T. Baenas[1], J.M. Ferrándiz[1], A. Escapa[1,2] and J. Getino[3]

[1]Dept. of Applied Mathematics Dept. University of Alicante. Spain
[2]Dept. of Mechanical, Informatics and Aeroespace Engineering. University of León. Spain
[3]Dept. of Applied Mathematics. University of Valladolid. Spain

23rd Sept. 2014

EFFECTS OF THE TIDAL MASS REDISTRIBUTION ON THE EARTH ROTATION

Journées Systèmes de Référence Spatio-Temporels
22-24 September 2014. St. Petersburg

Index

1. Introduction

- The gravitational action of the Moon and the Sun on the deformable Earth originates a redistribution of its mass, and thus a variation in its kinetic energy related with the change of the Earth inertia tensor.

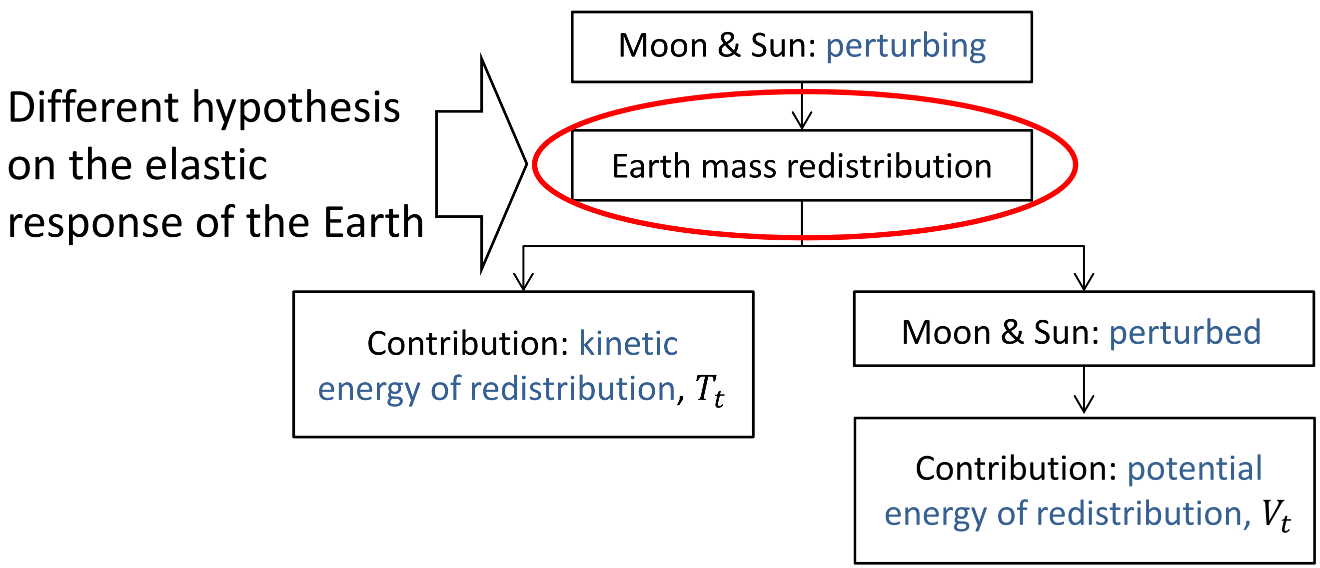
- Besides, the redistribution produces an additional term in the gravitational potential energy, commonly referred as tidal potential of redistribution.

- The effects on the Earth rotation were previously and partially exposed in

  ✓ Kubo (1991, 2011, 2012), Getino & Ferrándiz (1995, 2001), Souchay & Folgueira (2000), Escapa *et al.* (2004), Escapa (2011): Hamiltonian approach, simplified elastic response.

  ✓ Lambert *et al.* (2002), Lambert & Mathews (2006, 2008): SOS equations approach (Sasao et al. 1980), generalized elastic response.

- The basic scheme for the study of the rotation of the deformable Earth, with the Hamiltonian approach, is summarized in:
  - ✓ Use of the Hamiltonian formalism analogous to the rigid Earth (Kinoshita 1977), including the Lie-Hori perturbation method (Hori 1966).
  - ✓ Decoupling the elasticity problem from the rotational one: Moon and Sun are perturbed and perturbing bodies, with separated contributions in the Hamilton function.



- In Mathews *et al*. analytical formulation, the contributions of the Earth mass redistribution are calculated in two steps:

  ✓ The contribution of the angular momentum (equivalent to T_t) due to inertia tensor change is calculated by the transfer function method (MHB 2000).

  ✓ The potential energy of redistribution effect is derived by Lambert & Mathews (2006), based on SOS equations.

- In the Hamiltonian approach both contributions are terms of the Hamiltonian function at hand (therefore necessarly consistent each other):

$$H = T_0 + \underbrace{T_t + V_1 + E + V_t}_{\text{Different perturbations}} \Rightarrow \begin{cases} \text{- Hamilton equations} \\ \text{- Canonical perturbation methods} \end{cases}$$

- An *ab initio* reconstruction of the whole dynamical model has been done in order to include a more general Earth rheology, following the Love Number approach (Munk & MacDonald 1960).

- Let us recall that assuming a rheological model based in Smith (1974) , Wahr (1979) and IERS (2010) for the elastic/anelastic response of the Earth, the second order harmonic components of the redistribution potential admit a truncated development in complex spherical harmonics in the form

$$V_m(r,\theta,\phi) = k_{2m}\left(\frac{a_E}{r}\right)^3 Y_{2m}(\theta,\phi) + k_{2m}^{(+)}\left(\frac{a_E}{r}\right)^5 Y_{4m}(\theta,\phi)\,,\; r \geq a_E$$

Generalized Love Numbers, dependent on:

- ✓ Harmonic order, m
- ✓ Excitation frequencies

In magnitude,

$$k_{2m}^{(+)}\left(\frac{a_E}{r}\right)^5 \ll k_{2m}\left(\frac{a_E}{r}\right)^3$$

- The main elements for the construction are:
  - ✓ The expression of the tidal redistribution potential, given by the sum (over p and q, representing Moon and Sun) of terms of the form

$$\begin{aligned} V_{t;p,q} &= \frac{a_E^5}{r^3 r'^3} G m_p m_q \Big[k_{20} \mathscr{C}_{20} \mathscr{C}'_{20} \\ &\quad + k_{21} \frac{1}{3} \left(\mathscr{C}_{21} \mathscr{C}'_{21} + \mathscr{S}_{21} \mathscr{S}'_{21} \right) + k_{22} \frac{1}{12} \left(\mathscr{C}_{22} \mathscr{C}'_{22} + \mathscr{S}_{22} \mathscr{S}'_{22} \right) \Big] \end{aligned}$$

    where $\mathscr{C}_{2j}, \mathscr{S}_{2j}$ and $\mathscr{C}'_{2j}$, $\mathscr{S}'_{2j}$ stand for the second order real spherical harmonics, related to perturbed bodies and perturbing ones respectively.

  - ✓ The expression of the time dependent inertia tensor, $\mathbf{I}(t) = \mathbf{I}_0 + \mathbf{I}_1(t)$,

$$\mathbf{I}_1 = \left(\frac{a}{r'} \right)^3 \begin{pmatrix} \kappa_{20} \mathscr{C}'_{20} - \frac{1}{2} \kappa_{22} \mathscr{C}'_{22} & -\frac{1}{2} \kappa_{22} \mathscr{S}'_{22} & -\kappa_{21} \mathscr{C}'_{21} \\ -\frac{1}{2} \kappa_{22} \mathscr{S}'_{22} & \kappa_{20} \mathscr{C}'_{20} + \frac{1}{2} \kappa_{22} \mathscr{C}'_{22} & -\kappa_{21} \mathscr{S}'_{21} \\ -\kappa_{21} \mathscr{C}'_{21} & -\kappa_{21} \mathscr{S}'_{21} & -2\kappa_{20} \mathscr{C}'_{20} \end{pmatrix}$$

    where $\kappa_{m;p} = \frac{1}{3} k_{2m} m_p a_E^2 \left(\frac{a_E}{a_p} \right)^3$ is the constant defined by Kubo (1991).

- The modelling of the anelastic behaviour of the deformable Earth requires the *ab initio* introduction of complex functions that generalize the Love Numbers (following e.g. IERS Conventions 2010). The validity of the previous expressions with this substitution was provided e.g. in Baenas dissertation (2014).

$$\bar{k}_{2m,j} = \left|\bar{k}_{2m,j}\right| e^{i\varepsilon_{2m,j}}$$ (j subindex stands for orbital frequencies dependence)

- All the expressions must be expanded in terms of an Andoyer set of canonical variables to establish the transformation that relates the celestial and terrestrial (Tisserand mean) reference systems.

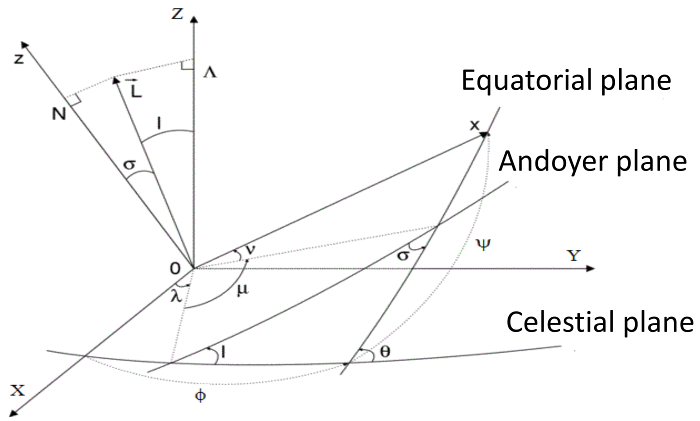


Fig. 1: Euler and Andoyer variables

Then we proceed like in the Kinoshita (1977) approach.

- The orbital motion of the perturbing bodies is expressed in trigonometric series giving the spherical harmonics,

$$
\begin{aligned}
\left(\frac{a}{r}\right)^3 \mathscr{C}_{20}(\eta',\alpha') &= -\sum_i A_i^{(0)} \cos\Theta_i,\\
\left(\frac{a}{r}\right)^3 \mathscr{C}_{21}(\eta',\alpha') &= 3\sum_i \bar{A}_i^{(1)} \sin\Theta_i,\ \left(\frac{a}{r}\right)^3 \mathscr{S}_{21}(\eta',\alpha') = 3\sum_i A_i^{(1)} \cos\Theta_i,\\
\left(\frac{a}{r}\right)^3 \mathscr{C}_{22}(\eta',\alpha') &= 3\sum_i A_i^{(2)} \cos\Theta_i,\ \left(\frac{a}{r}\right)^3 \mathscr{S}_{22}(\eta',\alpha') = 3\sum_i \bar{A}_i^{(2)} \sin\Theta_i,
\end{aligned}
$$

depending on the fundamental arguments of nutation (linear combinations of Delaunay variables of the orbital problem).

$$\Theta_i(t) = m_{1i} l + m_{2i} l' + m_{3i} F + m_{4i} D + m_{5i} \Omega$$

- The first order Lie-Hori perturbation equations are then applied (Baenas 2014). With this formulation, analytical expressions for the components of the solution can be obtained.

Index

Analytical expressions

- As an example, we include here the expression for the precession velocities (computed from the secular part of the Hamiltonian):

$$\delta_T n_\lambda = 0$$

$$\delta_T n_I = 0$$

$$\delta_V n_\lambda = -\frac{1}{\sin I^*}\frac{1}{CH_d}\sum_{p,q}^{M,S}\sum_{\substack{i,j;\tau,\epsilon \\ \tau\Theta_i^* - \epsilon\tilde{\Theta}_j = 0}}\sum_{m}^{0,1,2}\left|\bar{\kappa}_{2m,j;p}\right| k_q T_{ijpq,m}^{(n_\lambda)}(\tau,\epsilon)\cos\varepsilon_{2m,j}$$

$$\delta_V n_I = -\frac{1}{\sin I^*}\frac{1}{CH_d}\sum_{p,q}^{M,S}\sum_{\substack{i,j;\tau,\epsilon \\ \tau\Theta_i^* - \epsilon\tilde{\Theta}_j = 0}}\sum_{m}^{0,1,2}\left|\bar{\kappa}_{2m,j;p}\right| k_q T_{ijpq,m}^{(n_I)}(\tau,\epsilon)\sin\varepsilon_{2m,j}$$

- The subindexes T and V stand, respectively, for the contributions of the kinetic energy of redistribution and the potential energy of redistribution.
- $-\lambda^*$ and $-I^*$ are the precessional longitude and obliquity.
- The non-zero $\delta_V n_I$ contribution is a purely anelastic effect (stands for any $\varepsilon_{2m,j} \neq 0$) (in accordance with Lambert & Mathews 2006).

Analytical expressions

- As an example, we include here the expression for the precession velocities (computed from the secular part of the Hamiltonian):

$$\delta_T n_\lambda = 0$$

$$\delta_T n_I = 0$$

$$\delta_V n_\lambda = -\frac{1}{\sin I^*}\frac{1}{CH_d}\sum_{p,q}^{M,S}\sum_{\substack{i,j;\tau,\epsilon \\ \tau\Theta_i^*-\epsilon\tilde{\Theta}_j=0}}\sum_{m}^{0,1,2}\left|\bar{\kappa}_{2m,j;p}\right| k_q T^{(n_\lambda)}_{ijpq,m}(\tau,\epsilon)\cos\varepsilon_{2m,j}$$

$$\delta_V n_I = -\frac{1}{\sin I^*}\frac{1}{CH_d}\sum_{p,q}^{M,S}\sum_{\substack{i,j;\tau,\epsilon \\ \tau\Theta_i^*-\epsilon\tilde{\Theta}_j=0}}\sum_{m}^{0,1,2}\left|\bar{\kappa}_{2m,j;p}\right| k_q T^{(n_I)}_{ijpq,m}(\tau,\epsilon)\sin\varepsilon_{2m,j}$$

- $T_{ijpq,m}$ quantities are particular combinations of the B, C and D functions of the orbital coefficients $A_i^{(j)}$ defined by Kinoshita (1977).

$$T^{(n_\lambda)}_{ijpq,m}(\tau,\epsilon) = \frac{9}{4}\frac{\partial B^*_{i;p}}{\partial I^*}\tilde{B}_{j;q}\delta_{m0} + 3\frac{\partial C^*_{i;p}}{\partial I^*}\tilde{C}_{j;q}\delta_{m1} + \frac{3}{4}\frac{\partial D^*_{i;p}}{\partial I^*}\tilde{D}_{j;q}\delta_{m2}$$

$$T^{(n_I)}_{ijpq,m}(\tau,\epsilon) = \tau m_{5i}\left(\frac{9}{4}B^*_{i;p}\tilde{B}_{j;q}\delta_{m0} - 3C^*_{i;p}\tilde{C}_{j;q}\delta_{m1} - \frac{3}{4}D^*_{i;p}\tilde{D}_{j;q}\delta_{m2}\right)$$

Numerical representations for different models

Simplified elastic model

- Spherical non-perturbed state without rotation
- Real Love Numbers, frequency independent

$$\begin{aligned} \bar{k}_{2m,j} &= k \in \mathbb{R} \\ &\rightarrow \left|\bar{\kappa}_{2m,j;p}\right| = \kappa_p, \\ &\rightarrow \varepsilon_{2m,j} = 0. \end{aligned}$$

Simple anelastic model: constant time delay

- Ellipsoidal non-perturbed state with rotation
- Complex Love Numbers, frequency dependent (*ad hoc* functional dependence)

$$\begin{aligned} \bar{k}_{2m,j} &= k_{2m} e^{i\varepsilon_{2m,j}} \in \mathbb{C} \\ &\rightarrow \left|\bar{\kappa}_{2m,j;p}\right| = \kappa_{2m;p}, \\ &\rightarrow \varepsilon_{2m,j} = -\Delta t \left\{ m\omega_E - \left[\epsilon - \delta_{m0}\left(1+\epsilon\right)\right] \tilde{n}_j \right\}. \end{aligned}$$

Generalized elastic model

- Ellipsoidal non-perturbed state with rotation
- Real Love Numbers, with either nominal value for frequency band, or frequency dependent

$$\begin{aligned} \bar{k}_{2m,j} &= k_{2m,j} \in \mathbb{R} \\ &\rightarrow \left|\bar{\kappa}_{2m,j;p}\right| = \kappa_{2m,j;p}, \\ &\rightarrow \varepsilon_{2m,j} = 0. \end{aligned}$$

Generalized anelastic model

- Ellipsoidal non-perturbed state with rotation
- Complex Love Numbers, with either a nominal value for each frequency band, or frequency dependent

$$\begin{aligned} \bar{k}_{2m,j} &= \left|\bar{k}_{2m,j}\right| e^{i\varepsilon_{2m,j}} \in \mathbb{C} \\ &\rightarrow \bar{\kappa}_{2m,j;p} = \left|\bar{\kappa}_{2m,j;p}\right| e^{i\varepsilon_{2m,j}} \end{aligned}$$

Numerical representations: precession velocities

Precession in longitude and obliquity: tidal redistribution potential contribution.

	Zonal B_0	Zonal $B - B_0$	Tesseral C	Sectorial D	Total
Case 1: Elastic Earth (simplified model)					
δn_λ [mas/cent]	42.0639	-3.9604	-64.1250	26.0214	0.0000
δH	-27.34×10^{-9}	2.59×10^{-9}	41.75×10^{-9}	-16.94×10^{-9}	0.00×10^{-9}
δn_I [mas/cent]	0.0000	0.0000	0.0000	0.0000	0.0000
Case 2: Elastic Earth. Real Love Numbers with nominal value for frequency band (IERS 2010)					
δn_λ [mas/cent]	42.8254	-4.0321	-65.1642	26.7401	0.3693
δH	-27.88×10^{-9}	2.62×10^{-9}	42.42×10^{-9}	-17.41×10^{-9}	-0.24×10^{-9}
δn_I [mas/cent]	0.0000	0.0000	0.0000	0.0000	0.0000
Case 3: Anelastic Earth. Constant time delay model					
δn_λ [mas/cent]	42.8254	-4.0321	-65.1513	26.7194	0.3615
δH	-27.87×10^{-9}	2.62×10^{-9}	42.41×10^{-9}	-17.39×10^{-9}	-0.24×10^{-9}
δn_I [mas/cent]	0.0000	0.0000	0.5103	6.0390	6.5493
Case 4a: Anelastic Earth. Complex Love Numbers with nominal value for frequency band (IERS 2010)					
δn_λ [mas/cent]	43.7900	-4.1229	-65.9603	27.0102	0.7171
δH	-28.51×10^{-9}	2.68×10^{-9}	42.94×10^{-9}	-17.58×10^{-9}	-0.47×10^{-9}
δn_I [mas/cent]	0.0000	0.0000	0.1301	0.6656	0.7957
Case 4b: Anelastic Earth. Complex Love Numbers frequency dependent (IERS 2010)					
δn_λ [mas/cent]	43.7900	-4.1389	-60.6554	27.0102	6.0059
δH	-28.51×10^{-9}	2.69×10^{-9}	39.49×10^{-9}	-17.58×10^{-9}	-3.91×10^{-9}
δn_I [mas/cent]	0.0000	-0.0118	0.1209	0.6656	0.7748

Numerical representations: precession velocities

Precession in longitude and obliquity: tidal redistribution potential contribution.

	Zonal		Tesseral	Sectorial	Total
	B_0	$B-B_0$	C	D	
Case 1: Elastic Earth (simplified model)					
δn_λ [mas/cent]	42.0639	−3.9604	−64.1250	26.0214	0.0000
δH	-27.34×10^{-9}	2.59×10^{-9}	41.75×10^{-9}	-16.94×10^{-9}	0.00×10^{-9}
δn_I [mas/cent]	0.0000	0.0000	0.0000	0.0000	0.0000
Case 2: Elastic Earth. Real Love Numbers with nominal value for frequency band (IERS 2010)					
δn_λ [mas/cent]	42.8254	−4.0321	−65.1642	26.7401	0.3693
				[illegible]	-0.24×10^{-9}
δn_I [illegible]				0.0000	0.0000
Case [illegible]					
δn_λ [illegible]				26.7194	0.3615
				[illegible]	-0.24×10^{-9}
δn_I [illegible]				6.0390	6.5493
Case [illegible] frequency band (IERS 2010)					
δn_λ [illegible]				27.0102	0.7171
				[illegible]	-0.47×10^{-9}
δn_I [illegible]				0.6656	0.7957
Case 4b: Anelastic Earth. Complex Love Numbers frequency dependent (IERS 2010)					
δn_λ [mas/cent]	43.7900	−4.1389	−60.6554	27.0102	6.0059
δH	-28.51×10^{-9}	2.69×10^{-9}	39.49×10^{-9}	-17.58×10^{-9}	-3.91×10^{-9}
δn_I [mas/cent]	0.0000	−0.0118	0.1209	0.6656	0.7748

The permanent tide contribution is computed separately. B_0 contribution must be included or removed, depending on the dynamical model considered for the rigid part of the Earth inertia tensor (zero tide or tide free).

Total here corresponds to tide free.

Numerical representations: precession velocities

Precession in longitude and obliquity: tidal redistribution potential contribution.

		Zonal		Tesseral	Sectorial	Total
		B_0	$B-B_0$	C	D	
Case 1: Elastic Earth (simplified model)						
δn_λ [mas/cent]		42.0639	−3.9604	−64.1250	26.0214	0.0000
	δH	-27.34×10^{-9}	2.59×10^{-9}	41.75×10^{-9}	-16.94×10^{-9}	0.00×10^{-9}
δn_I [mas/cent]		[illegible]	[illegible]	[illegible]	[illegible]	0.0000
Case 2: Elastic Earth. [illegible] 010)						
δn_λ [mas/cent]		[illegible]	[illegible]	[illegible]	[illegible]	0.3693
	δH	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
δn_I [mas/cent]		[illegible]	[illegible]	[illegible]	[illegible]	0.0000
Case 3: Anelastic Eart[illegible]						
δn_λ [mas/cent]		[illegible]	[illegible]	[illegible]	[illegible]	0.3615
	δH	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
δn_I [mas/cent]		[illegible]	[illegible]	[illegible]	[illegible]	6.5493
Case 4a: Anelastic Ea[illegible] (IERS 2010)						
δn_λ [mas/cent]		[illegible]	[illegible]	[illegible]	[illegible]	0.7171
	δH	-28.51×10^{-9}	2.68×10^{-9}	42.94×10^{-9}	-17.58×10^{-9}	-0.47×10^{-9}
δn_I [mas/cent]		0.0000	0.0000	0.1301	0.6656	0.7957
Case 4b: Anelastic Earth. Complex Love Numbers frequency dependent (IERS 2010)						
δn_λ [mas/cent]		43.7900	−4.1389	−60.6554	27.0102	6.0059
	δH	-28.51×10^{-9}	2.69×10^{-9}	39.49×10^{-9}	-17.58×10^{-9}	-3.91×10^{-9}
δn_I [mas/cent]		0.0000	−0.0118	0.1209	0.6656	0.7748

An additive correction δH to the dynamical ellipticity related with δn_λ contribution, must be considered in order to ensure consistency between observational and theoretical magnitudes

$$\frac{\delta H}{H_d} = -\frac{\delta n_\lambda}{n_{\lambda,\mathrm{obs}}}$$

Numerical representations: nutations, T_t contribution

Typical order of magnitude: a few mas

Nutations: contribution of the kinetic energy of redistribution.

Argument					Longitude [μas]					Obliquity [μas]				
l	l'	F	D	Ω	Case 1	Case 2	Case 3	Case 4a	Case 4b	Case 1	Case 2	Case 3	Case 4a	Case 4b
								In-phase terms						
+0	+0	+0	+0	+1	+1053.97	+1071.05	+1070.83	+1084.14	+933.35	−312.32	−317.38	−317.32	−321.26	−274.99
+0	+0	+0	+0	+2	−20.58	−20.92	−20.91	−21.17	−18.00	+7.51	+7.63	+7.63	+7.73	+6.55
+0	+1	+0	+0	+0	+0.29	+0.29	+0.29	+0.30	−43.06	−42.22	−42.90	−42.89	−43.43	−58.19
+0	−1	+2	−2	+2	+19.64	+19.96	+19.95	+20.20	+19.71	−7.17	−7.29	−7.29	−7.38	−6.69
+0	+0	+2	−2	+2	−2346.96	−2385.00	−2384.50	−2414.13	−2338.50	+857.33	+871.23	+871.05	+881.87	+844.09
+0	+1	+2	−2	+2	−137.89	−140.13	−140.10	−141.84	−138.48	+50.40	+51.21	+51.20	+51.84	+50.26
+1	+0	+0	+0	+0	+27.40	+27.85	+27.85	+28.19	+21.65	−302.19	−307.08	−307.02	−310.83	−311.39
+0	+0	+2	+0	+2	−5395.72	−5483.16	−5482.17	−5550.15	−5537.17	+1992.35	+2024.64	+2024.28	+2049.37	+2043.26
+0	+0	+2	+0	+1	−1105.11	−1123.02	−1122.81	−1136.74	−1134.83	+341.03	+346.56	+346.50	+350.79	+349.43
+1	+0	+2	+0	+2	−1072.18	−1089.56	−1089.37	−1102.87	−1101.47	+397.97	+404.42	+404.36	+409.36	+408.67
								Out-of-phase terms						
+0	+0	+0	+0	+1			−21.89	−5.23	−0.40			−6.49	−1.55	+0.04
+0	+0	+0	+0	+2			+0.43	+0.10	−0.01			+0.16	+0.04	−0.01
+0	+1	+0	+0	+0			+0.00	−0.00	−1.04			−0.88	−0.21	+0.26
+0	−1	+2	−2	+2			−0.41	−0.10	−0.05			−0.15	−0.04	−0.03
+0	+0	+2	−2	+2			+48.48	+11.65	+9.14			+17.69	+4.26	+3.06
+0	+1	+2	−2	+2			+2.84	+0.68	+0.57			+1.04	+0.25	+0.20
+1	+0	+0	+0	+0			+0.00	−0.14	+0.07			−6.27	−1.50	−1.51
+0	+0	+2	+0	+2			+104.44	+26.79	+26.34			+38.11	+9.89	+9.69
+0	+0	+2	+0	+1			+21.64	+5.49	+5.42			+6.41	+1.69	+1.65
+1	+0	+2	+0	+2			+19.99	+5.32	+5.28			+7.30	+1.98	+1.96

Numerical representations: nutations, V_t contribution

Typical order of magnitude: a few μas

Nutations: contribution of the tidal potential of redistribution.

Argument					Longitude [μas]					Obliquity [μas]				
l	l'	F	D	Ω	Case 1	Case 2	Case 3	Case 4a	Case 4b	Case 1	Case 2	Case 3	Case 4a	Case 4b
							In-phase terms							
+0	+0	+0	+0	+1	+0.0000	−0.8459	−0.8101	−0.5385	+5.4095	+0.0000	−0.8633	−0.8434	−1.4738	−11.5748
+0	+0	+0	+0	+2	+0.0000	−0.0245	−0.0246	−0.0679	−1.0599	−0.0000	+0.0138	+0.0138	+0.0381	+0.5897
+0	+1	+0	+0	+0	+0.0000	−0.0110	−0.0110	−0.0307	+0.1294	+0.0000	+0.0000	+0.0000	+0.0000	−0.1903
+0	−1	+2	−2	+2	+0.0000	−0.0002	−0.0002	−0.0002	+0.0025	−0.0000	+0.0001	+0.0001	+0.0001	−0.0033
+0	+0	+2	−2	+2	−0.0000	+0.1484	+0.1424	+0.2017	1.8798	+0.0000	−0.0723	−0.0697	−0.1094	−0.9666
+0	+1	+2	−2	+2	−0.0000	+0.0066	+0.0064	+0.0090	+0.0282	+0.0000	−0.0032	−0.0031	−0.0048	−0.0150
+1	+0	+0	+0	+0	+0.0000	+0.0134	+0.0130	+0.0218	+0.0938	+0.0000	+0.0002	+0.0002	+0.0003	+0.0037
+0	+0	+2	+0	+2	+0.0000	−0.0262	−0.0252	−0.0357	−0.2686	−0.0000	+0.0125	+0.0120	+0.0186	+0.1373
+0	+0	+2	+0	+1	+0.0000	+0.0020	+0.0020	+0.0016	−0.0027	−0.0000	+0.0019	+0.0018	+0.0030	+0.0157
+1	+0	+2	+0	+2	+0.0000	+0.0045	+0.0044	+0.0062	+0.0372	+0.0000	−0.0021	−0.0021	−0.0032	−0.0186
+0	+2	−2	+2	−2	−0.0000	−1.0799	−1.0359	−1.4682	−22.9684	−0.0000	+0.5279	+0.5088	+0.7994	+8.5358
							Out-of-phase terms							
+0	+0	+0	+0	+1			+1.4342	+0.0643	+0.1377			+0.2474	+0.3697	+0.0393
+0	+0	+0	+0	+2			−0.0612	−0.0225	+0.0072			−0.0341	−0.0126	+0.0028
+0	+1	+0	+0	+0			−0.0280	−0.0102	+0.0049			+0.0000	+0.0000	+0.0045
+0	−1	+2	−2	+2			+0.0001	+0.0000	+0.0008			+0.0000	+0.0000	+0.0005
+0	+0	+2	−2	+2			−0.0967	+0.0421	−0.0089			−0.0226	+0.0256	−0.0001
+0	+1	+2	−2	+2			−0.0043	+0.0019	−0.0013			−0.0010	+0.0011	−0.0005
+1	+0	+0	+0	+0			−0.0007	+0.0054	+0.0032			−0.0001	−0.0001	+0.0001
+0	+0	+2	+0	+2			+0.0156	−0.0075	−0.0009			+0.0043	−0.0043	−0.0008
+0	+0	+2	+0	+1			−0.0030	+0.0000	+0.0001			−0.0002	−0.0007	−0.0002
+1	+0	+2	+0	+2			−0.0024	+0.0013	+0.0004			−0.0007	+0.0008	+0.0003
+0	+2	−2	+2	−2			+0.7160	−0.3070	−0.4111			+0.1678	−0.1868	−0.1500

Numerical representations: comparative of nutations

$T_t + V_t$: **DIFFERENCE** between **simplified elastic** and **generalized anelastic** models									
Argument					Period (days)	Nut. longitude [μas]		Nut. obliquity [μas]	
l	l'	F	D	Ω		In	Out	In	Out
+0	+0	+0	+0	+1	−6793,48	−115,2105	−0,2623	25,7552	0,0793
+0	+0	+0	+0	+2	−3396,74	1,5201	−0,0028	−0,3703	−0,0072
+0	+1	+0	+0	+0	365,26	−43,2206	−1,0351	−16,1603	0,2645
+0	−1	+2	−2	+2	365,25	0,0725	−0,0492	0,4767	−0,0295
+0	+0	+2	−2	+2	182,63	10,3398	9,1311	−14,2066	3,0599
+0	+1	+2	−2	+2	121,75	−0,5618	0,5687	−0,155	0,1995
+1	+0	+0	+0	+0	27,55	−5,6562	0,0732	−9,1963	−1,5099
+0	+0	+2	+0	+2	13,66	−141,7186	26,3391	51,0473	9,6892
+0	+0	+2	+0	+1	13,63	−29,7227	5,4201	8,4157	1,6498
+1	+0	−2	+0	−2	9,13	−29,2528	5,2804	10,6814	1,9603

Index

- The Hamiltonian approach to Earth rotation has been extended in a consistent way, including more general rheological models for the mantle elasticity.
- New analytical expressions for the full motion of the Earth figure axis have been derived, including a treatment of the effects of the tidal mass redistribution.
- The numerical results show a significant contribution with the frequency dependence of the Love Numbers. This is due to the existence of resonance processes in the diurnal frequency band (FCN, see e.g. IERS Conv. 2010).
- New non-negligible secular and periodic contributions have been found, the differences with respect to the simplified elastic case reach the following orders of magnitude:

Velocity of precession in longitude:	~ 6 mas/cJ
Velocity of precession in obliquity:	~ 0.8 mas/cJ
Nutation in longitude:	$\sim 140\ \mu$as
Nutation in obliquity:	$\sim 50\ \mu$as

This work has been partially supported by the Spanish government through the MINECO projects I+D+I AYA201022039-C02-01, AYA2010-22039-C02-02, and Generalitat Valenciana project GV/2014/072.