Statistical inversion method for binary asteroids orbit determination

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Introduction

We focus on the study of binary asteroids, which are common in the Solar system from its inner to its outer regions. These objects provide fundamental physical parameters such as mass and density, and hence clues on the early Solar System, or other processes that are affecting asteroid over time. The present method of orbit computation for resolved binaries is based on Markov Chain Monte-Carlo statistical inversion technique. Particularly, we use the Metropolis-Hasting algorithm with Thiele-Innes equation for sampling the orbital elements and system mass through the sampling of observations. The method requires a minimum of four observations, made at the same tangent plane; it is of particular interest for orbit determination over short arcs or with sparse data. The observations are sampled within their observational errors with an assumed distribution. The sampling yields the whole region of possible orbits, including the one that is most probable.

Orbit determination

The astrometric observations are related to the theoretical positions through the observational equation:

$$\boldsymbol{\varphi} = \psi(\boldsymbol{X}) + \varepsilon \quad \rightarrow \quad p(X|\varphi) = \frac{p(\varphi|X)p(X)}{p(\varphi)} \quad \rightarrow$$

- Observations: $\boldsymbol{\varphi} = (\rho_1, \theta_1; \dots; \rho_N, \theta_N)$
- Sky-plane position: $\psi(\boldsymbol{X})$
- Orbital elements + system's mass: $\boldsymbol{X} = (a, e, i, \Omega, \omega, M, m_{sys})$
- Observational errors: $\varepsilon = (\varepsilon_{\alpha 1}, \varepsilon_{\delta 1}; \dots; \varepsilon_{\alpha N}, \varepsilon_{\delta N})$

$$p(X|\varphi) \propto p(\varphi|X)p(X)$$

Where

- $p(X) \propto \sqrt{det\Lambda^{-1}}$
- $p(\boldsymbol{\varphi}|\boldsymbol{X}) = p_\varepsilon(\text{observational error p.d.f.})$
  $= \exp[-\frac{1}{2}(\boldsymbol{\varphi} - \psi(\boldsymbol{X}))^T \Lambda^{-1}(\boldsymbol{\varphi} - \psi(\boldsymbol{X}))]$

Markov Chain Monte-Carlo method

The Metropolis–Hastings algorithm will be used for sampling the parameters $\boldsymbol{X}$.

N observations → $\boldsymbol{S} = (\rho_1, \theta_1; \rho_2, \theta_2; \rho_3, \theta_3; \rho_4)$ Select a random set of 4 observations

For each *t* iteration:

$\boldsymbol{S}'$- proposal set of observations
$\boldsymbol{S}^t$- last accepted set of observations

$\mathrm{p}_t(\boldsymbol{S}'; S^t)$- proposal density
$\alpha_i' \propto G(\alpha_i^t, \sigma(\alpha_i)),\ \delta_i' \propto G(\delta_i^t, \sigma(\delta_i))$

→ Thiele – Innes method [1]:

$$t_i - t_j - \frac{\Delta_{ij}}{c} = \frac{1}{\mu}[(E_i - E_j) - \sin(E_i - E_j)]$$
$$i, j = 1, 2, 3, 4$$
$$\Delta ij = \rho_i \rho_j \sin(\theta_j - \theta_i)$$

=> $\boldsymbol{X} = (a, e\ \ i, \omega, \Omega, M, T)$

→ Orbit $\boldsymbol{X}'$+ monitor the fit to all observations

$Acceptence\ criteria$

$$a = \frac{p(X'|\varphi)|J^t|}{p(X^t|\varphi)|J'|}$$

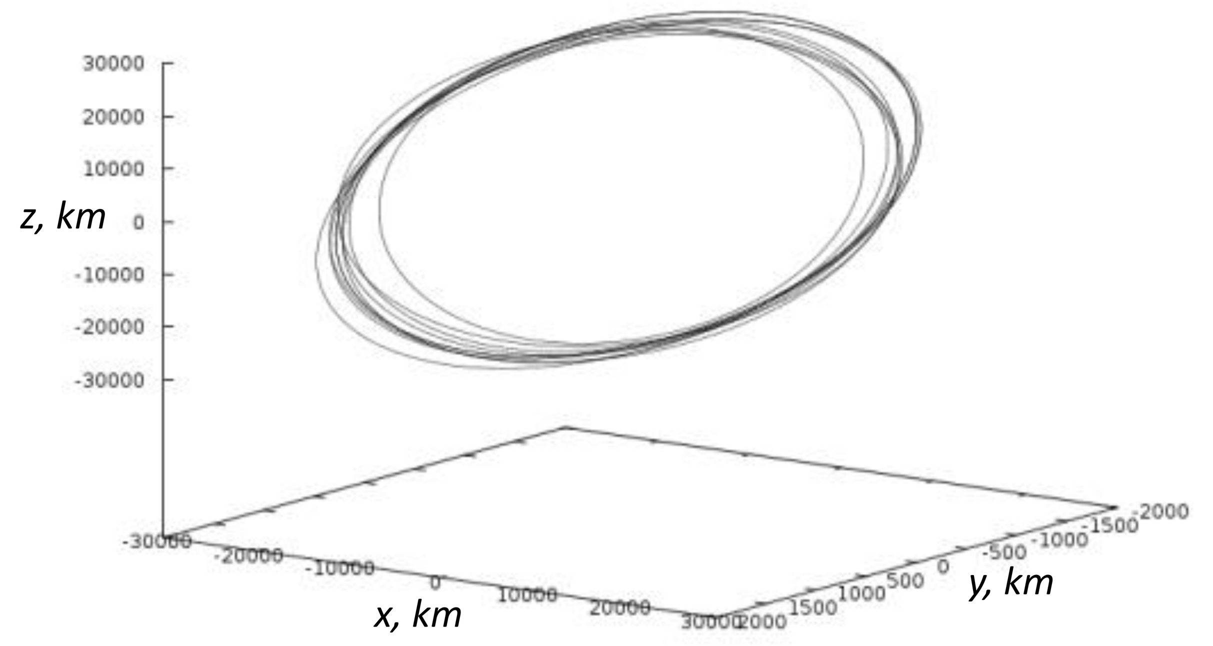
[2] J' and J^t - the determinants of Jacobians from coordinates to orbital parameters $J = \det\left|\frac{\partial s}{\partial x}\right|$

$If\ a \geq 1$: $\boldsymbol{X}^{t+1} = \boldsymbol{X}'$

$If\ a < 1$: $\boldsymbol{X}^{t+1} = \boldsymbol{X}'$, with probability a
or $\boldsymbol{X}^{t+1} = \boldsymbol{X}^t$,with probability $1 - a$

This process is repeated until the stationary a posteriori density is reached.

The algorithm is run for a large number of iterations until the entire possible orbital-element space is mapped



References
1) R. Palacios 1958 AJ 63, 395 2) D. Oszkiewicz et al. 2013 ,SF2A 237

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