EFFECTS OF THE TIDAL MASS REDISTRIBUTION ON THE EARTH ROTATION

T. Baenas\textsuperscript{1}, J.M. Ferrández\textsuperscript{1}, A. Escapa\textsuperscript{1,2} and J. Getino\textsuperscript{3}

\textsuperscript{1}Dept. of Applied Mathematics Dept. University of Alicante. Spain
\textsuperscript{2}Dept. of Mechanical, Informatics and Aerospace Engineering. University of León. Spain
\textsuperscript{3}Dept. of Applied Mathematics. University of Valladolid. Spain

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• The gravitational action of the Moon and the Sun on the **deformable Earth** originates a redistribution of its mass, and thus a variation in its kinetic energy related with the change of the Earth inertia tensor.

• Besides, the redistribution produces an additional term in the gravitational potential energy, commonly referred as **tidal potential of redistribution**.

• The effects on the Earth rotation were previously and partially exposed in


• The basic scheme for the study of the rotation of the deformable Earth, with the Hamiltonian approach, is summarized in:

✓ Use of the Hamiltonian formalism analogous to the rigid Earth (Kinoshita 1977), including the Lie-Hori perturbation method (Hori 1966).

✓ Decoupling the elasticity problem from the rotational one: Moon and Sun are perturbed and perturbing bodies, with separated contributions in the Hamilton function.

Different hypothesis on the elastic response of the Earth

Contribution: kinetic energy of redistribution, $T_t$

Moon & Sun: perturbed

Moon & Sun: perturbing

Earth mass redistribution

Contribution: potential energy of redistribution, $V_t$
• In Mathews et al. analytical formulation, the contributions of the Earth mass redistribution are calculated in two steps:

✓ The contribution of the angular momentum (equivalent to $T_t$) due to inertia tensor change is calculated by the transfer function method (MHB 2000).

✓ The potential energy of redistribution effect is derived by Lambert & Mathews (2006), based on SOS equations.

• In the Hamiltonian approach both contributions are terms of the Hamiltonian function at hand (therefore necessarily consistent each other):

\[
H = T_0 + \frac{T_t}{2} + V_1 + E + V_t
\]

Different perturbations

- Hamilton equations
- Canonical perturbation methods
• An *ab initio* reconstruction of the whole dynamical model has been done in order to include a more general Earth rheology, following the Love Number approach (Munk & MacDonald 1960).

• Let us recall that assuming a rheological model based in Smith (1974), Wahr (1979) and IERS (2010) for the elastic/anelastic response of the Earth, the second order harmonic components of the redistribution potential admit a truncated development in complex spherical harmonics in the form

\[ V_m(r, \theta, \phi) = k_{2m} \left( \frac{a_E}{r} \right)^3 Y_{2m}(\theta, \phi) + k_{2m}^{(+)} \left( \frac{a_E}{r} \right)^5 Y_{4m}(\theta, \phi), \quad r \geq a_E \]

Generalized Love Numbers, dependent on:

- Harmonic order, \( m \)
- Excitation frequencies

In magnitude,

\[ k_{2m}^{(+)} \left( \frac{a_E}{r} \right)^5 \ll k_{2m} \left( \frac{a_E}{r} \right)^3 \]
• The main elements for the construction are:

- The expression of the tidal redistribution potential, given by the sum (over \( p \) and \( q \), representing Moon and Sun) of terms of the form

\[
V_{t;p,q} = \frac{a_E^5}{r^3p^3} G m_p m_q \left[ k_{20} C_{20}' C_{20} + k_{21} \frac{1}{3} (C_{21}' C_{21} + I_{21}' I_{21}) + k_{22} \frac{1}{12} (C_{22}' C_{22} + I_{22}' I_{22}) \right]
\]

where \( C_{2j}, I_{2j} \) and \( C_{2j}', I_{2j}' \) stand for the second order real spherical harmonics, related to perturbed bodies and perturbing ones respectively.

- The expression of the time dependent inertia tensor, \( I(t) = I_0 + I_1(t) \),

\[
I_1 = \left( \frac{a}{r'} \right)^3 \begin{pmatrix}
\kappa_{20} C_{20}' - \frac{1}{2} \kappa_{22} C_{22}' \\
-\frac{1}{2} \kappa_{22} I_{22}' \\
-\kappa_{21} C_{21}'
\end{pmatrix} \begin{pmatrix}
\kappa_{20} C_{20}' - \frac{1}{2} \kappa_{22} C_{22}' \\
-\frac{1}{2} \kappa_{22} I_{22}' \\
-\kappa_{21} C_{21}'
\end{pmatrix} \begin{pmatrix}
\kappa_{21} C_{21}' \\
-\kappa_{21} I_{21}' \\
-2 \kappa_{20} C_{20}'
\end{pmatrix}
\]

where \( \kappa_{m;p} = \frac{1}{3} k_{2m} m_p a_E^2 \left( \frac{a_E}{a_p} \right)^3 \) is the constant defined by Kubo (1991).
• The modelling of the anelastic behaviour of the deformable Earth requires the \textit{ab initio} introduction of complex functions that generalize the Love Numbers (following e.g. IERS Conventions 2010). The validity of the previous expressions with this substitution was provided e.g. in Baenas dissertation (2014).

\[ \tilde{k}_{2m,j} = |\tilde{k}_{2m,j}| e^{i\varepsilon_{2m,j}} \quad (j \text{ subindex stands for orbital frequencies dependence}) \]

• All the expressions must be expanded in terms of an Andoyer set of canonical variables to establish the transformation that relates the celestial and terrestrial (Tisserand mean) reference systems.

Then we proceed like in the Kinoshita (1977) approach.
• The **orbital motion** of the perturbing bodies is expressed in trigonometric series giving the spherical harmonics,

\[
\left(\frac{a}{r}\right)^3 C_{20}(\eta', \alpha') = - \sum_i A_i^{(0)} \cos \Theta_i,
\]

\[
\left(\frac{a}{r}\right)^3 C_{21}(\eta', \alpha') = 3 \sum_i \bar{A}_i^{(1)} \sin \Theta_i, \quad \left(\frac{a}{r}\right)^3 S_{21}(\eta', \alpha') = 3 \sum_i A_i^{(1)} \cos \Theta_i,
\]

\[
\left(\frac{a}{r}\right)^3 C_{22}(\eta', \alpha') = 3 \sum_i A_i^{(2)} \cos \Theta_i, \quad \left(\frac{a}{r}\right)^3 S_{22}(\eta', \alpha') = 3 \sum_i \bar{A}_i^{(2)} \sin \Theta_i,
\]

depending on the **fundamental arguments** of nutation (linear combinations of Delaunay variables of the orbital problem).

\[
\Theta_i(t) = m_{1i} l + m_{2i} l' + m_{3i} F + m_{4i} D + m_{5i} \Omega
\]

• The first order Lie-Hori perturbation equations are then applied (Baenas 2014). With this formulation, **analytical expressions** for the components of the solution can be obtained.
2. Results

Analytical expressions

- As an example, we include here the expression for the precession velocities (computed from the secular part of the Hamiltonian):

\[
\begin{align*}
\delta_T n_\lambda &= 0 \\
\delta_T n_I &= 0 \\
\delta_V n_\lambda &= -\frac{1}{\sin I^*} \frac{1}{CH_d} \sum_{p,q}^{M,S} \sum_{i,j;\tau,\xi}^{0,1,2} \sum_m |\bar{F}_{2m,j;p}| \left| T_{ijpq,m}^{(n_\lambda)} (\tau, \xi) \cos \varepsilon_{2m,j} \right| \\
\delta_V n_I &= -\frac{1}{\sin I^*} \frac{1}{CH_d} \sum_{p,q}^{M,S} \sum_{i,j;\tau,\xi}^{0,1,2} \sum_m |\bar{F}_{2m,j;p}| \left| T_{ijpq,m}^{(n_I)} (\tau, \xi) \sin \varepsilon_{2m,j} \right|
\end{align*}
\]

- The subindexes \( T \) and \( V \) stand, respectively, for the contributions of the kinetic energy of redistribution and the potential energy of redistribution.

- \(-\lambda^* \) and \(-I^* \) are the precessional longitude and obliquity.

- The non-zero \( \delta_V n_I \) contribution is a purely anelastic effect (stands for any \( \varepsilon_{2m,j} \neq 0 \)) (in accordance with Lambert & Mathews 2006).
2. Results

Analytical expressions

- As an example, we include here the expression for the precession velocities (computed from the secular part of the Hamiltonian):

\[
\begin{align*}
\delta_T n_\lambda &= 0 \\
\delta_T n_I &= 0 \\
\delta_V n_\lambda &= - \frac{1}{\sin I^*} \frac{1}{CH_d} \sum_{p,q} \sum_{i,j;\ell,\epsilon} \sum_{m,0,1,2} \left| k_{ijpq,m} \right| T_{ijpq,m}^{(n_\lambda)} (\tau, \epsilon) \cos \varepsilon_{2m,j} \\
\delta_V n_I &= - \frac{1}{\sin I^*} \frac{1}{CH_d} \sum_{p,q} \sum_{i,j;\ell,\epsilon} \sum_{m,0,1,2} \left| k_{ijpq,m} \right| T_{ijpq,m}^{(n_I)} (\tau, \epsilon) \sin \varepsilon_{2m,j}
\end{align*}
\]

- \( T_{ijpq,m} \) quantities are particular combinations of the \( B, C \) and \( D \) functions of the orbital coefficients \( A^{(j)}_i \) defined by Kinoshita (1977).

\[
\begin{align*}
T_{ijpq,m}^{(n_\lambda)} (\tau, \epsilon) &= \frac{9}{4} \frac{\partial B^{*}_{i;p}}{\partial I^*} \tilde{B}_{ji;q} \delta_{m0} + \frac{3}{4} \frac{\partial C^{*}_{i;p}}{\partial I^*} \tilde{C}_{ji;q} \delta_{m1} + \frac{3}{4} \frac{\partial D^{*}_{i;p}}{\partial I^*} \tilde{D}_{ji;q} \delta_{m2} \\
T_{ijpq,m}^{(n_I)} (\tau, \epsilon) &= \tau m_{5i} \left( \frac{9}{4} B^{*}_{i;p} \tilde{B}_{ji;q} \delta_{m0} - 3 C^{*}_{i;p} \tilde{C}_{ji;q} \delta_{m1} - \frac{3}{4} D^{*}_{i;p} \tilde{D}_{ji;q} \delta_{m2} \right)
\end{align*}
\]

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2. Results

Numerical representations for different models

**Simplified elastic model**
- Spherical non-perturbed state without rotation
- Real Love Numbers, frequency independent

\[
\begin{align*}
\bar{k}_{2m,j} &= k \in \mathbb{R} \\
\Rightarrow |\bar{k}_{2m,j;p}| &= \kappa_p, \\
\Rightarrow \varepsilon_{2m,j} &= 0.
\end{align*}
\]

**Simple anelastic model:**
constant time delay
- Ellipsoidal non-perturbed state with rotation
- Complex Love Numbers, frequency dependent (*ad hoc* functional dependence)

\[
\begin{align*}
\bar{k}_{2m,j} &= k_2m e^{i\varepsilon_{2m,j}} \in \mathbb{C} \\
\Rightarrow |\bar{k}_{2m,j;p}| &= \kappa_{2m;p}, \\
\Rightarrow \varepsilon_{2m,j} &= -\Delta t \left\{ m\omega_E - [\epsilon - \delta_m (1 + \epsilon)] \tilde{n}_j \right\}.
\end{align*}
\]

**Generalized elastic model**
- Ellipsoidal non-perturbed state with rotation
- Real Love Numbers, with either nominal value for frequency band, or frequency dependent

\[
\begin{align*}
\bar{k}_{2m,j} &= k_{2m,j} \in \mathbb{R} \\
\Rightarrow |\bar{k}_{2m,j;p}| &= \kappa_{2m,j;p}, \\
\Rightarrow \varepsilon_{2m,j} &= 0.
\end{align*}
\]

**Generalized anelastic model**
- Ellipsoidal non-perturbed state with rotation
- Complex Love Numbers, with either a nominal value for each frequency band, or frequency dependent

\[
\begin{align*}
\bar{k}_{2m,j} &= |\bar{k}_{2m,j}| e^{i\varepsilon_{2m,j}} \in \mathbb{C} \\
\Rightarrow \bar{k}_{2m,j;p} &= |\bar{k}_{2m,j;p}| e^{i\varepsilon_{2m,j}}
\end{align*}
\]
## 2. Results

### Numerical representations: precession velocities

Precession in longitude and obliquity: tidal redistribution potential contribution.

<table>
<thead>
<tr>
<th>Case 1: Elastic Earth (simplified model)</th>
<th>Zonal</th>
<th>Tesseral</th>
<th>Sectorial</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta n_\lambda$ [mas/cent]</td>
<td>42.0639</td>
<td>$-3.9604$</td>
<td>$-64.1250$</td>
<td>26.0214</td>
</tr>
<tr>
<td>$\delta H$</td>
<td>$-27.34 \times 10^{-9}$</td>
<td>$2.59 \times 10^{-9}$</td>
<td>$41.75 \times 10^{-9}$</td>
<td>$-16.94 \times 10^{-9}$</td>
</tr>
<tr>
<td>$\delta n_I$ [mas/cent]</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Case 2: Elastic Earth. Real Love Numbers with nominal value for frequency band (IERS 2010)</th>
<th>Zonal</th>
<th>Tesseral</th>
<th>Sectorial</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta n_\lambda$ [mas/cent]</td>
<td>42.8254</td>
<td>$-4.0321$</td>
<td>$-65.1642$</td>
<td>26.7401</td>
</tr>
<tr>
<td>$\delta H$</td>
<td>$-27.88 \times 10^{-9}$</td>
<td>$2.62 \times 10^{-9}$</td>
<td>$42.42 \times 10^{-9}$</td>
<td>$-17.41 \times 10^{-9}$</td>
</tr>
<tr>
<td>$\delta n_I$ [mas/cent]</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Case 3: Anelastic Earth. Constant time delay model</th>
<th>Zonal</th>
<th>Tesseral</th>
<th>Sectorial</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta n_\lambda$ [mas/cent]</td>
<td>42.8254</td>
<td>$-4.0321$</td>
<td>$-65.1513$</td>
<td>26.7194</td>
</tr>
<tr>
<td>$\delta H$</td>
<td>$-27.87 \times 10^{-9}$</td>
<td>$2.62 \times 10^{-9}$</td>
<td>$42.41 \times 10^{-9}$</td>
<td>$-17.39 \times 10^{-9}$</td>
</tr>
<tr>
<td>$\delta n_I$ [mas/cent]</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.5103</td>
<td>6.0390</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>Case 4a: Anelastic Earth. Complex Love Numbers with nominal value for frequency band (IERS 2010)</th>
<th>Zonal</th>
<th>Tesseral</th>
<th>Sectorial</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta n_\lambda$ [mas/cent]</td>
<td>43.7900</td>
<td>$-4.1229$</td>
<td>$-65.9603$</td>
<td>27.0102</td>
</tr>
<tr>
<td>$\delta H$</td>
<td>$-28.51 \times 10^{-9}$</td>
<td>$2.68 \times 10^{-9}$</td>
<td>$42.94 \times 10^{-9}$</td>
<td>$-17.58 \times 10^{-9}$</td>
</tr>
<tr>
<td>$\delta n_I$ [mas/cent]</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.1301</td>
<td>0.6656</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Case 4b: Anelastic Earth. Complex Love Numbers frequency dependent (IERS 2010)</th>
<th>Zonal</th>
<th>Tesseral</th>
<th>Sectorial</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta n_\lambda$ [mas/cent]</td>
<td>43.7900</td>
<td>$-4.1389$</td>
<td>$-60.6554$</td>
<td>27.0102</td>
</tr>
<tr>
<td>$\delta H$</td>
<td>$-28.51 \times 10^{-9}$</td>
<td>$2.69 \times 10^{-9}$</td>
<td>$39.49 \times 10^{-9}$</td>
<td>$-17.58 \times 10^{-9}$</td>
</tr>
<tr>
<td>$\delta n_I$ [mas/cent]</td>
<td>0.0000</td>
<td>$-0.0118$</td>
<td>0.1209</td>
<td>0.6656</td>
</tr>
</tbody>
</table>
2. Results

Numerical representations: precession velocities

<table>
<thead>
<tr>
<th></th>
<th>Zonal</th>
<th></th>
<th>Tesseral</th>
<th></th>
<th>Sectorial</th>
<th></th>
<th>Total</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>B_0</td>
<td>42.0639</td>
<td>-3.9604</td>
<td>-64.1250</td>
<td>26.0214</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>B - B_0</td>
<td>-27.34 x 10^{-9}</td>
<td>2.59 x 10^{-9}</td>
<td>41.75 x 10^{-9}</td>
<td>-16.94 x 10^{-9}</td>
<td>0.00 x 10^{-9}</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

The **permanent tide contribution** is computed separately. B_0 contribution must be included or removed, depending on the dynamical model considered for the rigid part of the Earth inertia tensor (zero tide or tide free).

Total here corresponds to tide free.
An additive **correction** $\delta H$ to the **dynamical ellipticity** related with $\delta n_\lambda$ contribution, must be considered in order to ensure **consistency** between observational and theoretical magnitudes

\[
\frac{\delta H}{H_d} = - \frac{\delta n_\lambda}{n_{\lambda,\text{obs}}}
\]
## 2. Results

### Numerical representations: nutations, $T_t$ contribution

Nutations: contribution of the kinetic energy of redistribution.

<table>
<thead>
<tr>
<th>Argument</th>
<th>Longitude [μas]</th>
<th>Obliquity [μas]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Case 1</td>
<td>Case 2</td>
</tr>
<tr>
<td>$l$</td>
<td>$l'$</td>
<td>$F$</td>
</tr>
<tr>
<td>$+0$</td>
<td>$+0$</td>
<td>$+0$</td>
</tr>
<tr>
<td>$+0$</td>
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<tr>
<td>$+0$</td>
<td>$+1$</td>
<td>$+0$</td>
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<td>$+0$</td>
<td>$-1$</td>
<td>$+2$</td>
</tr>
<tr>
<td>$+0$</td>
<td>$+0$</td>
<td>$+2$</td>
</tr>
<tr>
<td>$+0$</td>
<td>$+1$</td>
<td>$+2$</td>
</tr>
<tr>
<td>$+1$</td>
<td>$+0$</td>
<td>$+0$</td>
</tr>
<tr>
<td>$+0$</td>
<td>$+0$</td>
<td>$+2$</td>
</tr>
<tr>
<td>$+0$</td>
<td>$+0$</td>
<td>$+2$</td>
</tr>
<tr>
<td>$+1$</td>
<td>$+0$</td>
<td>$+2$</td>
</tr>
</tbody>
</table>

**Typical order of magnitude: a few mas**
2. Results

Numerical representations: nutations, $V_t$ contribution

<table>
<thead>
<tr>
<th>Argument</th>
<th>Longitude $[\mu\text{as}]$</th>
<th>Obliquity $[\mu\text{as}]$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Case 1</td>
<td>Case 2</td>
</tr>
<tr>
<td>+0 +0 +0 +0 +1</td>
<td>+0.0000</td>
<td>-0.8459</td>
</tr>
<tr>
<td>+0 +0 +0 +0 +2</td>
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<td>-0.0245</td>
</tr>
<tr>
<td>+0 +1 +0 +0 +0</td>
<td>+0.0000</td>
<td>-0.0110</td>
</tr>
<tr>
<td>+0 -1 +2 -2 +2</td>
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<td>-0.0002</td>
</tr>
<tr>
<td>+0 +0 +2 -2 +2</td>
<td>-0.0000</td>
<td>+0.1484</td>
</tr>
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</tr>
<tr>
<td>+1 +0 +0 +0 +0</td>
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<tr>
<td>+0 +0 +2 +0 +2</td>
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<td>-0.0262</td>
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<td>+0 +0 +2 +0 +1</td>
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<tr>
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<tr>
<td>+0 +2 -2 +2 -2</td>
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<td>-1.0799</td>
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</table>

Typical order of magnitude: a few $\mu\text{as}$
## 2. Results

Numerical representations: comparative of nutations

### $T_t + V_t$: DIFFERENCE between simplified elastic and generalized anelastic models

<table>
<thead>
<tr>
<th>Argument</th>
<th>Period (days)</th>
<th>Nut. longitude [μas]</th>
<th>Nut. obliquity [μas]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l$</td>
<td>$l'$</td>
<td>$F$</td>
<td>$D$</td>
</tr>
<tr>
<td>+0</td>
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</tr>
<tr>
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</tr>
<tr>
<td>+1</td>
<td>+0</td>
<td>−2</td>
<td>+0</td>
</tr>
</tbody>
</table>
1. Introduction
2. Results
3. Conclusions
3. Conclusions

- The **Hamiltonian approach** to Earth rotation has been extended in a consistent way, including more general rheological models for the mantle elasticity.

- New **analytical expressions** for the full motion of the **Earth figure axis** have been derived, including a treatment of the effects of the **tidal mass redistribution**.

- The numerical results show a **significant contribution** with the **frequency dependence of the Love Numbers**. This is due to the existence of resonance processes in the diurnal frequency band (FCN, see e.g. IERS Conv. 2010).

- New **non-negligible secular and periodic contributions** have been found, the differences with respect to the **simplified elastic case** reach the following orders of magnitude:
  
  - **Velocity of precession in longitude**: \( \sim 6 \text{ mas/cJ} \)
  - **Velocity of precession in obliquity**: \( \sim 0.8 \text{ mas/cJ} \)
  - **Nutation in longitude**: \( \sim 140 \mu\text{as} \)
  - **Nutation in obliquity**: \( \sim 50 \mu\text{as} \)
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