Estimating the Period and Q of the Chandler Wobble from Observations and Models of its Excitation

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Abstract. Any irregularly shaped solid body rotating about some axis that is not aligned with its figure axis will freely wobble as it rotates. For the Earth, this free wobble is known as the Chandler wobble in honor of S. C. Chandler, Jr. who first observed it in 1891. Unlike the forced wobbles of the Earth, such as the annual wobble, whose periods are the same as the periods of the forcing mechanisms, the period of the free Chandler wobble is a function of the internal structure and rheology of the Earth, and its decay time constant, or quality factor Q, is a function of the dissipation mechanism(s), like mantle anelasticity, that are acting to dampen it. Improved estimates of the period and Q of the Chandler wobble can therefore be used to improve our understanding of these properties of the Earth. Here, estimates of the period and Q of the Chandler wobble are obtained by finding those values that minimize the power within the Chandler band of the difference between observed and modeled polar motion excitation spanning 1962-2010.

Introduction

- Polar motion
  - Earth not rotating about figure axis
    - So, Earth wobbles as it rotates
  - Forced wobbles
    - Forced by changes in relative motion (winds and currents), surficial loading processes, glacial isostatic adjustment, etc.
    - Frequency of wobble same as frequency of forcing mechanism
  - Free wobbles
    - Chandler wobble of 14-month period
    - Nearly Diurnal Free Wobble (Free Core Nutation) of retrograde diurnal period
    - Unlike forced wobbles, frequencies of free wobbles depend on Earth's interior structure and dissipation processes
- Estimate period and Q of Chandler wobble
  - Observed polar motion and polar motion rate estimates
  - Modeled atmospheric, oceanic, and hydrologic excitation
  - During 1962–2010

Data Sets

- Observed polar motion variations
  - COMB2010 combined EOP series
    - Combination of optical astrometric, LLR, SLR, VLBI, & GPS observations
    - Polar motion rate observations not used (contaminated by tidal artifacts)
    - Daily values at midnight spanning January 20, 1962 to July 15, 2011
- Helmholtz Centre Potsdam – GFZ
  - Consistent estimates of AAM, OAM, & HAM computed at GFZ
    - AAM computed from European Centre for Medium-Range Weather Forecasts
    - OAM computed from Ocean Model for Circulation and Tides (OMCT)
    - HAM computed from Land-Surface Discharge Model (LSDM)
    - Ocean and hydrology models driven by ECMWF fields
    - Global atmosphere/oceans/hydrology mass conservation imposed
  - Merge AAM, OAM, & HAM from ERA-40 / ERA-Interim
    - Adjust bias of ERA-40 series to agree with that of ERA-Interim series
    - Merged series spans January 1, 1958 to December 31, 2010 at daily intervals

Estimation Strategy

- **Polar motion equation:**

$$\mathrm{p}(t) + \frac{i}{\sigma_{cw}} \frac{\partial \mathrm{p}}{\partial t} = \chi(t)$$

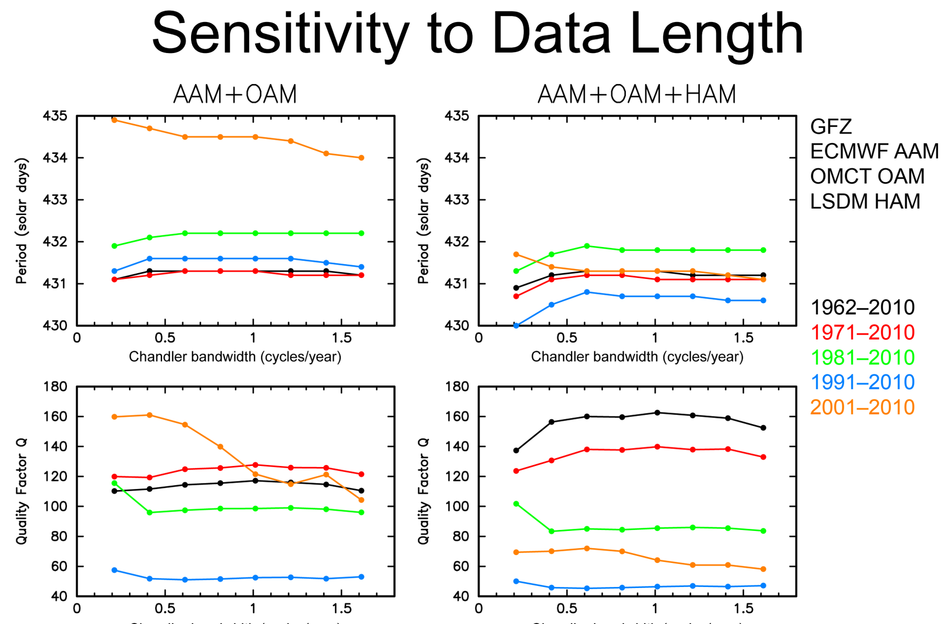
where: polar motion $\mathrm{p}(t) \equiv x_p(t) - i\, y_p(t)$

excitation function $\chi(t) \equiv \chi_1(t) + i\, \chi_2(t)$

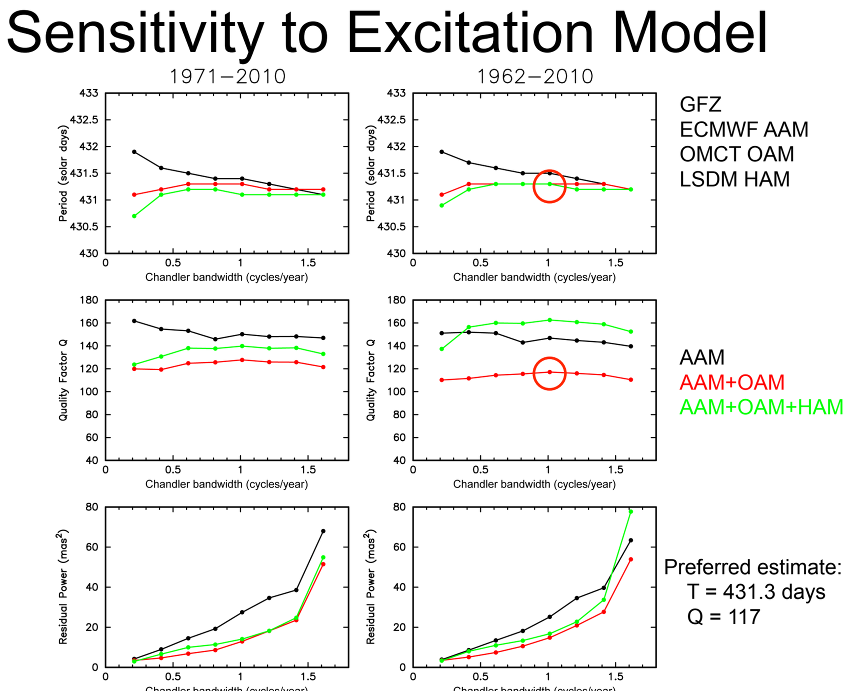
σ_{cw} is complex-valued frequency of Chandler wobble

- **Recover σ_{cw} using**
  - Observed values of polar motion $\mathrm{p}(t)$ and polar motion rate $d\mathrm{p}/dt$
    - Polar motion rate $d\mathrm{p}/dt$ determined using Kalman filter
  - Models of excitation functions
    - Atmospheric, oceanic and hydrologic angular momentum
- **Frequency domain**
  - Isolate Chandler band
  - Minimizes effects of polar motion measurement errors
  - Find that value of σ_{cw} which minimizes the difference in power between observed and modeled excitation functions (Furuya and Chao, 1996)
    - Observed excitation functions computed using polar motion equation specifying different values for Chandler period and Q

Sensitivity to Data Length



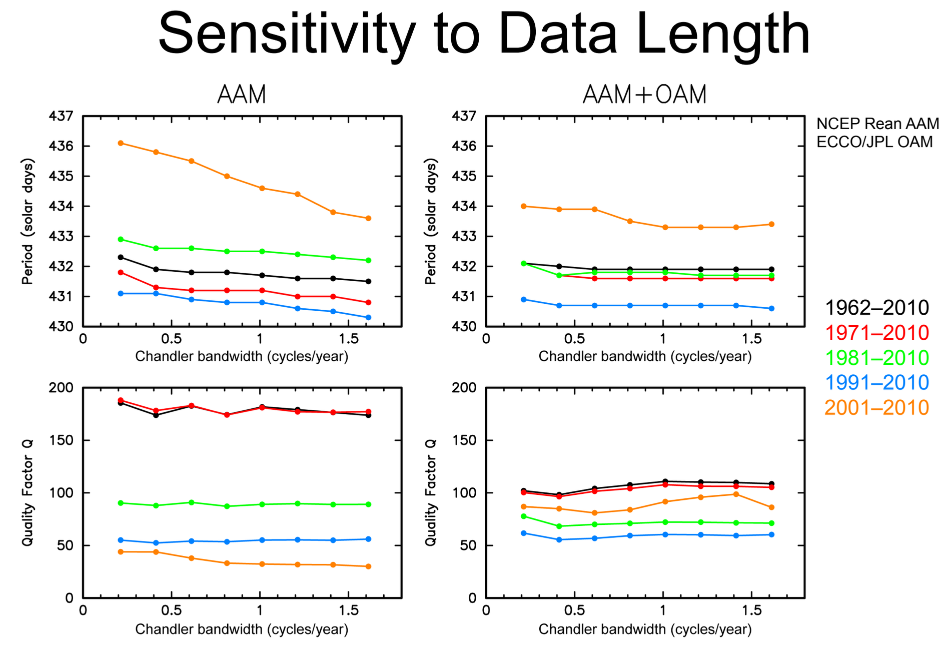
Sensitivity to Excitation Model



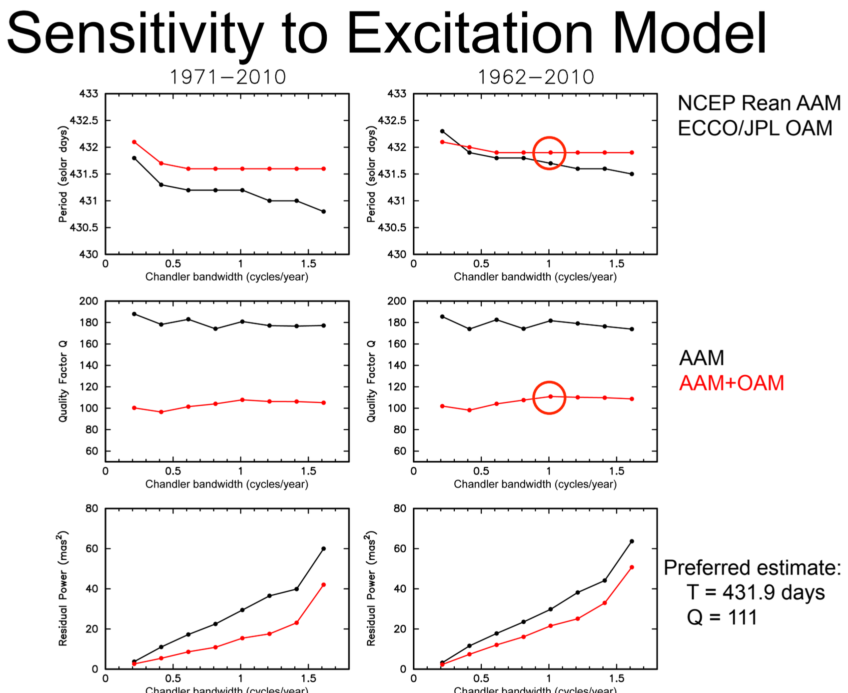
Independent Excitation Model

- Oceanic angular momentum
  - ECCO/JPL 50-year simulation
    - 10-day values at midnight spanning 1949-2002
    - Sum of current and bottom pressure terms
  - ECCO/JPL data assimilating model (kf080)
    - Hourly values spanning January 1, 1993 to present
    - Averaged to 10-day values at midnight
    - Sum of current and bottom pressure terms
  - Merge 50-year and kf080 series
    - Adjust bias of 50-year series to agree with that of kf080 series
    - Merged series spans 1949 to present at 10-day intervals
- Atmospheric angular momentum
  - NCEP/NCAR Reanalysis
    - 6-hour values spanning January 1, 1948 to present
    - Averaged to 10-day values at midnight
    - Sum of wind and inverted barometer pressure terms

Sensitivity to Data Length



Sensitivity to Excitation Model



Estimates of Chandler Frequency

Table 12 Estimated period and *Q* of Chandler wobble

Period (solar days)	*Q*	*Data span (years)*	*Source*
Statistical excitation			
433.2 ± 2.2	63 (36, 192)	67.6	(a)
434.0 ± 2.6	100 (50, 400)	70	(b)
434.8 ± 2.0	96 (50, 300)	76	(c)
433.3 ± 3.1	170 (47, 1000)	78	(d)
433.0 ± 1.1	**179 (74, 789)**	**86**	**(e)**
433.1 ± 1.7	—	93	(f)
Atmospheric excitation			
439.5 ± 2.1	72 (30, 500)	8.6	(g)
433.7 ± 1.8	49 (35, 100)	10.8	(h)
430.8	41	10	(i)
Atmospheric and oceanic excitation			
429.4	107	10	(i)
431.9	83	51	(i)
432.98	97	60	(j)
Semi-analytic			
430.3	88.4	20	(k)
433.03	100.20	20	(l)

The recommended estimate is given in bold. The 1σ confidence interval for the *Q* estimates is given in parentheses. Sources: (a) Jeffreys (1972); (b) Wilson and Haubrich (1976); (c) Ooe (1978); (d) Wilson and Vicente (1980); (e) Wilson and Vicente (1990); (f) Vicente and Wilson (1997); (g) Kuehne *et al.* (1996); (h) Furuya and Chao (1996); (i) Gross (2005b); (j) Seitz *et al.* (2012); (k) Mathews *et al.* (2002); (l) Chen and Shen (2010).

Gross (2014)

GFZ ECMWF
Preferred estimate:
T = 431.3 days
Q = 117

NCEP & ECCO/JPL
Preferred estimate:
T = 431.9 days
Q = 111

Summary

- Estimate period and Q of Chandler Wobble
  - From polar motion and polar motion rate observations
    - COMB2010 combination of astrometric, LLR, SLR, VLBI, & GPS observations
  - And models of atmospheric, oceanic, & hydrologic excitation
    - Consistent ECMWF AAM, OMCT OAM, and LSDM HAM from GFZ
    - NCEP/NCAR Reanalysis AAM and ECCO/JPL OAM
- Estimated values
  - Sensitive to accuracy of modeled excitation
    - For longest data span, best fit to observations in Chandler band with AAM+OAM
  - Sensitive to length of data analyzed
    - Most consistent estimates obtained using longest data spans

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