The Time Transfer Function as a tool to compute range, Doppler and astrometric observables

A. Hees - Rhodes University, South Africa
S. Bertone - University of Bern, Switzerland
C. Le Poncin-Lafitte and P. Teyssandier - LNE-SYRTE, Paris Obs.

RHODES UNIVERSITY
Where leaders learn

JSR14, Pulkovo Observatory
September 22, 2014

l'Observatoire de Paris SYRTE
Systèmes de Référence Temps-Espace

Light propagation is crucial in the modelling of Sol. Sys. observations

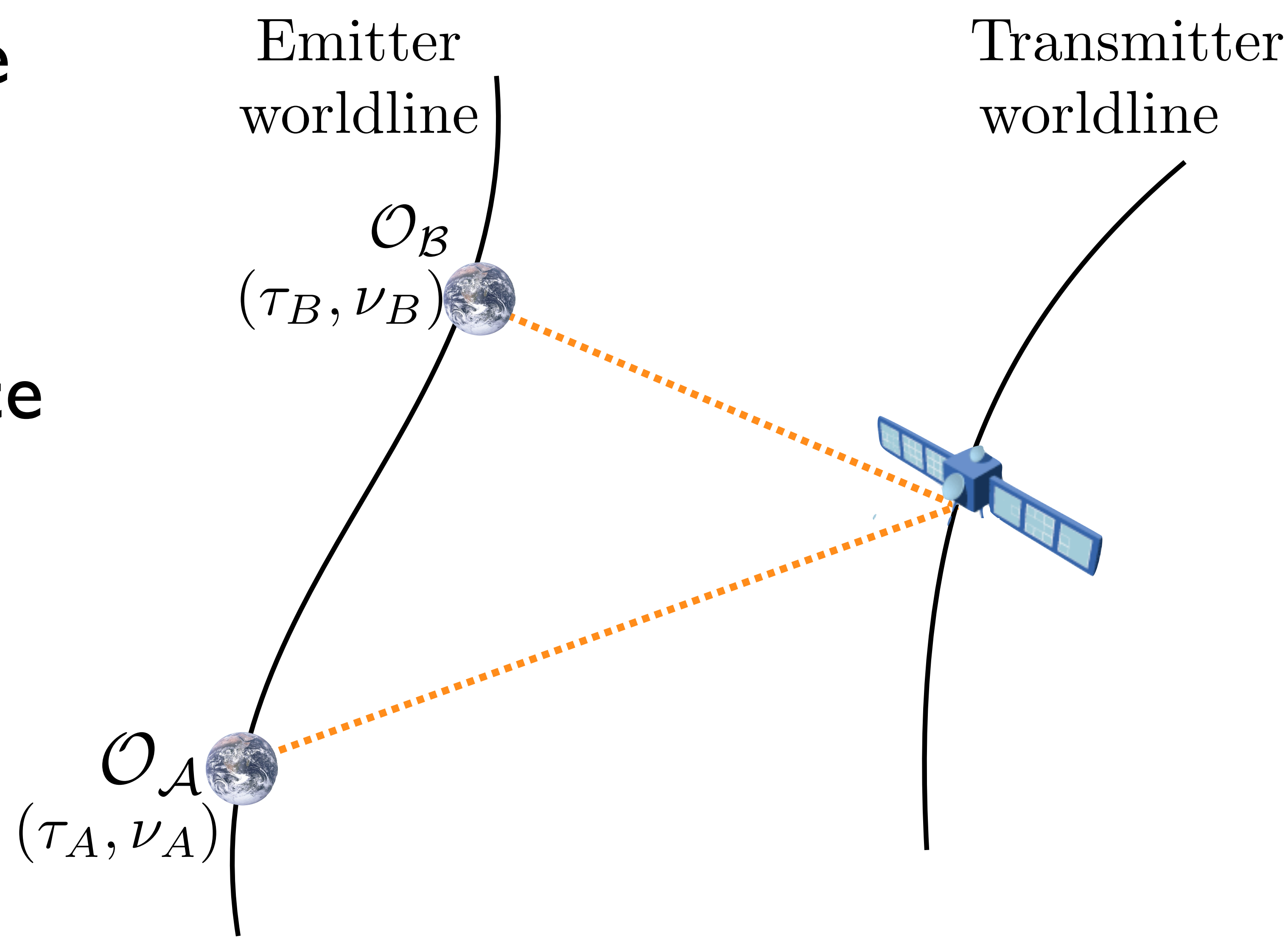
1) Range observable

- Difference in proper time

$$\text{Range} = c(\tau_B - \tau_A)$$

- Depends on the difference in coord. time (amongst other parameters)

$$t_B - t_A$$



Light propagation is crucial in the modelling of Sol. Sys. observations

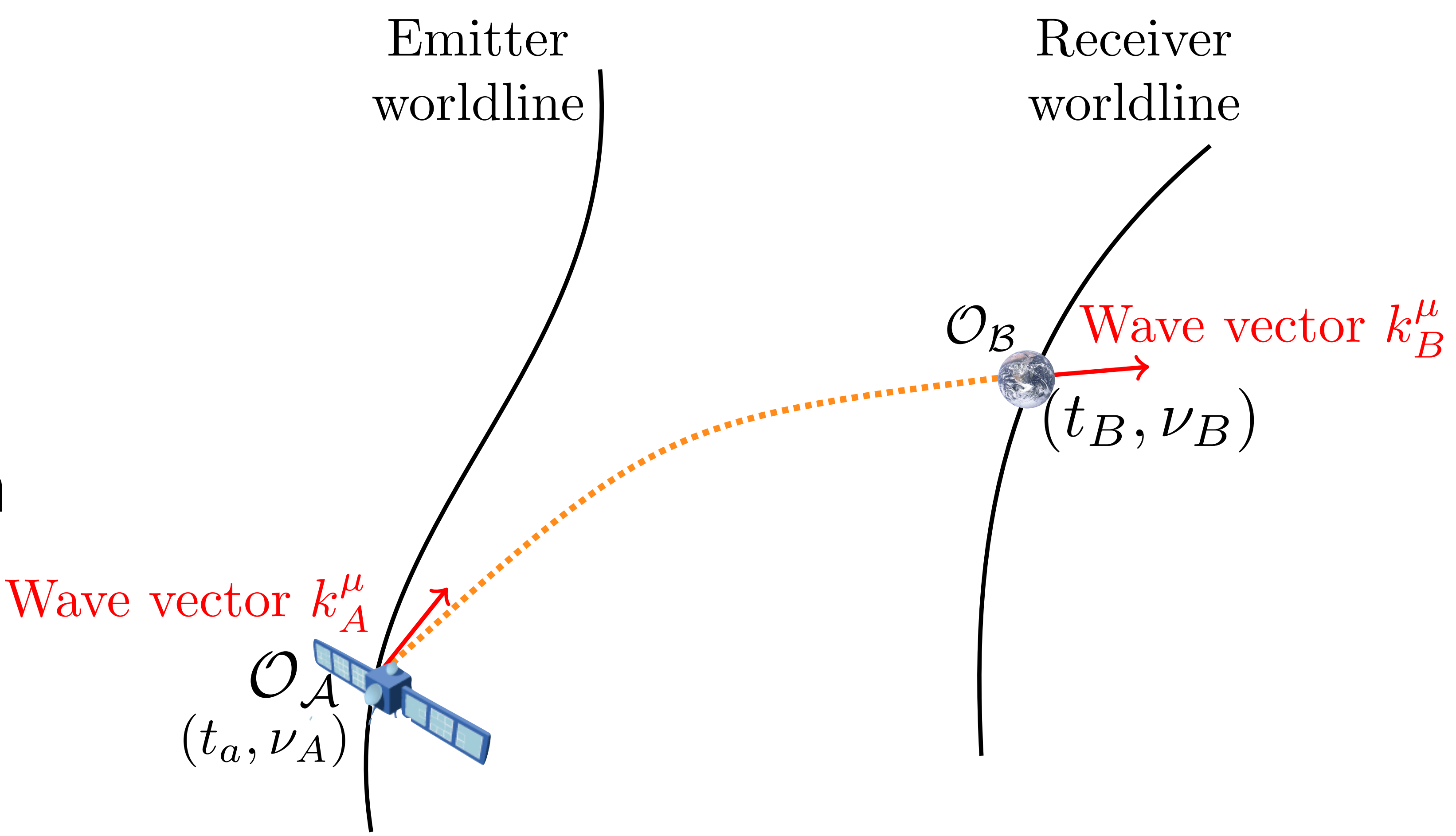
2) Doppler observable

- Ratio of proper frequency $D = \frac{\nu_B}{\nu_A} = \left(\frac{d\tau}{dt}\right)_A \left(\frac{d\tau}{dt}\right)_B^{-1} \frac{k_0^B}{k_0^A} \frac{1 + \beta^i_B \hat{k}_i^B}{1 + \beta^i_A \hat{k}_i^A}$

  with $\beta^i = v^i/c$ and

  $$\hat{k}_i = \frac{k_i}{k_0}$$

- Wave vector at emission and reception needed



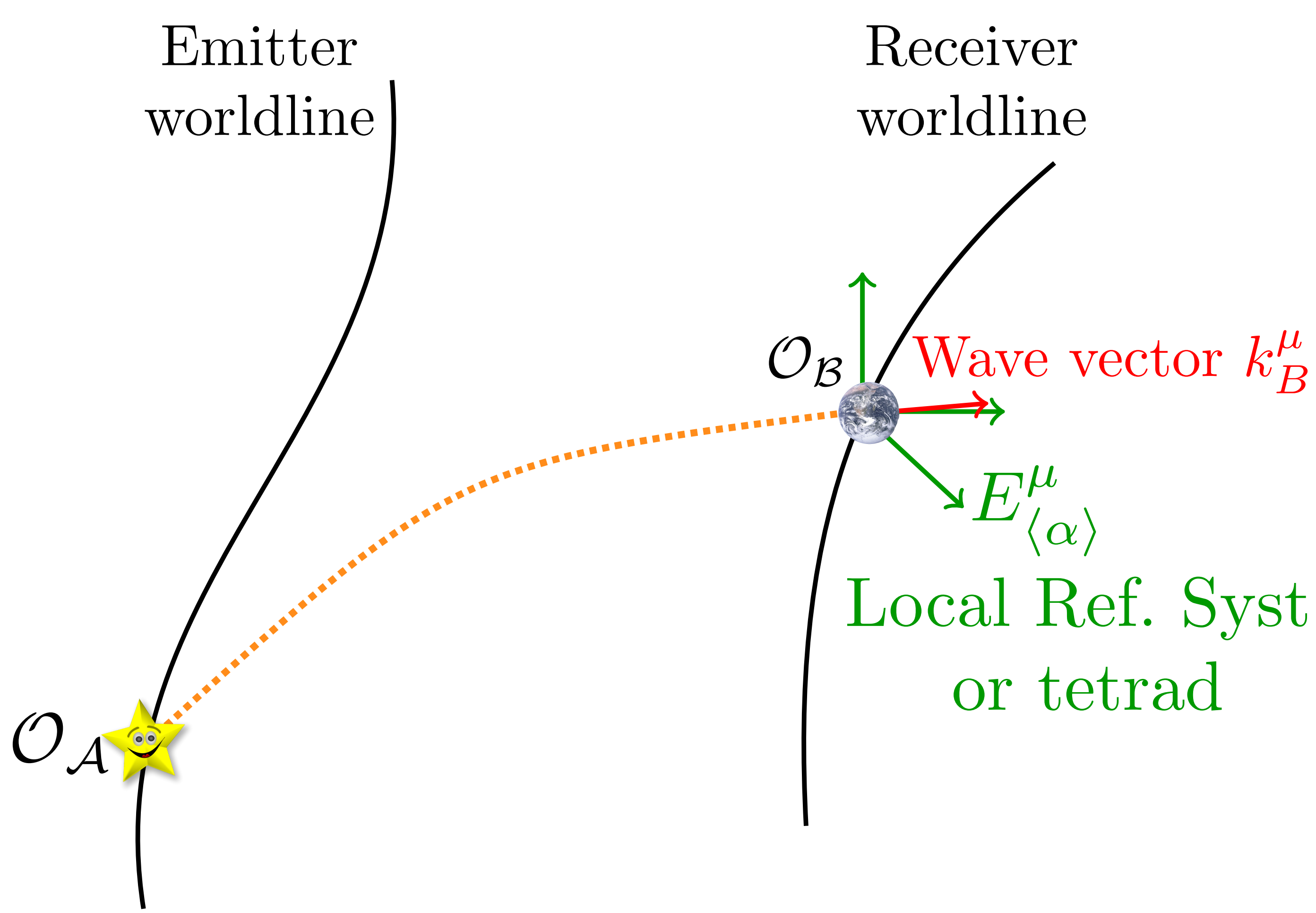
Light propagation is crucial in the modelling of Sol. Sys. observations

3) Astrometric observable & VLBI

- Direction of observation of the light ray in a local reference system (or tetrad)

$$n^{\langle i\rangle} = -\frac{E^0_{\langle i\rangle} + E^j_{\langle i\rangle}\hat{k}^B_j}{E^0_{\langle 0\rangle} + E^j_{\langle 0\rangle}\hat{k}^B_j}$$

- Wave vector at reception needed

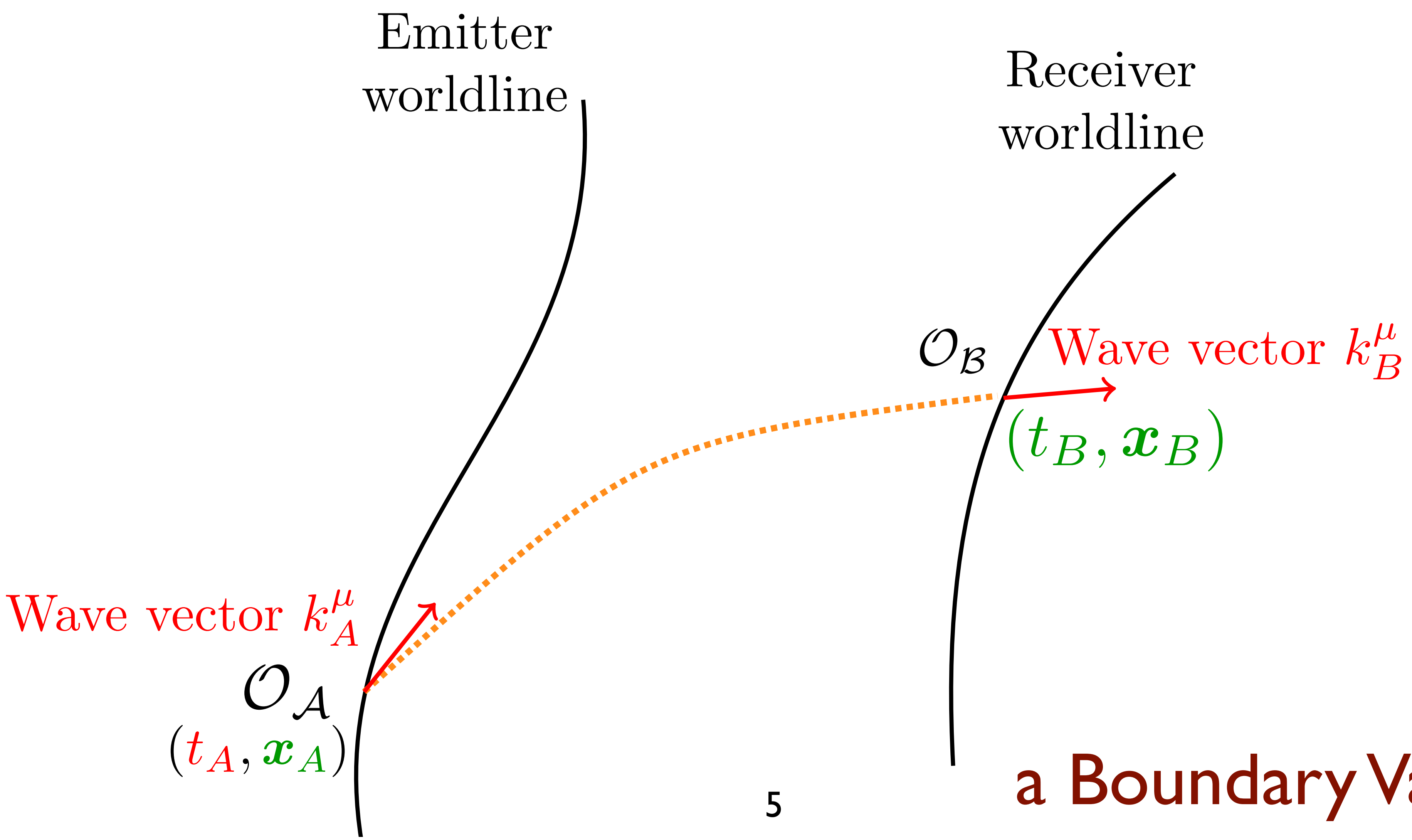


How to determine the light propagation ?

- At the geometric optics approximation: photons follow null geodesics

$$\frac{dk^\mu}{d\lambda} + \Gamma^\mu_{\alpha\beta} k^\alpha k^\beta = 0 \qquad\qquad k^\mu k_\mu = 0$$

with $k^\mu = \dfrac{dx^\mu}{d\lambda}$ the tangent vector



a Boundary Value Problem

Methods to solve the null geodesic eqs.

- Full numerical integration of the null geodesic eqs. with a shooting method

see A. San Miguel, Gen. Rel. Grav. 39, 2025, 2007

Methods to solve the null geodesic eqs.

- Full numerical integration of the null geodesic eqs. with a shooting method

see A. San Miguel, Gen. Rel. Grav. 39, 2025, 2007

- Exact analytical solution for some metrics: Schwarzschild and Kerr (solution with Jacobian/Weierstrass elliptic functions)

see for example: de Jans, Mem. de l'Ac. Roy. de Bel., 1922
B. Carter, Com. in Math. Phys. 10, 280, 1968

A. Cadez, U. Kostic, PRD 72, 104024, 2005
A. Cadez, et al, New Astr. 3, 647, 1998

Methods to solve the null geodesic eqs.

- Full numerical integration of the null geodesic eqs. with a shooting method
  see A. San Miguel, Gen. Rel. Grav. 39, 2025, 2007
- Exact analytical solution for some metrics: Schwarzschild and Kerr (solution with Jacobian/Weierstrass elliptic functions)

see for example: de Jans, Mem. de l'Ac. Roy. de Bel., 1922
B. Carter, Com. in Math. Phys. 10, 280, 1968
A. Cadez, U. Kostic, PRD 72, 104024, 2005
A. Cadez, et al, New Astr. 3, 647, 1998

- Analytical solutions for low gravitational field:
  - 1 pM Schwarzschild metric — see E. Shapiro, PRL 13, 26, 789, 1964
  - moving monopoles at 1pM order — see S. Kopeikin, G. Schäffer, PRD 60, 124002, 1999; S. Klioner, A & A, 404, 783, 2003
  - static extended bodies with multipolar expansion at 1pM — see S. Kopeikin, J. of Math. Phys., 38, 2587
  - 2 pM Schwarzschild metric — see G. Richter, R. Matzner, PRD 28, 3007, 1983; S. Klioner, S. Zschocke, CQG 27, 075015, 2010

Methods to solve the null geodesic eqs.

- Full numerical integration of the null geodesic eqs. with a shooting method

see A. San Miguel, Gen. Rel. Grav. 39, 2025, 2007

- Exact analytical solution for some metrics: Schwarzschild and Kerr (solution with Jacobian/Weierstrass elliptic functions)

see for example: de Jans, Mem. de l'Ac. Roy. de Bel., 1922
B. Carter, Com. in Math. Phys. 10, 280, 1968
A. Cadez, U. Kostic, PRD 72, 104024, 2005
A. Cadez, et al, New Astr. 3, 647, 1998

- Analytical solutions for low gravitational field:
  - 1 pM Schwarzschild metric — see E. Shapiro, PRL 13, 26, 789, 1964
  - moving monopoles at 1pM order — see S. Kopeikin, G. Schäffer, PRD 60, 124002, 1999; S. Klioner, A & A, 404, 783, 2003
  - static extended bodies with multipolar expansion at 1pM — see S. Kopeikin, J. of Math. Phys., 38, 2587
  - 2 pM Schwarzschild metric — see G. Richter, R. Matzner, PRD 28, 3007, 1983; S. Klioner, S. Zschocke, CQG 27, 075015, 2010
- Use of the eikonal equation:
  - perturbative solution for spherically symmetric space-time

see for example N. Ashby, B. Bertotti, CQG 27, 145013, 2010

… and the Time Transfer Functions

see C. Le Poncin-Lafitte, et al, CQG 21, 4463, 2004
P. Teyssandier and C. Le Poncin-Lafitte, CQG 25, 145020, 2008

- The (reception) Time Transfer Function - TTF - is defined by

$$t_B - t_A = \mathcal{T}_r(\boldsymbol{x}_A, t_B, \boldsymbol{x}_B)$$

- The TTF is solution of an eikonal equation well adapted to a perturbative expansion

… and the Time Transfer Functions

see C. Le Poncin-Lafitte, et al, CQG 21, 4463, 2004
P. Teyssandier and C. Le Poncin-Lafitte, CQG 25, 145020, 2008

- The (reception) Time Transfer Function - TTF - is defined by

$$t_B - t_A = \mathcal{T}_r(\boldsymbol{x}_A, t_B, \boldsymbol{x}_B)$$

- The TTF is solution of an eikonal equation well adapted to a perturbative expansion
- The derivatives of the TTF are of crucial interest since

$$\hat{k}_i^A = c\frac{\partial \mathcal{T}_r}{\partial x_A^i} \qquad \hat{k}_i^B = -c\frac{\partial \mathcal{T}_r}{\partial x_B^i}\left[1 - \frac{\partial \mathcal{T}_r}{\partial t_B}\right]^{-1} \qquad \frac{k_0^B}{k_0^A} = 1 - \frac{\partial \mathcal{T}_r}{\partial t_B}$$

Range, Doppler, astrometric observables can be written in terms of the TTF and its derivatives

Post-Minkowskian expansion of the TTF

see P. Teyssandier and C. Le Poncin-Lafitte, CQG 25, 145020, 2008

- A pM expansion of the TTF: $\mathcal{T}_r(\boldsymbol{x}_A, t_B, \boldsymbol{x}_B) = \dfrac{R_{AB}}{c} + \sum_{n>1} \mathcal{T}_r^{(n)}$
- Computation with an iterative procedure involving integrations over a straight line between the emitter and the spatial position of the receiver !
- Example at 1 pM: $\mathcal{T}_r^{(1)} = \dfrac{R_{AB}}{2c} \displaystyle\int_0^1 \left[g_{(1)}^{00} - 2N_{AB}^i g_{(1)}^{0i} + N_{AB}^i N_{AB}^j g_{(1)}^{ij} \right]_{z^\alpha(\lambda)} d\lambda$

  with $z^\alpha(\lambda)$ the straight Mink. null path between em. and rec.
- Main advantages:

  - analytical computations relatively easy

  - very well adapted to numerical evaluation

Analytical results in Schwarzschild space-time

see B. Linet and P. Teyssandier, CQG 30, 175008, 2014
P. Teyssandier, 2014, arXiv: 1407.4361

- A "simplified" iterative method has been developed for static spherically symmetric geometry

$$ds^2 = \left(1 - 2\frac{m}{r} + 2\beta\frac{m^2}{r^2} - \frac{3}{2}\beta_3\frac{m^3}{r^3} + \ldots\right)dt^2 - \left(1 + 2\gamma\frac{m}{r} + \frac{3}{2}\epsilon\frac{m^2}{r^2} + \frac{1}{2}\gamma_3\frac{m^3}{r^3} + \ldots\right)d\boldsymbol{x}^2$$

- In GR: $\gamma = \beta = \epsilon = \beta_3 = \gamma_3 = 1$

- A pM expansion of the TTF: $\mathcal{T} = \dfrac{R_{AB}}{c} + \displaystyle\sum_{n>1}\mathcal{T}^{(n)}$

and the corresponding derivatives have been computed up to the 3rd pM order

Analytical results in Schwarzschild space-time

- A pM expansion of the TTF: $\mathcal{T} = \dfrac{R_{AB}}{c} + \sum_{n>1} \mathcal{T}^{(n)}$

$$\mathcal{T}^{(1)} = \frac{(1+\gamma)m}{c} \ln \frac{r_A + r_B + |\boldsymbol{x}_B - \boldsymbol{x}_A|}{r_A + r_B - |\boldsymbol{x}_B - \boldsymbol{x}_A|}$$

see E. Shapiro, PRL 13, 26, 789, 1964

$$\mathcal{T}^{(2)} = \frac{m^2}{r_A r_B} \frac{|\boldsymbol{x}_B - \boldsymbol{x}_A|}{c} \left[\kappa \frac{\arccos \boldsymbol{n}_A.\boldsymbol{n}_B}{|\boldsymbol{n}_A \times \boldsymbol{n}_B|} - \frac{(1+\gamma)^2}{1 + \boldsymbol{n}_A.\boldsymbol{n}_B} \right]$$

see C. Le Poncin-Lafitte, et al, CQG 21, 4463, 2004
S. Klioner, S. Zschocke, CQG 27, 075015, 2010

$$\mathcal{T}^{(3)} = \frac{m^3}{r_A r_B} \left(\frac{1}{r_A} + \frac{1}{r_B} \right) \frac{|\boldsymbol{x}_B - \boldsymbol{x}_A|}{c(1 + \boldsymbol{n}_A.\boldsymbol{n}_B)} \left[\kappa_3 - (1+\gamma)\kappa \frac{\arccos \boldsymbol{n}_A.\boldsymbol{n}_B}{|\boldsymbol{n}_A \times \boldsymbol{n}_B|} + \frac{(1+\gamma)^3}{1 + \boldsymbol{n}_A.\boldsymbol{n}_B} \right]$$

see B. Linet and P. Teyssandier, CQG 30, 175008, 2014

with $\kappa = 2 + 2\gamma - \beta + \dfrac{3}{4}\epsilon$

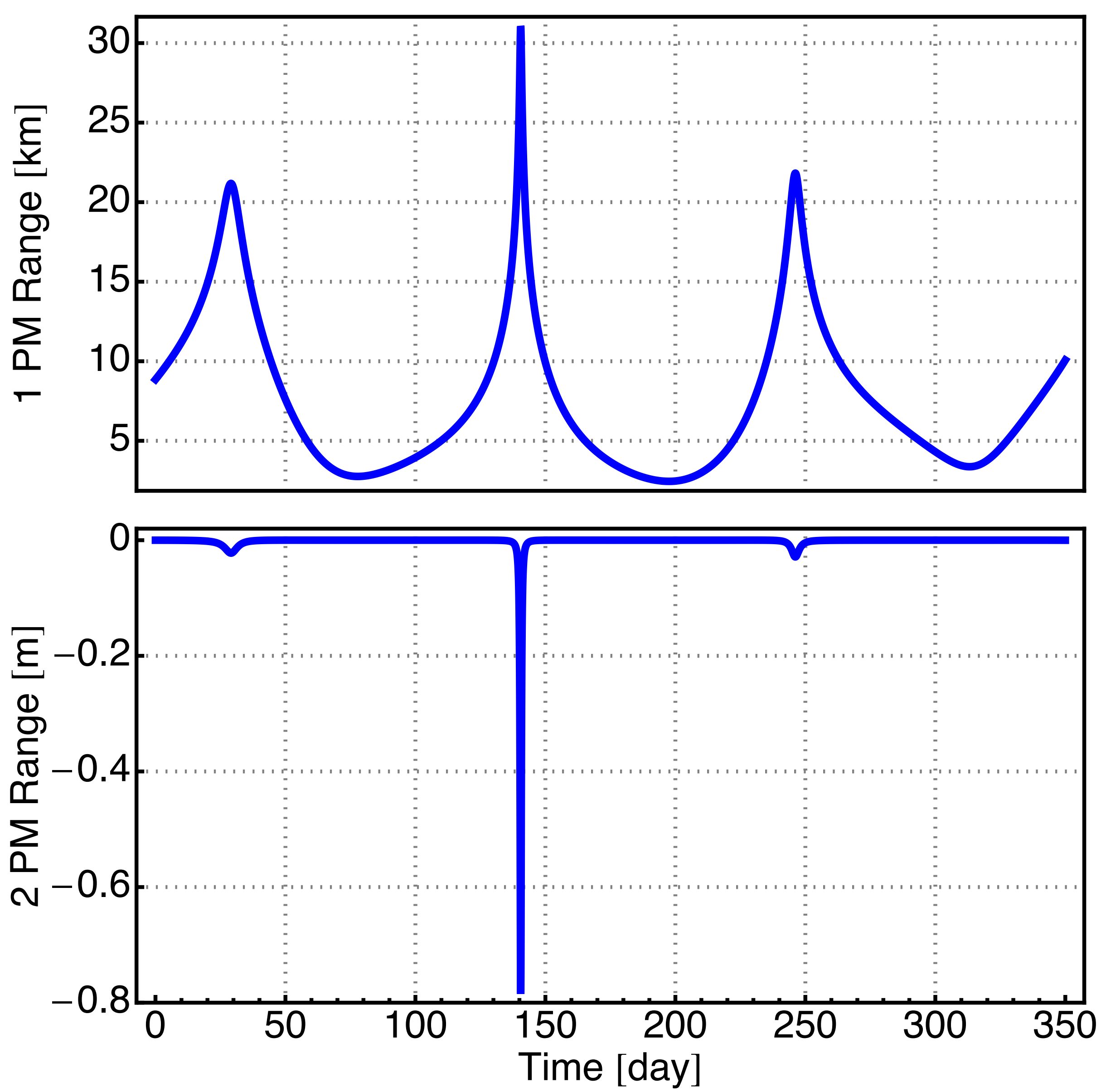
$\kappa_3 = 2\kappa - 2\beta(1+\gamma) + \dfrac{1}{4}(3\beta_3 + \gamma_3)$

and $\boldsymbol{n}_{A/B} = \dfrac{\boldsymbol{x}_{A/B}}{r_{A/B}}$

Is it necessary to go to the 3rd order?

- In a conjunction geometry, at each order n, there are enhanced terms proportional to $(1+\gamma)^n$

- Ex. with Earth-BepiColombo range (accuracy ~ 10 cm) $\Rightarrow$ 2pM term needed



- Ex. with SAGAS: link between spacecraft in the outer Solar System to measure γ at 10^{-8} $\Rightarrow$ accuracy at the mm level

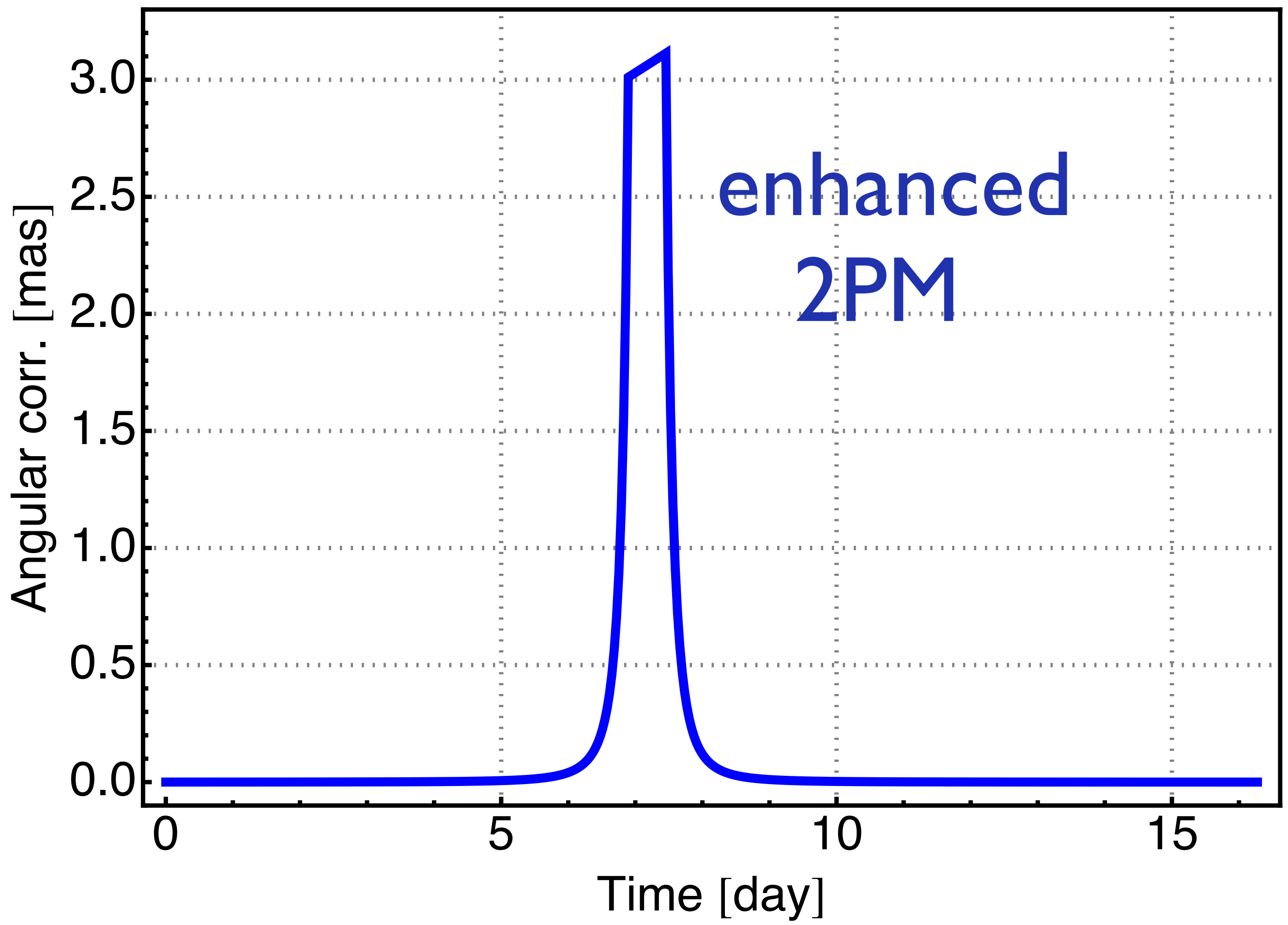
  $\Rightarrow$ 3pM term needed

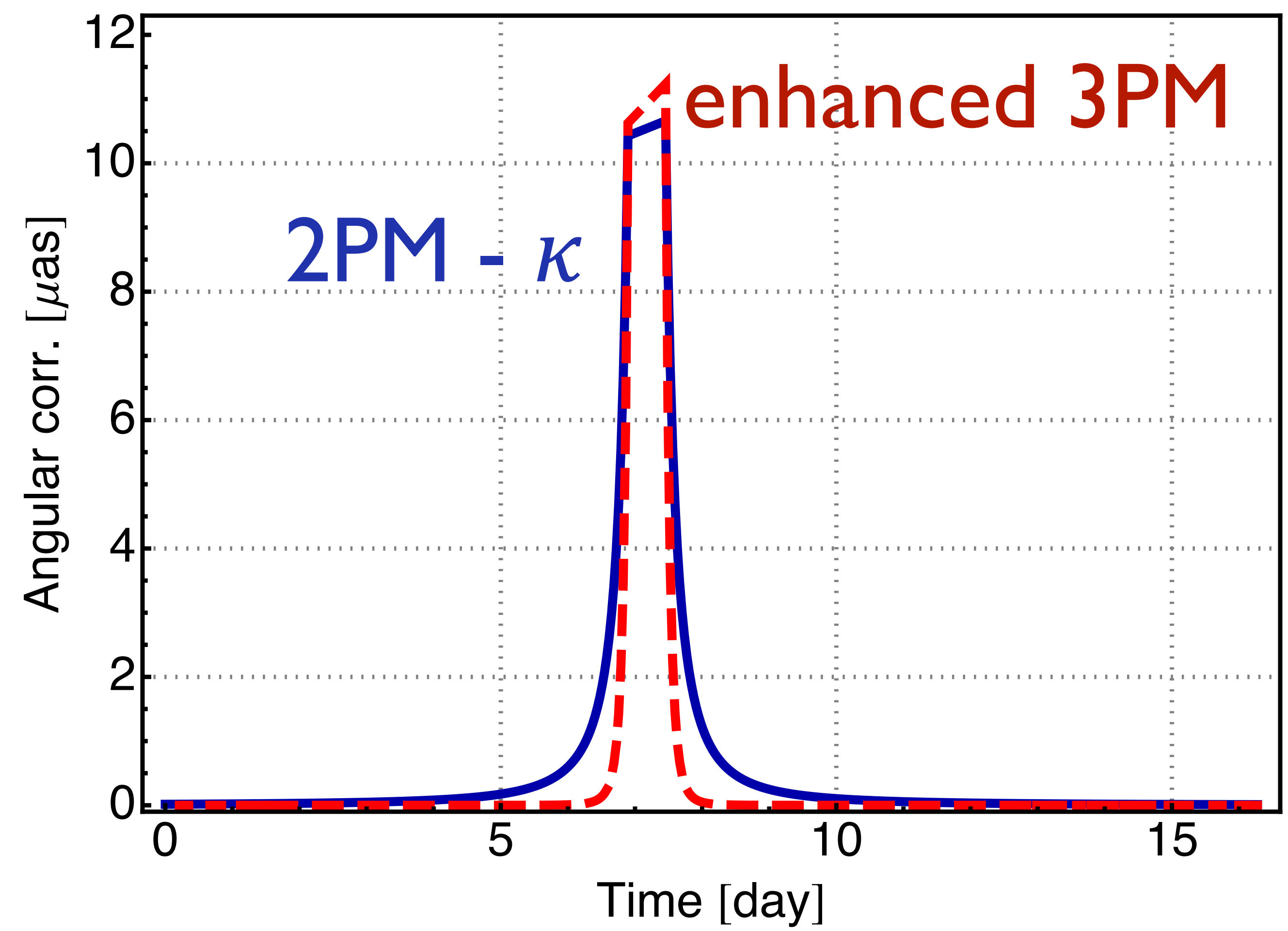
$r_c/R_\odot$	$\mathcal{T}^{(2)}_{\rm enh}$	$\mathcal{T}^{(2)}_{\kappa}$	$\mathcal{T}^{(3)}_{\rm enh}$
1	-5 m	37 cm	10 cm
2	-1.3 m	18 cm	0.6 mm
5	-21 cm	7 mm	15 μm

see P. Teyssandier, 2014, arXiv: 1407.4361
A. Hees, S. Bertone, C. Le Poncin-Lafitte, PRD 89, 064045, 2014

Is it necessary to go to the 3rd order?

- In a conjunction geometry, at each order n, there are enhanced terms proportional to $(1+\gamma)^n$
- Ex. with light deflection for Sun grazing rays: AGP space mission (old GAME). Expected accuracy: μas
  $\Rightarrow$ 3pM term needed





see A. Hees, S. Bertone, C. Le Poncin-Lafitte, PRD 89, 064045, 2014
P. Teyssandier, B. Linet, proceedings of JSR 2013, arXiv:1312.3510

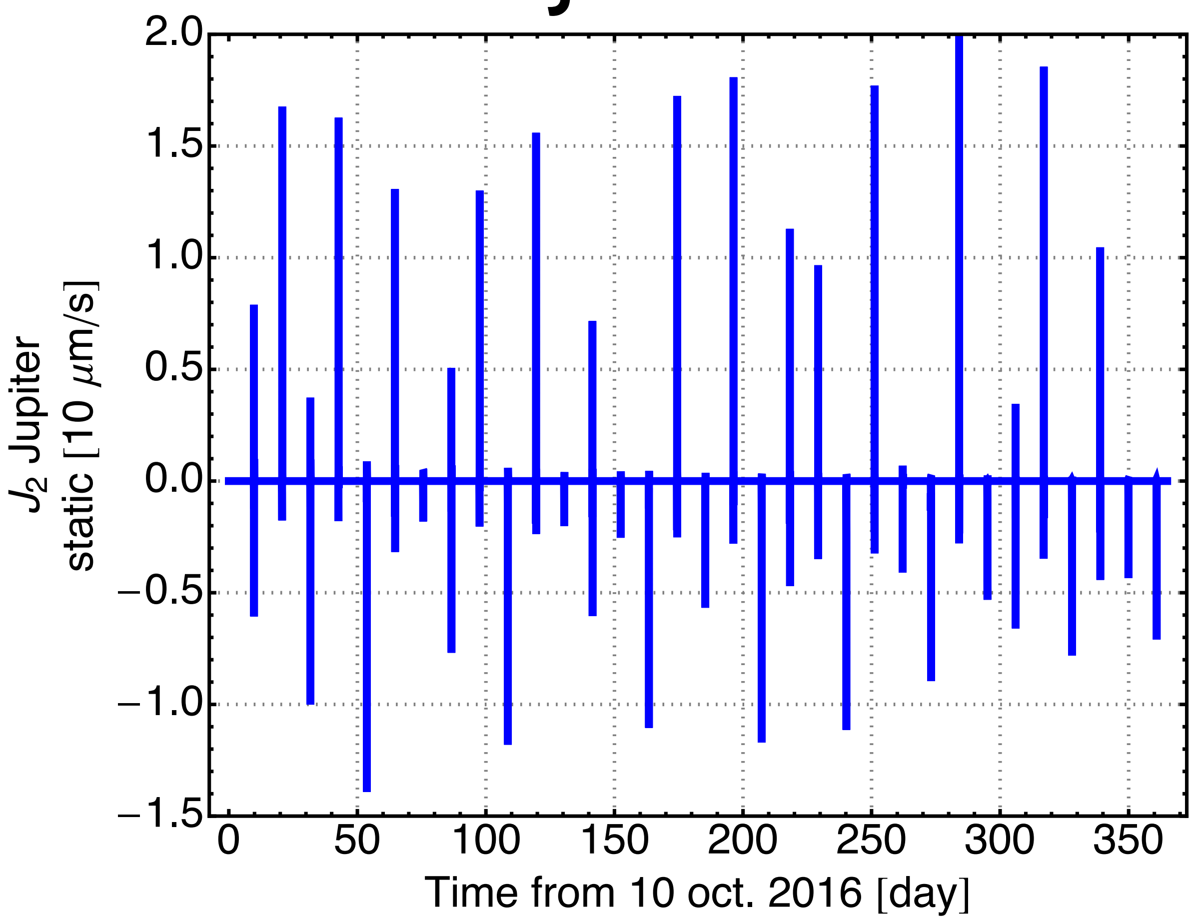
Analytical result around axisymmetric bodies

- Influence of all the multipole moments J_n from the grav. potential

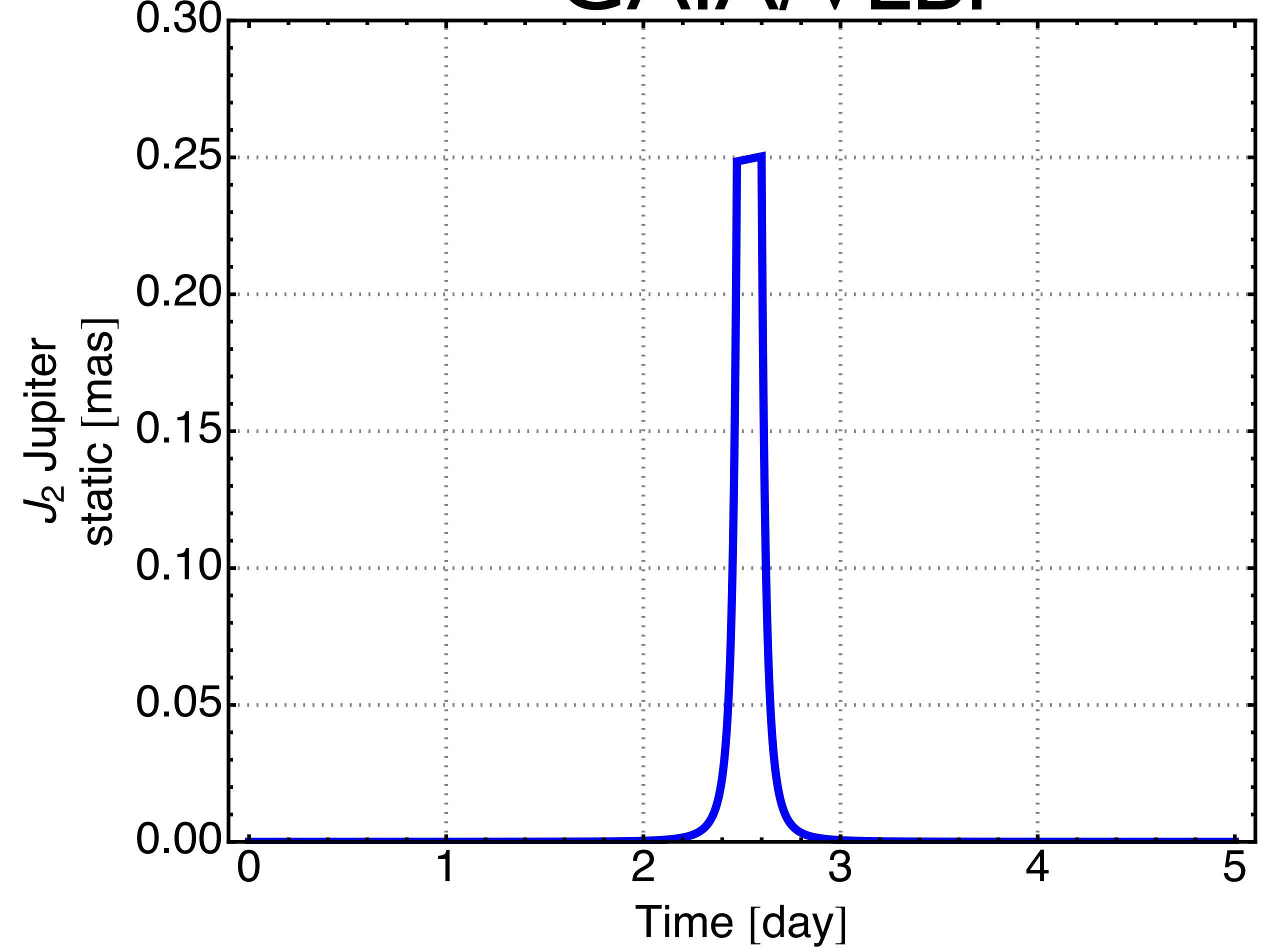
  see C. Le Poncin-Lafitte, P. Teyssandier, PRD 77, 044029, 2008 for a computation with the TTF
  or S. Kopeikin, J. of Math. Physics 38, 2587, 1997 for another approach

- Influence of Jupiter J_2 on the JUNO Doppler (1 μm/s accuracy) and for GAIA (10 μas acc.)

JUNO



GAIA/VLBI



- terms important for the data analysis for these missions

see A. Hees, et al, accepted in PRD, 2014, arXiv:1406.6600

What happens if the body is moving ?

- At first pM order, the TTF for uniformly moving bodies can easily be derived from the TTF generated by a static body

$$\Delta(\boldsymbol{x}_A, t_B, \boldsymbol{x}_B) = \gamma(1 - \boldsymbol{N}_{AB}.\boldsymbol{\beta})\tilde{\Delta}(\boldsymbol{R}_A + \gamma\boldsymbol{\beta}R_{AB}, \boldsymbol{R}_B)$$

TTF in the moving case

static TTF

with $\boldsymbol{\beta} = \boldsymbol{v}/c, \quad \gamma = (1-\beta^2)^{-1/2}$

and $\boldsymbol{R}_X$ depends on $\boldsymbol{x}_X, \boldsymbol{\beta}$

- All the analytical results computed for a static source can be extended in the case of a uniformly moving source

see A. Hees, et al, accepted in PRD, 2014, arXiv:1406.6600

Time Transfer around a moving body

- moving monopole:
  - using the previous result:

$$\Delta_M(\boldsymbol{x}_A, t_B, \boldsymbol{x}_B) = 2\frac{GM_p}{c^2}\gamma_p\,(1-\boldsymbol{N}_{AB}\cdot\boldsymbol{\beta}_p)\ln\frac{|\boldsymbol{R}_{pA}+\gamma_p\boldsymbol{\beta}_p R_{AB}|+R_{pB}+\gamma_p R_{AB}(1-\boldsymbol{\beta}_p\cdot\boldsymbol{N}_{AB})}{|\boldsymbol{R}_{pA}+\gamma_p\boldsymbol{\beta}_p R_{AB}|+R_{pB}-\gamma_p R_{AB}(1-\boldsymbol{\beta}_p\cdot\boldsymbol{N}_{AB})}$$

see A. Hees, et al, accepted in PRD, 2014, arXiv:1406.6600
S. Bertone et al, CQG 31, 015021, 2014 for a pN expansion

- also determined by other methods

see S. Kopeikin, G. Schäffer, PRD 60, 124002, 1999
S. Klioner, A & A, 404, 783, 2003

Time Transfer around a moving body

- moving monopole:
  - using the previous result:

$$\Delta_M(\boldsymbol{x}_A, t_B, \boldsymbol{x}_B) = 2\frac{GM_p}{c^2}\gamma_p\,(1-\boldsymbol{N}_{AB}\cdot\boldsymbol{\beta}_p)\ln\frac{|\boldsymbol{R}_{pA}+\gamma_p\boldsymbol{\beta}_p R_{AB}|+R_{pB}+\gamma_p R_{AB}(1-\boldsymbol{\beta}_p\cdot\boldsymbol{N}_{AB})}{|\boldsymbol{R}_{pA}+\gamma_p\boldsymbol{\beta}_p R_{AB}|+R_{pB}-\gamma_p R_{AB}(1-\boldsymbol{\beta}_p\cdot\boldsymbol{N}_{AB})}$$

see A. Hees, et al, accepted in PRD, 2014, arXiv:1406.6600
S. Bertone et al, CQG 31, 015021, 2014 for a pN expansion

  - also determined by other methods see S. Kopeikin, G. Schäffer, PRD 60, 124002, 1999
S. Klioner, A & A, 404, 783, 2003

- moving quadrupole: - using the TTF see A. Hees, et al, acc. in PRD, 2014, arXiv:1406.6600
  - with another method

see S. Kopeikin, V. Makarov, PRD, 75, 062002, 2007

Time Transfer around a moving body

- moving monopole:

  - using the previous result:

$$\Delta_M(\boldsymbol{x}_A, t_B, \boldsymbol{x}_B) = 2\frac{GM_p}{c^2}\gamma_p\left(1 - \boldsymbol{N}_{AB}\cdot\boldsymbol{\beta}_p\right)\ln\frac{|\boldsymbol{R}_{pA} + \gamma_p\boldsymbol{\beta}_p R_{AB}| + R_{pB} + \gamma_p R_{AB}(1 - \boldsymbol{\beta}_p\cdot\boldsymbol{N}_{AB})}{|\boldsymbol{R}_{pA} + \gamma_p\boldsymbol{\beta}_p R_{AB}| + R_{pB} - \gamma_p R_{AB}(1 - \boldsymbol{\beta}_p\cdot\boldsymbol{N}_{AB})}$$

see A. Hees, et al, accepted in PRD, 2014, arXiv:1406.6600
S. Bertone et al, CQG 31, 015021, 2014 for a pN expansion

  - also determined by other methods

see S. Kopeikin, G. Schäffer, PRD 60, 124002, 1999
S. Klioner, A & A, 404, 783, 2003

- moving quadrupole: - using the TTF see A. Hees, et al, acc. in PRD, 2014, arXiv:1406.6600
  - with another method

see S. Kopeikin, V. Makarov, PRD, 75, 062002, 2007

- moving axisymmetric bodies: see A. Hees, et al, acc. in PRD, 2014, arXiv:1406.6600

Time Transfer around a moving body

- moving monopole:
  - using the previous result:

$$\Delta_M(\boldsymbol{x}_A, t_B, \boldsymbol{x}_B) = 2\frac{GM_p}{c^2}\gamma_p\left(1 - \boldsymbol{N}_{AB}\cdot\boldsymbol{\beta}_p\right)\ln\frac{|\boldsymbol{R}_{pA} + \gamma_p\boldsymbol{\beta}_p R_{AB}| + R_{pB} + \gamma_p R_{AB}(1 - \boldsymbol{\beta}_p\cdot\boldsymbol{N}_{AB})}{|\boldsymbol{R}_{pA} + \gamma_p\boldsymbol{\beta}_p R_{AB}| + R_{pB} - \gamma_p R_{AB}(1 - \boldsymbol{\beta}_p\cdot\boldsymbol{N}_{AB})}$$

see A. Hees, et al, accepted in PRD, 2014, arXiv:1406.6600
S. Bertone et al, CQG 31, 015021, 2014 for a pN expansion

  - also determined by other methods see S. Kopeikin, G. Schäffer, PRD 60, 124002, 1999
S. Klioner, A & A, 404, 783, 2003

- moving quadrupole: - using the TTF see A. Hees, et al, acc. in PRD, 2014, arXiv:1406.6600
  - with another method

see S. Kopeikin, V. Makarov, PRD, 75, 062002, 2007

- moving axisymmetric bodies: see A. Hees, et al, acc. in PRD, 2014, arXiv:1406.6600

- moving body with arbitrary static multipoles: slow velocity app.

see M. Soffel, W.-B. Han, arXiv:1409.3743

Time Transfer around a moving body

- moving monopole:
  - using the previous result:

$$\Delta_M(\boldsymbol{x}_A, t_B, \boldsymbol{x}_B) = 2\frac{GM_p}{c^2}\gamma_p\left(1 - \boldsymbol{N}_{AB}\cdot\boldsymbol{\beta}_p\right)\ln\frac{|\boldsymbol{R}_{pA} + \gamma_p\boldsymbol{\beta}_p R_{AB}| + R_{pB} + \gamma_p R_{AB}(1 - \boldsymbol{\beta}_p\cdot\boldsymbol{N}_{AB})}{|\boldsymbol{R}_{pA} + \gamma_p\boldsymbol{\beta}_p R_{AB}| + R_{pB} - \gamma_p R_{AB}(1 - \boldsymbol{\beta}_p\cdot\boldsymbol{N}_{AB})}$$

see A. Hees, et al, accepted in PRD, 2014, arXiv:1406.6600
S. Bertone et al, CQG 31, 015021, 2014 for a pN expansion

  - also determined by other methods see S. Kopeikin, G. Schäffer, PRD 60, 124002, 1999
S. Klioner, A & A, 404, 783, 2003

- moving quadrupole: - using the TTF see A. Hees, et al, acc. in PRD, 2014, arXiv:1406.6600
  - with another method

see S. Kopeikin, V. Makarov, PRD, 75, 062002, 2007

- moving axisymmetric bodies: see A. Hees, et al, acc. in PRD, 2014, arXiv:1406.6600

- moving body with arbitrary static multipoles: slow velocity app.

see M. Soffel, W.-B. Han, arXiv:1409.3743

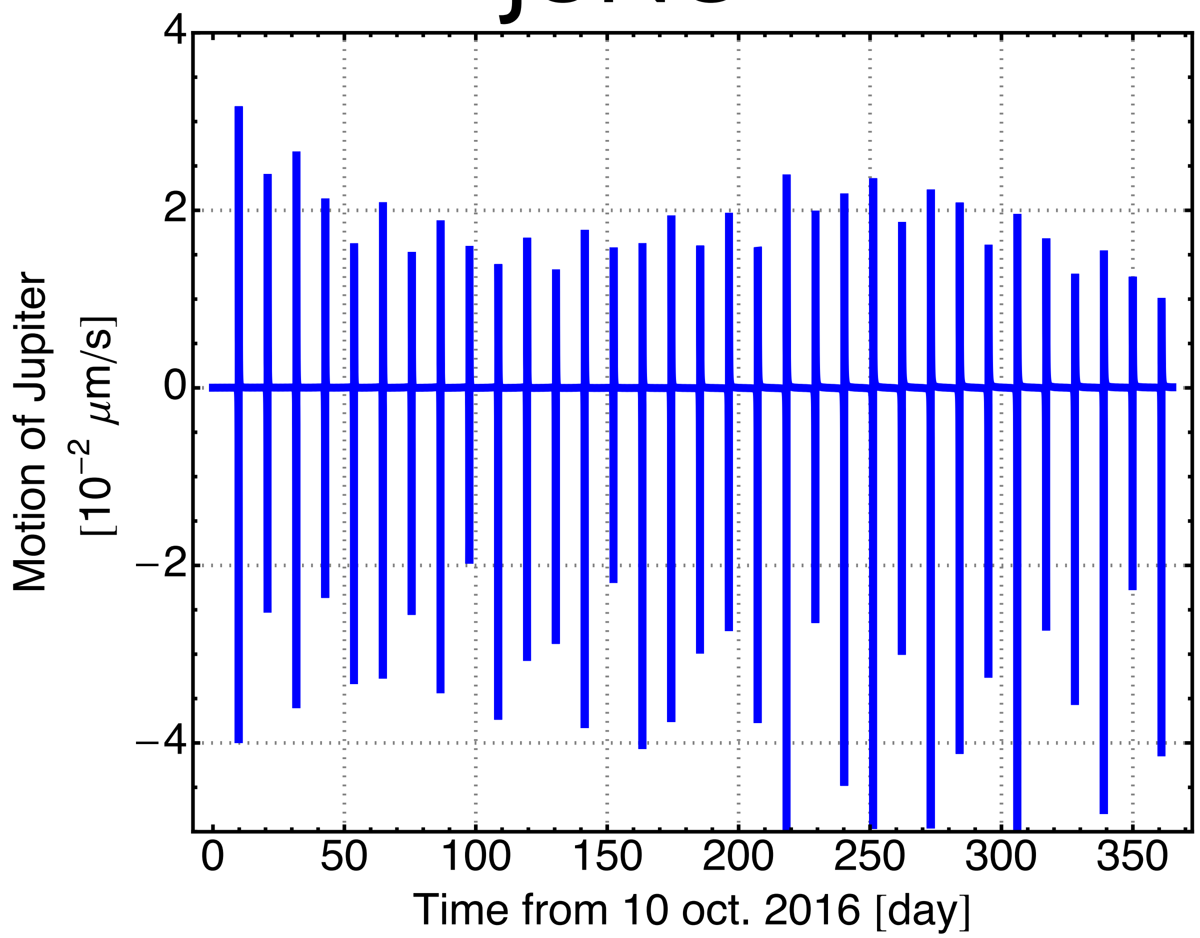
- arbitrarily moving point masses: numerical expression

see A. Hees, et al, acc. in PRD, 2014, arXiv:1406.6600

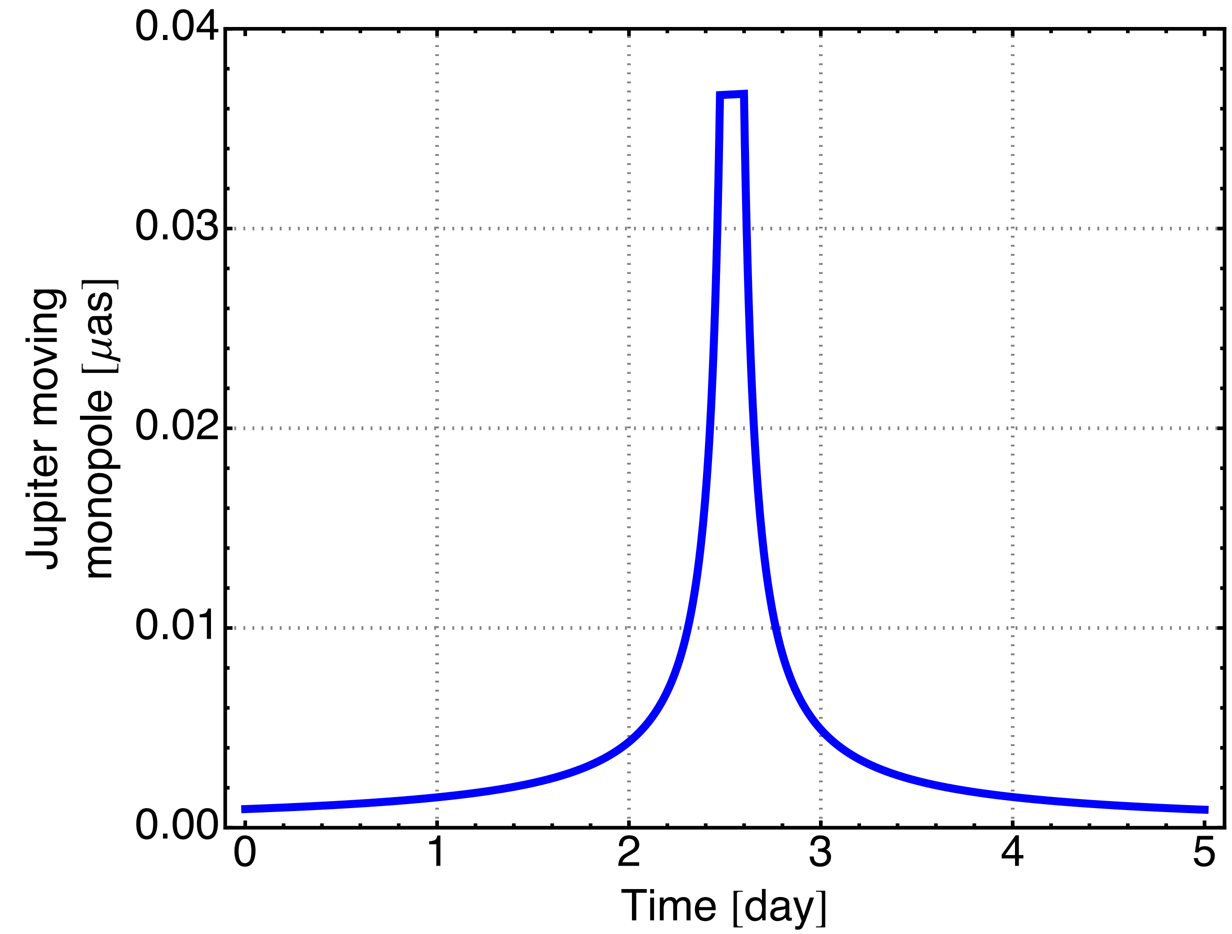
Ex.: motion of Jupiter

- Influence of Jupiter velocity on the JUNO Doppler (1μm/s accuracy) and for GAIA (10 μas acc.)

JUNO



GAIA/VLBI



- depend highly on the orbit geometry: conjunction and $\beta.N_{AB}$
- In particular: should be reassessed for JUICE orbit

see A. Hees, et al, accepted in PRD, 2014, arXiv:1406.6600

Numerical evaluation of the TTF

- Iterative procedure involving integrals over a straight line: appropriate for numerical evaluation

see A. Hees, et al, PRD 89, 064045, 2014

Numerical evaluation of the TTF

- Iterative procedure involving integrals over a straight line: appropriate for numerical evaluation
- At 1pM order: a simple integral to evaluate

$$\mathcal{T}^{(1)} = \int_0^1 m\left[z^\alpha(\mu);\ g^{(1)}_{\alpha\beta},\ \boldsymbol{x}_A, t_B, \boldsymbol{x}_B\right] d\mu$$

$$\frac{\partial \mathcal{T}^{(1)}}{\partial x^i_{A/B}} = \int_0^1 m_{A/B}\left[z^\alpha(\mu);\ g^{(1)}_{\alpha\beta},\ g^{(1)}_{\alpha\beta,\gamma},\ \boldsymbol{x}_A, t_B, \boldsymbol{x}_B\right] d\mu$$

see A. Hees, et al, PRD 89, 064045, 2014

Numerical evaluation of the TTF

- Iterative procedure involving integrals over a straight line: appropriate for numerical evaluation
- At 1pM order: a simple integral to evaluate

$$\mathcal{T}^{(1)} = \int_0^1 m\left[z^\alpha(\mu);\ g^{(1)}_{\alpha\beta},\ \boldsymbol{x}_A, t_B, \boldsymbol{x}_B\right] d\mu$$

$$\frac{\partial \mathcal{T}^{(1)}}{\partial x^i_{A/B}} = \int_0^1 m_{A/B}\left[z^\alpha(\mu);\ g^{(1)}_{\alpha\beta},\ g^{(1)}_{\alpha\beta,\gamma},\ \boldsymbol{x}_A, t_B, \boldsymbol{x}_B\right] d\mu$$

- At 2pM order: a double integral to evaluate

$$\mathcal{T}^{(2)} = \int_0^1 \int_0^1 n\left[z^\alpha(\mu\lambda);\ g^{(2)}_{\alpha\beta},\ g^{(1)}_{\alpha\beta},\ g^{(1)}_{\alpha\beta,\gamma},\ \boldsymbol{x}_A, t_B, \boldsymbol{x}_B\right] d\lambda d\mu$$

$$\frac{\partial \mathcal{T}^{(2)}}{\partial x^i_{A/B}} = \int_0^1 \int_0^1 n_{A/B}\left[z^\alpha(\mu\lambda);\ g^{(2)}_{\alpha\beta},\ g^{(2)}_{\alpha\beta,\gamma},\ g^{(1)}_{\alpha\beta},\ g^{(1)}_{\alpha\beta,\gamma},\ g^{(1)}_{\alpha\beta,\gamma\delta},\ \boldsymbol{x}_A, t_B, \boldsymbol{x}_B\right] d\lambda d\mu$$

see A. Hees, et al, PRD 89, 064045, 2014

Numerical evaluation of the TTF

- Iterative procedure involving integrals over a straight line: appropriate for numerical evaluation
- At 1pM order: a simple integral to evaluate

$$\mathcal{T}^{(1)} = \int_0^1 m\left[z^\alpha(\mu);\ g^{(1)}_{\alpha\beta},\ \boldsymbol{x}_A, t_B, \boldsymbol{x}_B\right] d\mu$$

$$\frac{\partial \mathcal{T}^{(1)}}{\partial x^i_{A/B}} = \int_0^1 m_{A/B}\left[z^\alpha(\mu);\ g^{(1)}_{\alpha\beta},\ g^{(1)}_{\alpha\beta,\gamma},\ \boldsymbol{x}_A, t_B, \boldsymbol{x}_B\right] d\mu$$

- At 2pM order: a double integral to evaluate

$$\mathcal{T}^{(2)} = \int_0^1 \int_0^1 n\left[z^\alpha(\mu\lambda);\ g^{(2)}_{\alpha\beta},\ g^{(1)}_{\alpha\beta},\ g^{(1)}_{\alpha\beta,\gamma},\ \boldsymbol{x}_A, t_B, \boldsymbol{x}_B\right] d\lambda d\mu$$

$$\frac{\partial \mathcal{T}^{(2)}}{\partial x^i_{A/B}} = \int_0^1 \int_0^1 n_{A/B}\left[z^\alpha(\mu\lambda);\ g^{(2)}_{\alpha\beta},\ g^{(2)}_{\alpha\beta,\gamma},\ g^{(1)}_{\alpha\beta},\ g^{(1)}_{\alpha\beta,\gamma},\ g^{(1)}_{\alpha\beta,\gamma\delta},\ \boldsymbol{x}_A, t_B, \boldsymbol{x}_B\right] d\lambda d\mu$$

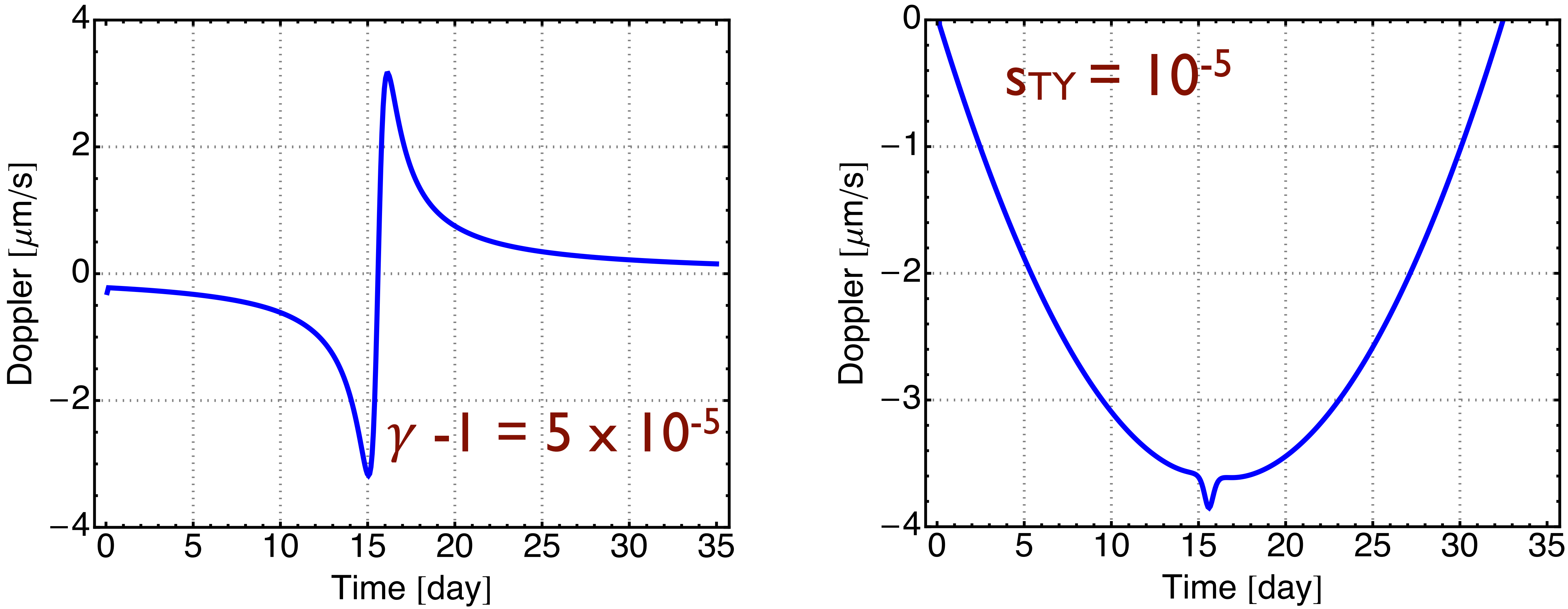
- Numerically efficient ; useful when no analytical solution can be found

see A. Hees, et al, PRD 89, 064045, 2014

Numerical evaluation of the TTF

- Numerical evaluation appropriate to evaluate effects due to alternative theories of gravitation
- Example: Doppler for 30 days of Cassini tracking between Jupiter and Saturn ("γ experiment")
- Effect of the γ PPN and of Standard Model Extension s_{TY} on Cassini Doppler

for SME, see Q. Bailey and A. Kostelecky, PRD 74, 045001, 2006



see A. Hees, et al, CQG 29, 235027, 2012

Conclusion

- The TTF is a very nice tool to compute the time transfer, the Doppler and astrometric (VLBI) observations
- Analytical results found (so far):
  - time transfer in Schwarzschild space-time at 1, 2, 3 pM order

    see B. Linet and P. Teyssandier, CQG 30, 175008, 2014
  - time transfer around static axisymmetric body

    see C. Le Poncin-Lafitte, P. Teyssandier, PRD 77, 044029, 2008
  - time transfer around a slowly moving monopole

    see S. Bertone et al, CQG 31, 015021, 2014
  - time transfer around uniformly moving axisymmetric body

    see A. Hees, et al, accepted in PRD, 2014, arXiv:1406.6600
- Very efficient from a numerical point of view

  see A. Hees, et al, PRD 89, 064045, 2014
- Useful to assess order of magnitude of different GR effects but also effects from alternative theories of gravitation

  see A. Hees, et al, CQG 29, 235027, 2012