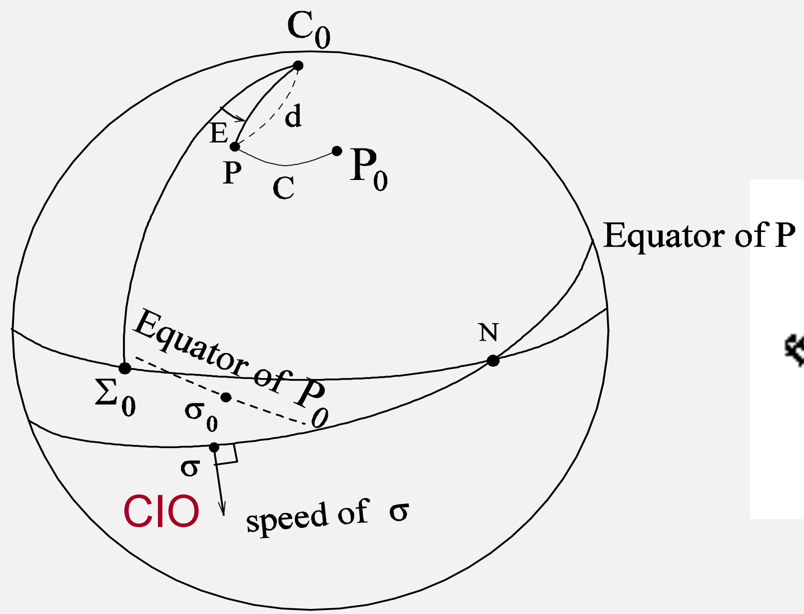
BCRS → GCRS $$X^a = \delta_{ai}\left[r_E^i + \frac{1}{c^2}\left(\frac{1}{2}v_E^i v_E^j r_E^j + w_{ext}(\mathbf{x}_E) r_E^i + r_E^i a_E^j r_E^j - \frac{1}{2} a_E^i r_E^2\right)\right] + O(c^{-4}),$$

TCB → TCG $$T = t - \frac{1}{c^2}\left[A(t) + v_E^i r_E^i\right] + \frac{1}{c^4}\left[B(t) + B^i(t) r_E^i + B^{ij}(t) r_E^i r_E^j + C(t,\mathbf{x})\right] + O(c^{-5}),$$

IAU 2006 Resolution B2: Fixing the default orientation of the BCRS

The BCRS orientation is such that for all practical applications, unless otherwise stated, the BCRS is assumed to be oriented according to the ICRS axes.

The Earth's rotation angle



kinematical definition of the CIO only dependent on the motion of the CIP

geometrical definition of the equinox, γ, dependent on both the equator (GCRS) and ecliptic (BCRS) motions

ERA: *Earth rotation angle*

ERA = Hour angle from the CIO

replaces

GST: Greenwhich sidereal time

GST = Hour angle from the equinox

GST = *ERA − EO*

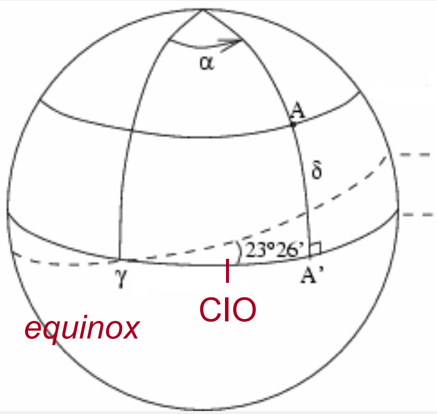
EO: *equation of the origins*

Not dependent of the precession-nutation model

Equatorial coordinates

New

- *Right ascension referred to the CIO*
- *Right ascension referred to the ICRS*



α	RA	right ascension	generic term	
α_i	RA_i	intermediate right ascension, CIO right ascension	ERA-compatible	CIO
α_e	RA_e	equinox right ascension, right ascension with respect to the equinox, apparent right ascension	ST-compatible	equinox
α_{ICRS}	RA_{ICRS}	ICRS right ascension		ICRS origin on the ICRS equator
δ	Dec, DEC	declination	generic term	CIO & equinox
δ_i	Dec, DEC	intermediate declination declination with respect to the CIP equator	identical to apparent declination (w.r.t. the true equator and equinox of date reference system)	CIO & equinox
δ_{app}	Dec, DEC	declination with respect the true equator of date apparent declination		equinox
δ_{ICRS}	Dec_{ICRS}	declination measured from the ICRS equator		

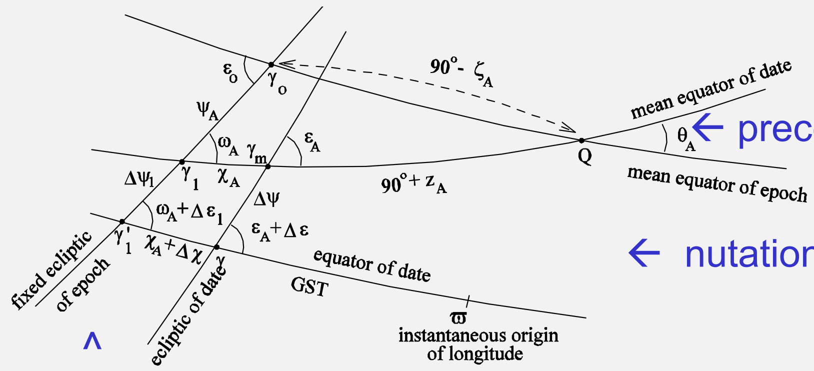
Precession-nutation parameters

(1) The equinox based case

Parameters referred to the ecliptic of date, or the ecliptic of epoch:

- precession quantities: ψ_A, ε_A, …
- nutation quantities: $\Delta\psi$, $\Delta\varepsilon$, …

Referring to the eclitpic of date mixes precession of the ecliptic and precession of the equator.



← precession of the equator

← nutation of the equator

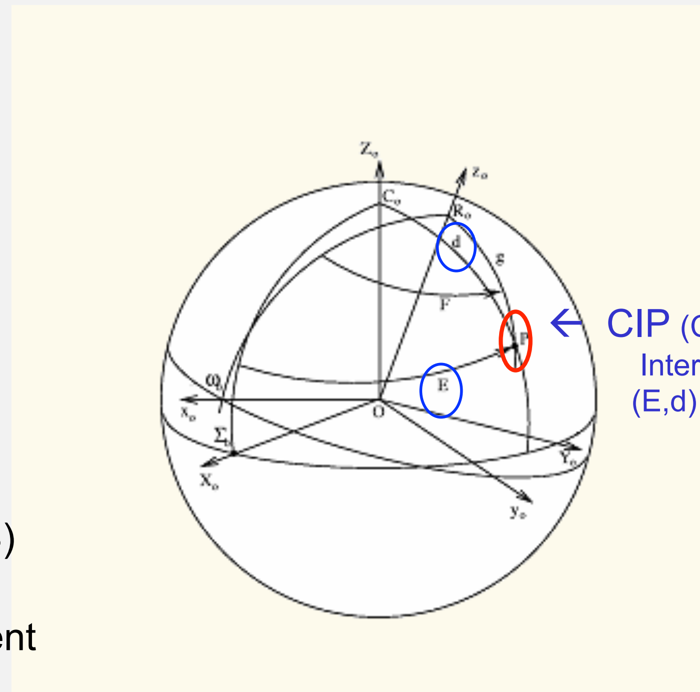
^
precession
of the ecliptic

(2) The CIO based case

Parameters referred to the GCRS ($OX_0Y_0Z_0$):

- x, y-coordinates of the CIP unit vector:
  X = sind cosE, Y = sind sinE

- provide where the pole is in the sky (the GCRS)
- contain precession-nutation-bias + cross terms
- The definition of these parameters is independent of ecliptic and equinox.



← CIP (Celestial Intermediate Pole): (E,d) or (X, Y)

IAU 2009 System of astronomical constants

Table 1 IAU 2009 System of Astronomical Constants

Constant	Description	Value
		Natural Defining Constant
c	Speed of Light	2.99792458×10^{8} ms^{-1}
		Auxiliary Defining Constant
k[a]	Gaussian gravitational constant	1.720209895×10^{-2}
L_G	1−d(TT)/d(TCG)	$6.969290134\times10^{-10}$
L_B	1−d(TDB)/d(TCB)	1.550519768×10^{-8}
TDB_0[b]	TDB−TCB at T_0	-6.55×10^{-5} s
θ_0	Earth rotation angle at J2000.0	0.7790572732640 revolutions
$\dot{\theta}$[c]	Rate of advance of Earth rotation angle	1.00273781191135448 revolutions UT1-day^{-1}

Replacement of the IAU 1976 system:

Improvements in:

- the classification of the constants,
- the accuracy, and uncertainties of the numerical values,
- the consistency with the SI units

(TDB/TCB/TT/TCG-compatible values)

Constant	Description	Value	Uncertainty
		Natural Measurable Constants	
G	Constant of gravitation	6.67428×10^{-11} m^{3}kg^{-1}s^{-2}	6.7×10^{-15} m^{3}kg^{-1}s^{-2}
		Other Constants	
au[d]	Astronomical unit	$1.49597870700\times10^{11}$ m	3 m
L_C	average value of 1−d(TCG)/d(TCB)	$1.48082686741\times10^{-8}$	2×10^{-17}
		Body Constants[e]	
GM_S	Heliocentric gravitational constant	$1.32712442099\times10^{20}$ m^{3}s^{-2} (TCB-compatible) $1.32712440041\times10^{20}$ m^{3}s^{-2} (TDB-compatible)	1.0×10^{10} m^{3}s (TCB-compatil 1.0×10^{10} m^{3}s (TDB-compatil
a_E[f]	Equatorial radius of the Earth	6.3781366×10^{6} m	1×10^{-1} m
J_2[f]	Dynamical form factor	1.0826359×10^{-3}	1×10^{-10}
$\dot{J}_2$	Time rate of change in J_2	-3.0×10^{-9} cy^{-1}	6×10^{-10} cy^{-1}
GM_E	Geocentric gravitational constant	3.986004418×10^{14} m^{3}s^{-2} (TCB-compatible) 3.986004415×10^{14} m^{3}s^{-2} (TT-compatible) 3.986004356×10^{14} m^3 s^{-2} (TDB-compatible)	8×10^{5} m^3 s^{-2} (TCB-compatil 8×10^{5} m^3 s^{-2} (TT-compatibl 8×10^{5} m^3 s^{-2} (TDB-compatil
W_0	Potential of the geoid	6.26368560×10^{7} m^{2}s^{-2}	5×10^{-1} m^{2}s
ω[g]	Nominal mean angular velocity of the Earth	7.292115×10^{-5} rad s^{-1}	

Constant	Description	Value	Uncertainty	
M_M/M_E	Ratio of the mass of the Moon to the Earth	1.23000371×10^{-2}	4×10^{-10}	[3
M_S/M_{Me}	Ratio of the mass of the Sun to Mercury	6.0236×10^{6}	3×10^{2}	[1
M_S/M_{Ve}	Ratio of the mass of the Sun to Venus	4.08523719×10^{5}	8×10^{-3}	[2
M_S/M_{Ma}	Ratio of the mass of the Sun to Mars	3.09870359×10^{6}	2×10^{-2}	[2
M_S/M_J	Ratio of the mass of the Sun to Jupiter	1.047348644×10^{3}	1.7×10^{-5}	[2
M_S/M_{Sa}	Ratio of the mass of the Sun to Saturn	3.4979018×10^{3}	1×10^{-4}	[2
M_S/M_U	Ratio of the mass of the Sun to Uranus	2.290298×10^{4}	3×10^{-2}	[2
M_S/M_N	Ratio of the mass of the Sun to Neptune	1.941226×10^{4}	3×10^{-2}	[2
M_S/M_P	Ratio of the mass of the Sun to (134340) Pluto	1.36566×10^{8}	2.8×10^{4}	[3
M_S/M_{Eris}	Ratio of the mass of the Sun to (136199) Eris	1.191×10^{8}	1.4×10^{6}	[3
M_{Ceres}/M_S	Ratio of the mass of (1) Ceres to the Sun	4.72×10^{-10}	3×10^{-12}	[3
M_{Pallas}/M_S	Ratio of the mass of (2) Pallas to the Sun	1.03×10^{-10}	3×10^{-12}	[3
M_{Vesta}/M_S	Ratio of the mass of (4) Vesta to the Sun	1.35×10^{-10}	3×10^{-12}	[3
		Initial Values at J2000.0		
ϵ_{J2000}[h]	Obliquity of the ecliptic at J2000.0	8.4381406×10^{4} ″	1×10^{-3} ″	[2

The solar system ephemerides - the GR framework -

- The post-Newtonian equations of motion for a set of "point-masses" (the Einstein-Infeld-Hoffmann (EIH) equations) are the basis of the 3 state-of-the-art numerical solar system ephemerides:
  - the American one, DE (Development Ephemeris; JPL),
  - the Russian one, EPM (Ephemerides of Planets and the Moon; IPA, St.Petersburg) and
  - the French one, INPOP (Intégrateur Numérique Planétaire de l'Observatoire de Paris).

- The EIH equations of motion are integrated numerically for the whole solar system including a set of selected minor planets.

- The equations are solved in the BCRS .

Ref: DE430 & DE431 (Folkner et al. 2014); EPM2014: Pitjeva et al. 2014; INPOP13b (Fienga et al. 2014)

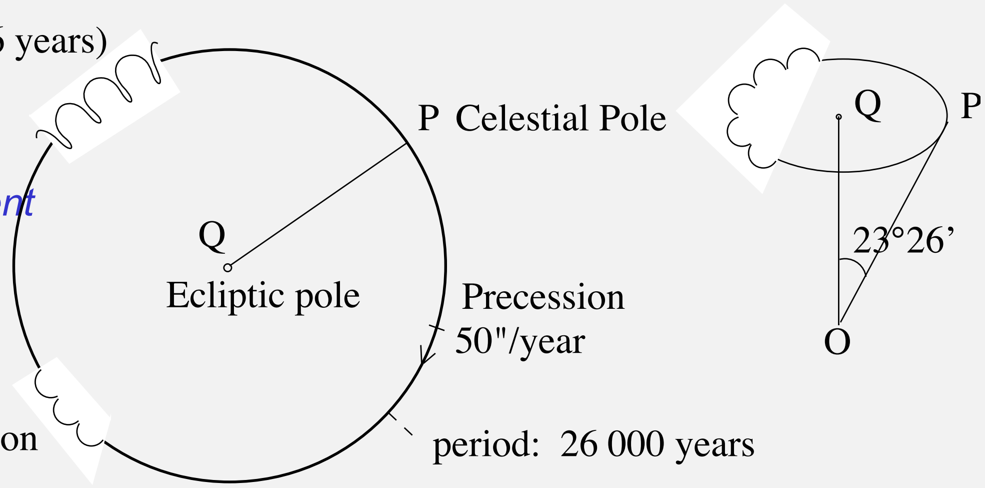
The numerical solar system ephemerides
- the current tie to the ICRF -

Table 3.1: Overview over the American (DE) and Russian (EPM) solar-system ephemerides; from Pitjeva (2005).

Ephemerides	Interval of integration	Reference frame	Mathematical model	Type of observations	Number of observations	Time interval
DE118	1599 ⟶ 2169	FK4	Integration:	Optical	44755	1911–1979
(1981)			the Sun, the Moon, nine planets +	Radar	1307	1964–1977
⇓		⇓	perturbations from three asteroids	Spacecraft and landers	1408	1971–1980
DE200		J2000.0	(two-body problem)	LLR (lunar laser ranging)	2954	1970–1980
		system		Total	50424	1911–1980
EPM87	1700 ⟶ 2020	FK4	Integration:	Optical	48709	1717–1980
(1987)			the Sun, the Moon, nine planets +	Radar	5344	1961–1986
			perturbations from five asteroids	Spacecraft and landers	–	–
			(two-body problem)	LLR (lunar laser ranging)	1855	1972–1980
				Total	55908	1717–1986
DE403	–1410 ⟶ 3000	ICRF	Integration:	Optical	26209	1911–1995
(1995)			the Sun, the Moon, nine planets +	Radar	1341	1964–1993
⇓	⇓		perturbations from 300 asteroids	Spacecraft and landers	1935	1971–1994
DE404	–3000 ⟶ 3000		(mean elements)	LLR (lunar laser ranging)	9555	1970–1995
				Total	39057	1911–1995
EPM98	1886 ⟶ 2006	DE403	Integration:	Optical	–	–
(1998)			the Sun, the Moon, nine planets,	Radar	55959	1961–1995
			five asteroids + perturbations from	Spacecraft and landers	1927	1971–1982
			295 asteroids (mean elements)	LLR (lunar laser ranging)	10000	1970–1995
				Total	67886	1961–1995
DE405	1600 ⟶ 2200	ICRF	Integration:	Optical	28261	1911–1996
(1997)			the Sun, the Moon, nine planets +	Radar	955	1964–1993
⇓			perturbations from 300 (integrated)	Spacecraft and landers	1956	1971–1995
DE406	–3000 ⟶ 3000		asteroids	LLR (lunar laser ranging)	11218	1969–1996
				Total	42410	1911–1996

(from Soffel & Langhans 2013)

Definition and description of precession-nutation



The celestial displacement of the CIP

Precession of the Earth’s equator and precession of the ecliptic

CIP equator: plane perpendicular to the CIP axis has a precession-nutation motion

Ecliptic: plane perpendicular to the mean orbital angular momentum vector of the Earth-Moon barycenter in the BCRS has a secular motion (called « precession of the ecliptic »)

Precession-nutation equations for a rigid Earth in the celestial reference system

(1) The equinox-based precession-nutation equations

variables: the Euler angles between the equinox and ecliptic of epoch and the ITRS

$$\ddot{\omega} + \frac{C}{A}\sin\omega_0\,\varphi_1\,\dot{\psi} = \frac{L}{A} + F_2$$
$$\sin\omega_0\,\ddot{\psi} - \frac{C}{A}\varphi_1\,\dot{\omega} = \frac{M}{A} + G_2$$
$$\ddot{\varphi} = \frac{N}{C} + H_2$$

L, M, N: Torque components in the true equator and equinox (CIP,γ_1) Woolard (1953), Bretagnon et al.(1997)

F_2, G_2, H_2: 2d order terms

(axially symmetric Earth)

(2) The CIO-based precession-nutation equations

variables: the GCRS CIP coordinates

$$\begin{cases} -\ddot{Y} + \frac{C\Omega}{A}\dot{X} = \frac{L}{A} + F'' \\ \ddot{X} + \frac{C\Omega}{A}\dot{Y} = \frac{M}{A} + G'' \end{cases}$$

L, M, N: Torque components in the celestial intermediate system (CIP, Σ) (Capitaine et al. 2006)

F'', G'': 2d order terms

(axially symmetric Earth)

Polynomial approximations for precession parameters

Almost all precession models are expressed in terms of polynomial developments of all the various precession parameters, which are intended for high-accuracy applications over a time span of a few centuries.

In fact, the osculating elements of the Earth-Moon barycenter (EMB) orbit are quasi-periodic functions of the time that can be expressed in the form of Poisson series whose arguments are linear combinations of the mean planetary longitudes (see next slide).

T*he* *precession of the ecliptic* can be defined as the secularly-moving ecliptic pole (i.e. mean EMB orbital angular momentum vector) in a fixed ecliptic frame.

The IAU 2006 precession of the ecliptic was computed as the part of the motion of the ecliptic covering periods longer than 300 centuries, while shorter ones are presumed to be included in the periodic component of the ecliptic motion (VSOP87 + fit to DE406).

IAU 2006 expressions for precession

(Capitaine et al. 2003)

ecliptic

equator (equinox based quantities)

Source		t^0	t	t^2	t^3	t^4	t^5
		mas	mas/cy	mas/cy^2	mas/cy^3	mas/cy^4	mas/cy^5
IAU 2000	P_A		4197.6	194.47	-0.179		
P03			4199.094	193.9873	-0.22466	-0.000912	0.0000120
IAU	Q_A		-46815.0	50.59	0.344		
P03			-46811.015	51.0283	0.52413	-0.000646	-0.0000172
IAU 2000	ψ_A		5038478.750	-1072.59	-1.147		
P03			5038481.507	-1079.0069	-1.14045	0.132851	-0.0000951
IAU 2000	ω_A	84381448.0	-25.240	51.27	-7.726		
P03		84381406.0	-25.754	51.2623	-7.72503	-0.000467	0.0003337

equator (CIO based quantities)

Source	t^0	t	t^2	t^3	t^4	t^5
X	-16.617	2004191.898	-429.7829	-198.61834	0.007578	0.0059285
Y	-6.951	-25.896	-22407.2747	1.90059	1.112526	0.0001358
$s+XY/2$	0.094	3.80865	-0.12268	-72.57411	0.02798	0.01562

$\psi_{A1} \times \sin\varepsilon$, ω_{A1}, X_1, Y_1 : equator rates

Polynomial coefficients for all the precession angles in Hilton et al. (2006)

Long term expressions for the precession of the ecliptic

(Vondrak et al. 2011)

Table 1. Periodic terms in P_A, Q_A.

Term	C/S	$P_A['']$	$Q_A['']$	P[cy]
σ_3	C_1	–5486.751211	–684.661560	708.15
	S_1	667.666730	–5523.863691	
$-s_1$	C_2	–17.127623	2446.283880	2309.00
	S_2	–2354.886252	–549.747450	
	C_3	–617.517403	399.671049	1620.00
	S_3	–428.152441	–310.998056	
$-s_6$	C_4	413.442940	–356.652376	492.20
	S_4	376.202861	421.535876	
	C_5	78.614193	–186.387003	1183.00
	S_5	184.778874	–36.776172	
	C_6	–180.732815	–316.800070	622.00
	S_6	335.321713	–145.278396	
	C_7	–87.676083	198.296071	882.00
	S_7	–185.138669	–34.744450	
	C_8	46.140315	101.135679	547.00
	S_8	–120.972830	22.885731	

Equinox: Concepts and definitions

- **equinox:** either of the two points at which the ecliptic intersects the celestial equator; also the time at which the Sun passes through either of these intersection points; i.e., when the apparent longitude of the Sun is 0° or 180°. When required, the equinox can be designated by the ephemeris of the Earth from which it is obtained (e.g. vernal equinox of DE 405). By 2100 the equinox will have moved 1.4° from the ICRS meridian, due to the precession of the equinoxes.

- **dynamical mean equinox:** the ascending node of the ecliptic on the mean equator. The mean equinox of epoch corresponds to the definition of the ecliptic in its "inertial" sense. It differs by 93.66 mas from the "rotational dynamical mean equinox of J2000.0", which was intended to coincide with the FK5 equinox.

*: *definitions from the IAU 2006 NFA Glossary, http://syrte.obspm.fr/iauWGnfa/*

Mixing BCRS and GCRS should be avoided

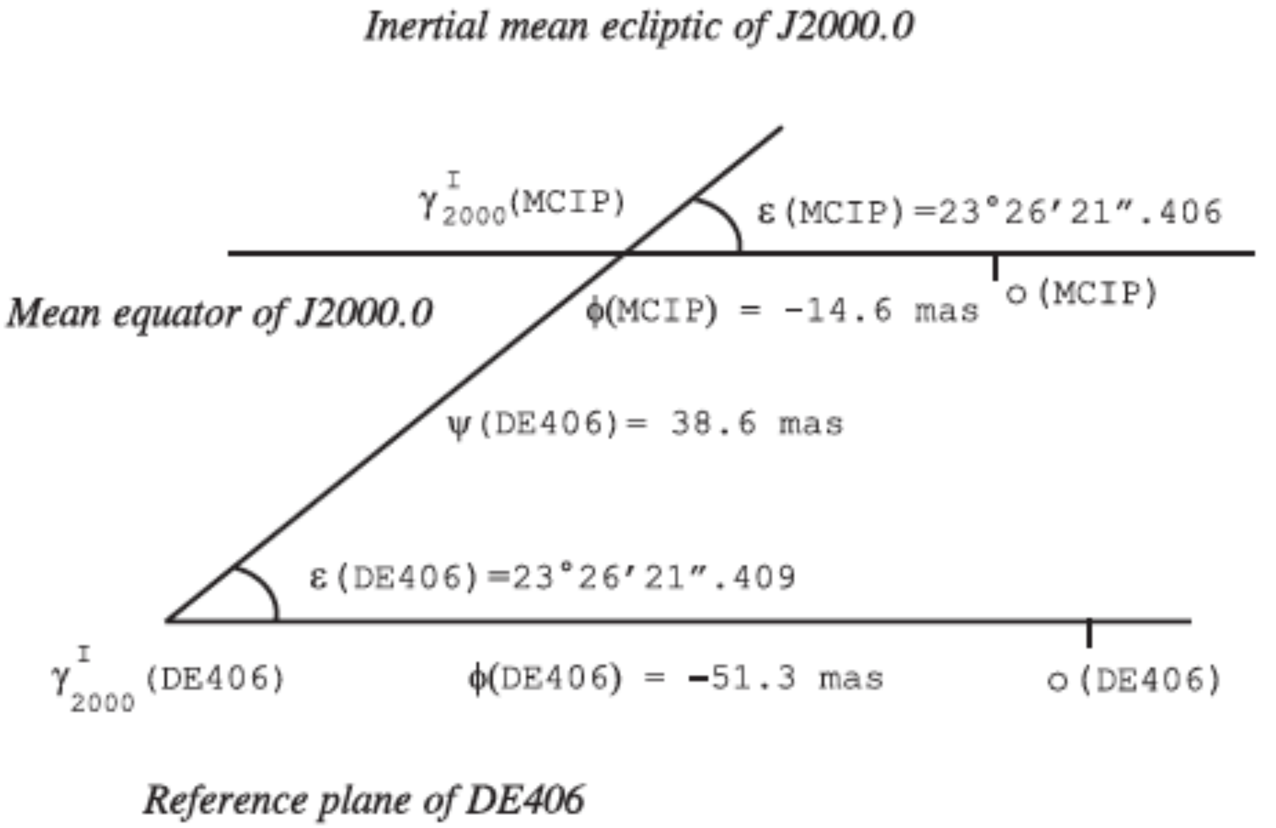


Fig. 1. Rotation angles ϕ and ϵ to rotate from equatorial to ecliptic frame.

The role of the ecliptic in the modern context

- No ecliptic is needed for the realization of the ICRS
- The time-dependent ecliptic is no more needed as reference for the astronomical coordinates
- The modern numerical barycentric ephemerides are referred to the ICRF
- The modern description of precession-nutation of the equator is the motion of the CIP in the GCRS without reference to the ecliptic .
- Numerical integration of precession-nutation does not use an ecliptic
- Several semi-analytical Earth's rotation models use the concept of a time dependent ecliptic as a practical intermediate plane.
- Precession distinguishes the precession of the ecliptic from the precession of the equator.

Definition of the ecliptic in GR ?

- In relativity, it is necessary to carefully distinguish between barycentric and geocentric quantities, so the calculation of a moving ecliptic presents a serious problem when it is used in the GCRS.

- Due to the loss of the importance of the ecliptic, the definition of the time-dependent ecliptic in GR is not required

- If this can be useful, for continuity to traditional approach, to define a conventional BCRS fixed ecliptic frame as realized by rotating the BCRS by a constant rotation according to some mean ecliptic an equinox J2000.