Numerical-analytical modeling of the Earth's pole oscillations

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Achievements

- Refined model based on the celestial-mechanics methods is presented that is a generalized version of the previously developed basic model
- Subtle effects are qualitatively explained based on the amplitude-frequency analysis and the computer numerical modelling of the oscillatory pole motion, where the key feature is the Chandler wobble

Basic Model of the Polar Oscillations

Model of the polar oscillations is developed based on the gravitational-tidal mechanism:

$$x_p = c_x(\tau) + a_x^c \cos(2\pi N\tau) + a_x^s \sin(2\pi N\tau) - Nd_x^c \cos(2\pi\tau) - d_x^s \sin(2\pi\tau)$$

$$y_p = c_y(\tau) + a_y^c \cos(2\pi N\tau) + a_y^s \sin(2\pi N\tau) - Nd_x^c \cos(2\pi\tau) + d_y^s \sin(2\pi\tau)$$

$$\mathrm{N} \cong 0.84 - 0.85$$

Model features:

- Coefficients should approximately match as $a_x^{c,s} \approx a_y^{s,c}, \quad d_x^{c,s} \approx d_y^{s,c}$
- Parameters are prone to significant changes due to the inertia tensor perturbations.
- Factors like mechanical and physical planet parameters like large-scale natural events in the ocean and atmosphere are not taken into account
- During the anomaly periods the interpolation and forecast isn’t sufficient.

Generalized model requirements

- Consist of small amount of parameters
- Agree qualitatively and quantitatively with previously developed basic model
- Has the same structure features as the basic model
- Averaging dynamic parameters correspond to the basic model parameters
- Working with the geopotential in general

Geopotential analysis

Earth figure –dynamic geoid due to the inertia tensor variations. The additional perturbing potential δW appears. Most significant component – perturbations caused by polar oblateness (second perturbing harmonic)

Normal spherical function

$$\delta W_2 = \frac{fm_E R_E^2}{r^3} \Delta \bar{Y}_2(\theta, \varphi),$$

$$fm_E = 3.98600442 \times 10^{14} \ \ м^3 / c^2,$$

$$R_E \cong 6.38 \times 10^6 \ м$$

Normal function variations:

Normal associated Legendre functions

$$\Delta \bar{Y}_2(\theta,\varphi) = \delta c_{20} \bar{P}_{20}(\cos\theta) + \left[\delta c_{21}\cos\varphi + \delta s_{21}\sin\varphi\right]\bar{P}_{21}(\cos\theta) +$$

$$+\left[\delta c_{22}\cos 2\varphi + \delta s_{22}\sin 2\varphi\right]\bar{P}_{22}(\cos\theta)$$

Polar coordinates differential equations: amplitude-phase form

Sometimes convenient to transform polar coordinates to amplitude-phase form:

$$x_p = c_x + a\cos\psi, \quad y_p = c_y + a\sin\psi$$

From the dynamic Euler-Lliouville equations the differential equations for the amplitude and phase of the polar oscillations are derived taking into account the geopotential expansion

$$\dot{a} = \frac{2m_E R_E^2}{A^*} r_0 \left[c_{22}^* \left(1 - \frac{C^*}{B^*}\right) + \delta c_{22} \right] a \sin 2\psi + \left[\mu_p \cos\psi + \mu_q \sin\psi \right],$$

$$\dot{\psi} = -N_q \cos^2\psi - N_p \sin^2\psi + a^{-1} \left[\mu_q \cos\psi - \mu_p \sin\psi \right].$$

Coefficients for the Chandler wobble frequency:

$$N_p = \frac{C^* - B^* + \delta C - \delta B}{A^* + \delta A} r_0, \quad N_q = \frac{C^* - A^* + \delta C - \delta A}{B^* + \delta B} r_0$$

External perturbations of the gravitational-tidal and geophysical nature:

$$\mu_{p,q} = \mu_{p,q}^{ch} + \mu_{p,q}^{h}$$

Chandler and annual components

Chandler wobble

Amplitude and phase of the Chandler wobble

$$a_{ch} = a_{ch}^{0} + a_{ch}^{var}\left(t, \frac{\pi}{N}\right), \quad \psi_{ch} = \psi_{ch}^{0} - N^{*}t + \int \phi(\delta C, \delta c_{20})dt + \psi_{ch}^{var}\left(t, \frac{\pi}{N}\right),$$

Chandler wobble for *p, q* components of the angular velocity

$$p_{ch} = \frac{a_{ch}^{0} \cos\psi_{ch}}{\sqrt{\dot{\psi}_{ch}}} r_{0}, \quad q_{ch} = \frac{a_{ch}^{0} \sin\psi_{ch}}{\sqrt{\dot{\psi}_{ch}}} r_{0}.$$

For example, equations for $p(t)$ of the perturbed motion of the Earth pole

$$p(t) = p_{ch} + p'_{ch} + p'_{h},$$

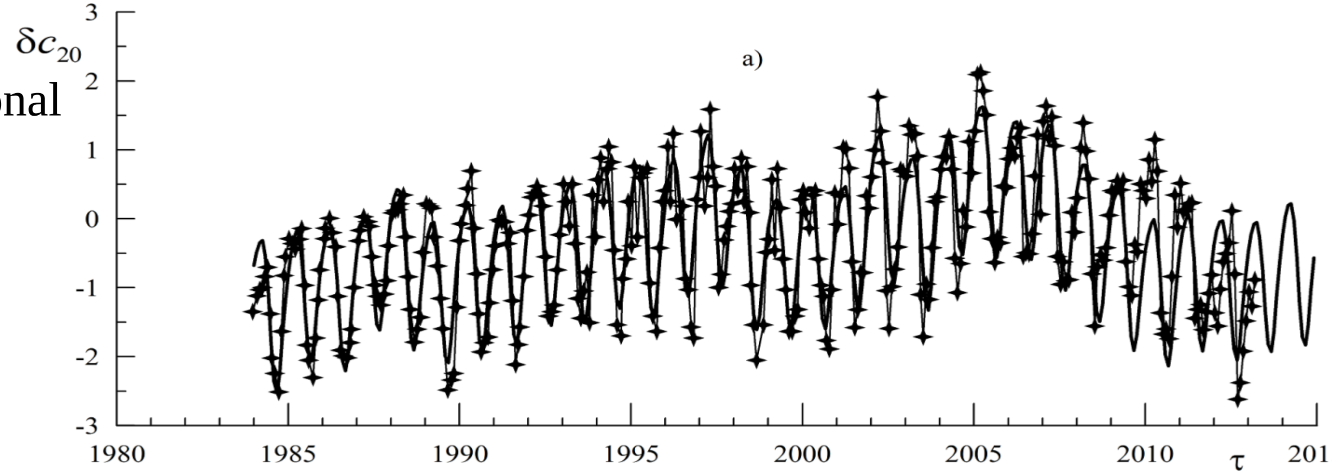
$$p'_{ch,h} = \frac{\sin\psi_{ch}}{\sqrt{\dot{\psi}_{ch}}} \int \frac{\cos\psi_{ch}}{\sqrt{\dot{\psi}_{ch}}} f_{p}^{ch,h}(t)dt - \frac{\cos\psi_{ch}}{\sqrt{\dot{\psi}_{ch}}} \int \frac{\sin\psi_{ch}}{\sqrt{\dot{\psi}_{ch}}} f_{p}^{ch,h}(t)dt,$$

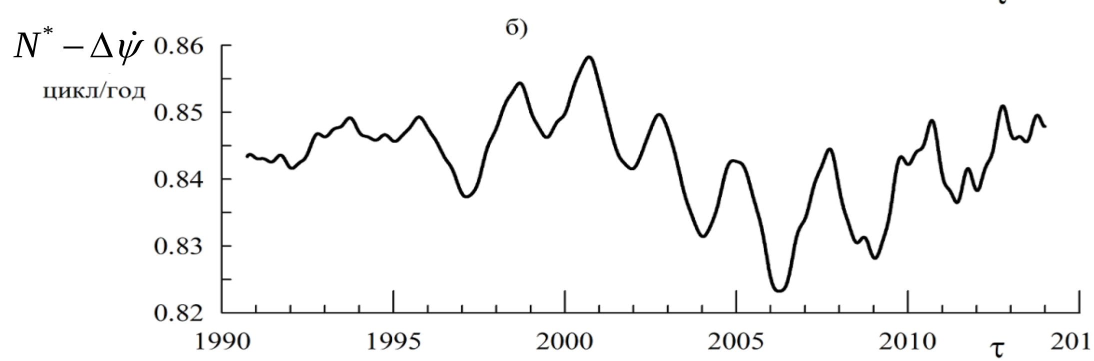
$$f_{p}^{ch,h} = -\mu_{p} N_{p}^{-1} \ddot{\psi}_{ch} - \mu_{q} \dot{\psi}_{ch} - \dot{\mu}_{p}.$$

Numerical modelling of perturbing factors

$$\dot{\psi}_{ch} = -N^* + \phi(\delta C, \delta c_{20}) + \dot{\psi}_{ch}^{var}\left(t, \frac{\pi}{N}\right)$$

Variations of the second zonal harmonic δc_{20} of the geopotential: interpolation 1984-2008 yy.,
forecast 2009-2014 yy.
comparing with SLR data





$N^* - \Delta\dot{\psi}$ frequency variation of the perturbed Chandler wobble of the Earth pole in 1990-2014 yy.

Generalized model

Chandler and annual wobble amplitude

$$a_{ch,h}^{p,q} = \frac{r_0}{a_{ch}^0 \sqrt{\dot{\psi}}} \left\{ \left[\int p_{ch} f_{p,q}^{ch,h}(t)dt \right]^2 + \left[\int q_{ch} f_{p,q}^{ch,h}(t)dt \right]^2 \right\}^{\frac{1}{2}}$$

Equations for polar coordinates assuming $\tilde{a}_{ch,h} \approx a_{ch,h}^{p,q}$

$$x_p = c_x + \tilde{a}_{ch} \cos(\psi_{ch}^0 - N^* t + \delta\psi + \Delta\psi) + a_h \cos(\psi_h^0 + \nu_h t + \chi)$$

$$y_p = c_y + \tilde{a}_{ch} \sin(\psi_{ch}^0 - N^* t + \delta\psi + \Delta\psi + \varepsilon) + a_h \sin(\psi_h^0 + \nu_h t)$$

$$\delta\psi = \int \phi(\delta C, \delta c_{20}) dt$$

For the numerical modelling purposes transform to:

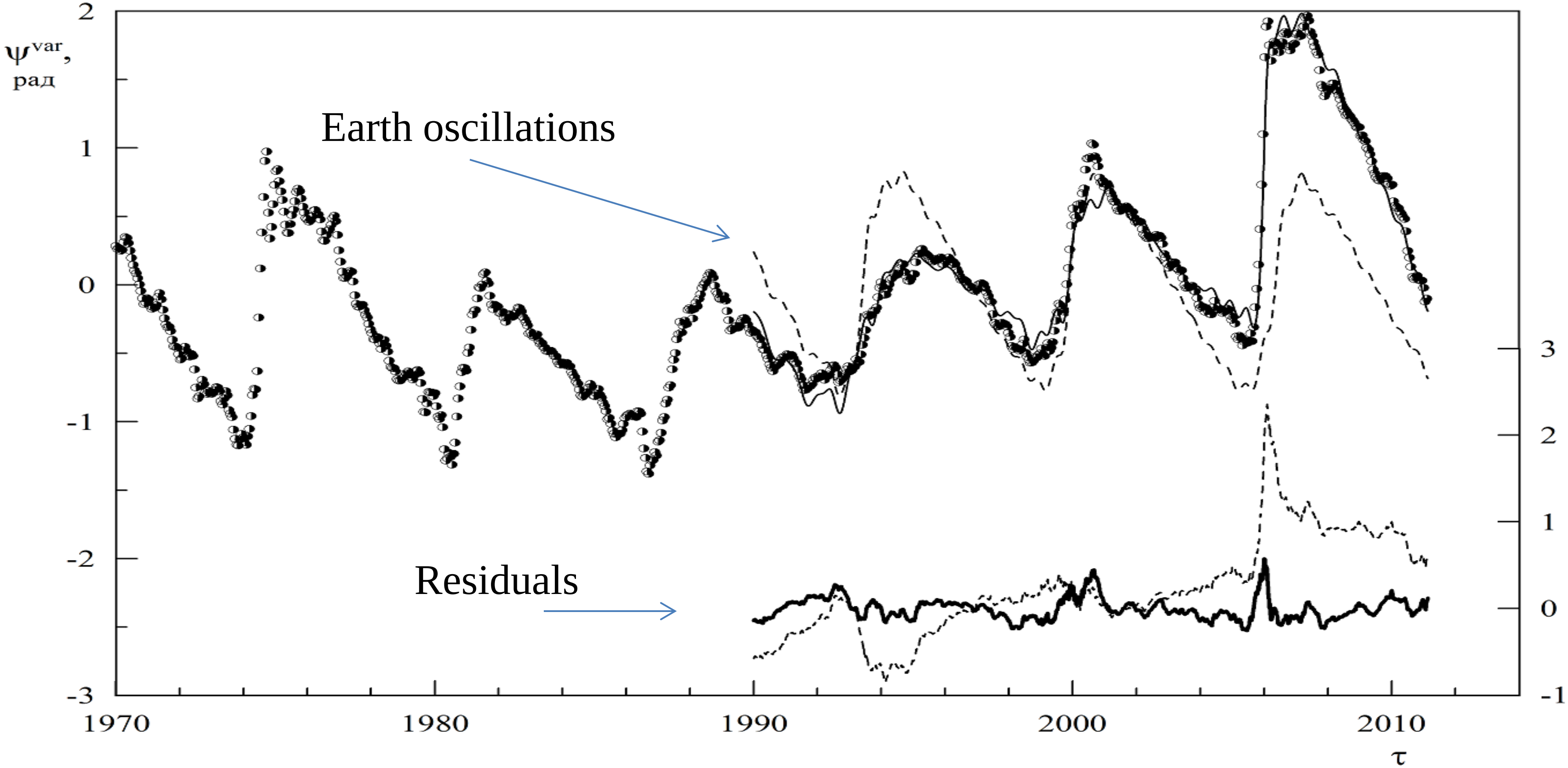
$$x_p = c_x(\tau) - a_x^c(\tau)\cos 2\pi\bar{\psi}_{ch}(\tau) + a_x^s(\tau)\sin 2\pi\bar{\psi}_{ch}(\tau) - Nd_x^c \cos 2\pi\tau - d_x^s \sin 2\pi\tau$$

$$y_p = c_y(\tau) + a_y^c(\tau)\cos 2\pi\bar{\psi}_{ch}(\tau) + a_y^s(\tau)\sin 2\pi\bar{\psi}_{ch}(\tau) - Nd_y^c \cos 2\pi\tau + d_y^s \sin 2\pi\tau$$

$$\bar{\psi}_{ch}(\tau) = N^*\tau - \delta\psi(\tau) - \Delta\psi(\tau); \quad N^* = 0.843 \text{ cycle/year}$$

Apply structure features of the basic model: $a_x^{c,s}(\tau) \approx a_y^{s,c}(\tau), \quad d_x^{c,s}(\tau) \approx d_y^{s,c}(\tau)$

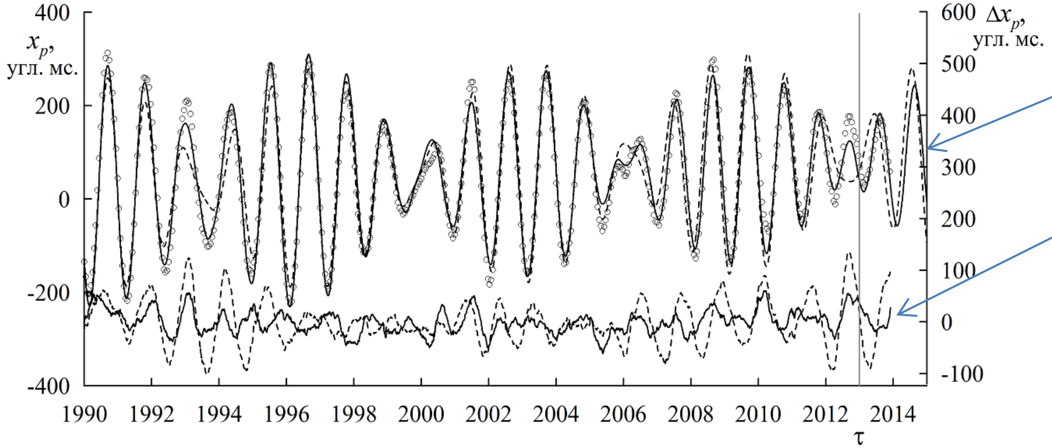
Phase variations of the Earth Pole

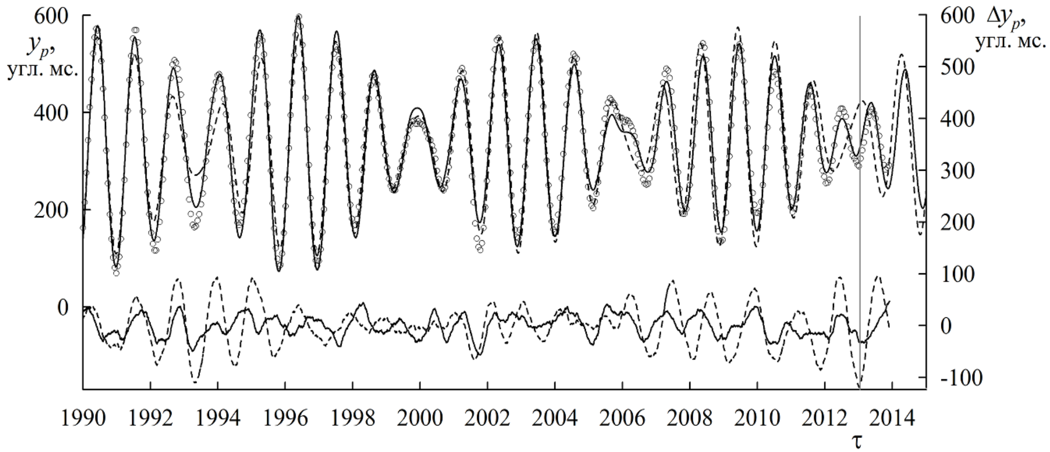


Polar phase variations ψ^{var}(interpolation) without linear component and residuals. Dots – IERS observations, dashed curve– basic model, solid line – generalized model.

Numerical modelling of the polar coordinates variations

Interpolation for 1990-2012 yy. and forecast for 2013-2014 yy. Basic model – dashed curve, generalized model – solid curve. Dots for observation data from IERS





Polar oscillations

Residuals

Mean square deviations for interpolation for the models: **main** and **generalized.**

$$\sigma_x^* = 44.3067 \quad \sigma_x = 24.1476,$$

$$\sigma_y^* = 43.3290 \quad \sigma_y = 20.2541,$$

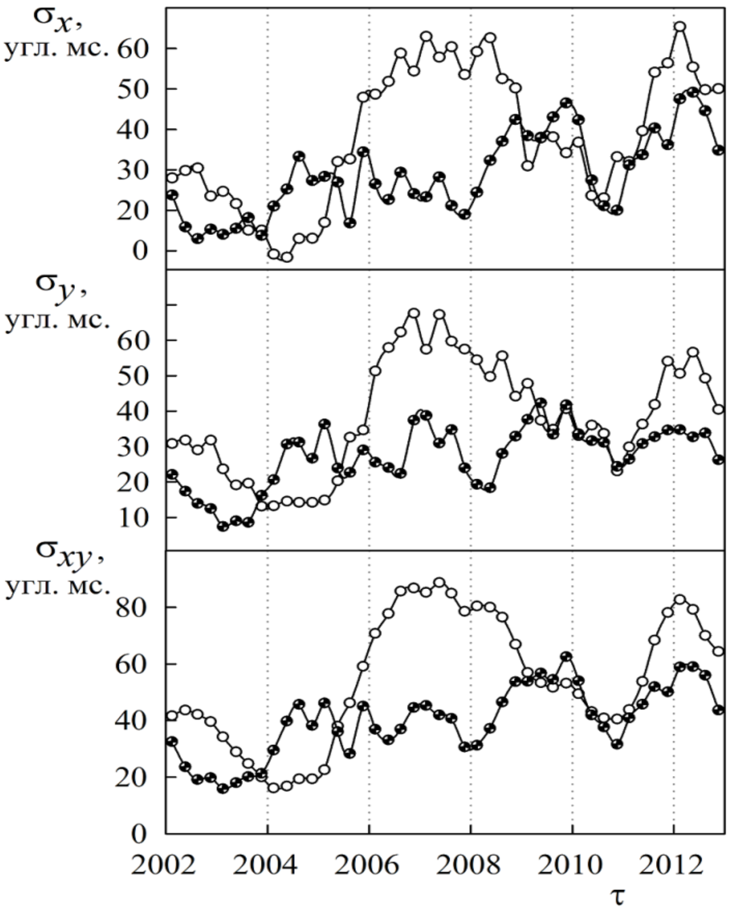
$$\sigma_{xy}^* = 61.9716, \quad \sigma_{xy} = 31.5173.$$

Forecast error analysis

Mean square deviation for the series of the annual forecasts of the Earth pole coordinates and polar trajectory. Gray line - for basic model, black line – for the generalized model. Dots for the iterations (fixing Chandler wobble value at the end of the interval).

Basic model gives sufficient forecast during the stable times (Chandler frequency is a constant) **- 2004-2005** yy.

When anomaly occurs, **generalized model** gives much more accurate forecast – 2**006-2009**, **2012-2013**

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