Long time dynamical evolution of highly elliptical satellites orbits

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Outline

Astronomical Observatory of the Ural Federal University

Orbital evolution of HEO objects is studied by

- both a positional observation method (SBG telescope)
- and theoretical methods (this work)
  - analytical
  - numerical

Motivation

Long-term dynamical evolution near HEO

- Safety of active satellites
- Secular perturbations of semi-major axes
  - Atmospheric drag
  - The Poynting–Robertson effect
- Long-term evolution of eccentricities and inclinations due to the Lidov–Kozai resonance
- Passage through high-order resonance zones
- Formation of stochastic trajectories

Methods

Analytical

- Resonant semi-major axis values
- Critical arguments

Numerical

- Positions and sizes of high-order resonance zones
- Estimation of semi-major axes secular perturbations
- Estimation of integrated autocorrelation function

Outline

1. Introduction
2. Analytical approximation
   - Critical arguments and their frequencies
   - p:q resonances
3. Numerical simulation
   - Numerical model
   - Dynamical evolution in region near the high-order resonance
4. Summary
   - Results

Critical arguments (Allan 1967)

$$\Phi_1 = p(M + \Omega + g) - q\omega t = \nu_1 t$$
$$\Phi_2 = p(M + g) + q(\Omega - \omega t) = \nu_2 t$$
$$\Phi_3 = pM + q(g + \Omega - \omega t) = \nu_3 t$$

Frequencies of the critical arguments

$$\nu_1 = p(n_M + n_\Omega + n_g) - q\omega$$
$$\nu_2 = p(n_M + n_g) + q(n_\Omega - \omega)$$
$$\nu_3 = pn_M + q(n_g + n_\Omega - \omega)$$

M, Ω, g are angular elements, n_M, n_Ω, n_g are mean motions,
ω is the angular velocity of the Earth
p, q are integers

Outline

Types of resonances

n-resonance

$\nu_1 \approx 0$

p:*q* resonance between the satellite's mean motion n_M and the Earth's angular velocity ω

i-resonance

$\nu_2 \approx 0$

The position of the ascending node of the orbit repeats periodically in a rotating coordinate system

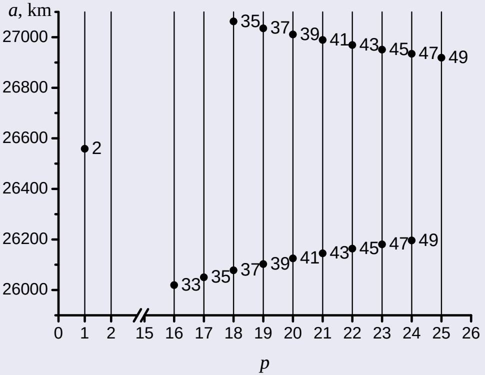
e-resonance

$\nu_3 \approx 0$

The position of the line of apsides of the orbit repeats periodically in a rotating coordinate system

17 high-order resonance relations *p:q*

Resonant semi-major axis values



$e = 0.65$ and
$i = 63.4^\circ$

$16 \leqslant |p| \leqslant 25$
$33 \leqslant |q| \leqslant 49$

Order of resonance:
$49 \leqslant |p| + |q| \leqslant 74$

Outline

Numerical Model of Artificial Earth Satellites Motion (Bordovitsyna et al. 2007)

Software developer

- Research Institute of Applied Mathematics and Mechanics of Tomsk State University

Integrator

- Everhart's method of the 19^{th} order

Interval

- 24 years

The model of perturbing forces (Kuznetsov and Kudryavtsev 2009)

- the Earth's gravitational field (EGM96, harmonics up to the 27^{th} order and degree inclusive)
- the attraction of the Moon and the Sun
- the tides in the Earth's body
- the direct radiation pressure, taking into account the shadow of the Earth (the reflection coefficient $k = 1.44$)
- the Poynting–Robertson effect
- the atmospheric drag

Initial conditions

High-elliptical orbits

- a_0 are consistent with resonant conditions arisen from the analytical approximation
- $e_0 = 0.65$
- Critical inclination $i_0 = 63.4^\circ$
- $g_0 = 270^\circ$
- $\Omega_0 = 0^\circ, 90^\circ, 180^\circ$, and 270°
- Ω_0 coincide with initial values of solar angle $\varphi_0 = \Omega_0 + g_0 = 270^\circ, 0^\circ, 90^\circ$, and 180°
- AMR $= 0.02$, 0.2, and $2\ \mathrm{m^2/kg}$

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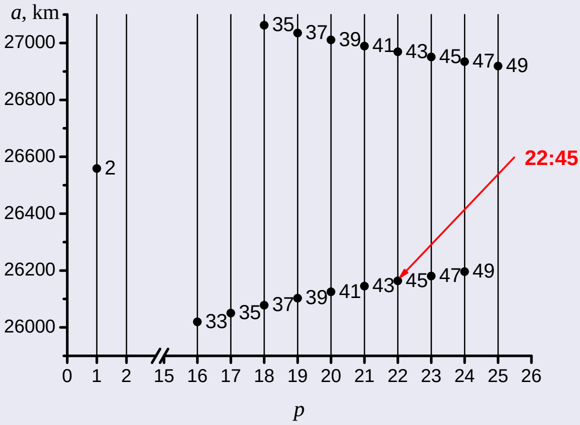
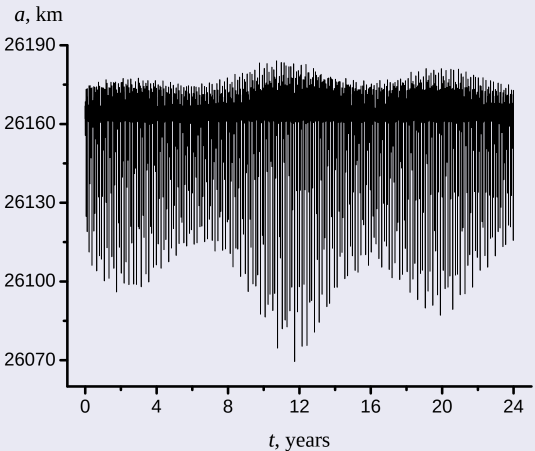
22:45 resonance region



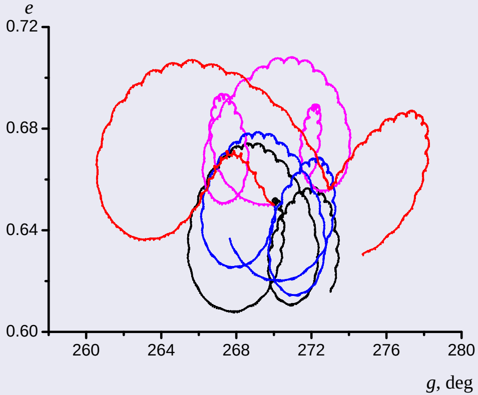
Evolution of the semi-major axis a near the 22:45 resonance region

$a_0 = 26162$ km, $\varphi_0 = 0°$, AMR is 0.02 m^2/kg



Evolution of the eccentricity e and argument of the pericenter g near the 22:45 resonance region

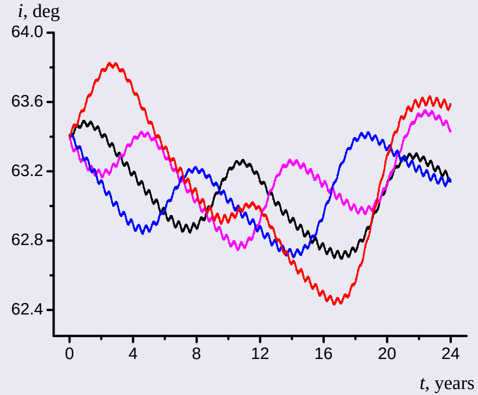
$a_0 = 26162$ km, AMR is 0.02 m^2/kg



$\varphi_0 = 0°$
$\varphi_0 = 90°$
$\varphi_0 = 180°$
$\varphi_0 = 270°$

Evolution of the inclination i near the 22:45 resonance region

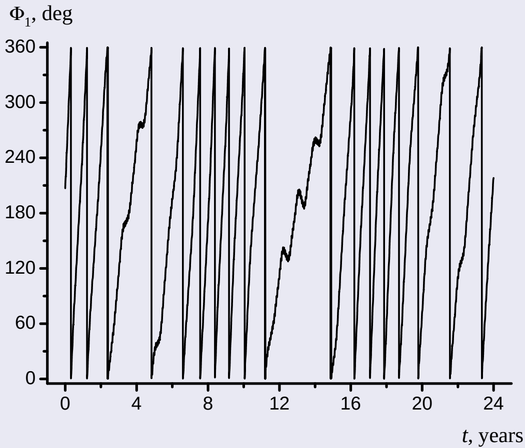
$a_0 = 26162$ km, AMR is 0.02 m^2/kg



$\varphi_0 = 0°$
$\varphi_0 = 90°$
$\varphi_0 = 180°$
$\varphi_0 = 270°$

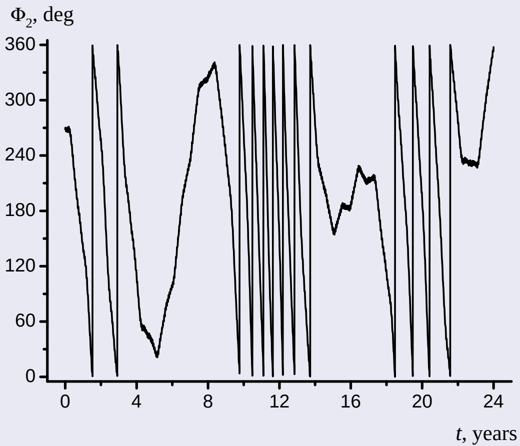
Evolution of the critical argument Φ_1 near the 22:45 resonance region

$a_0 = 26162$ km, $\varphi_0 = 90^\circ$, AMR is 0.02 m^2/kg



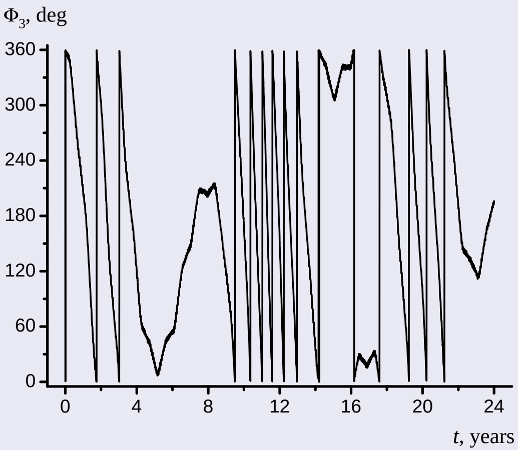
Evolution of the critical argument Φ_2 near the 22:45 resonance region

$a_0 = 26162$ km, $\varphi_0 = 0^\circ$, AMR is 0.02 m^2/kg



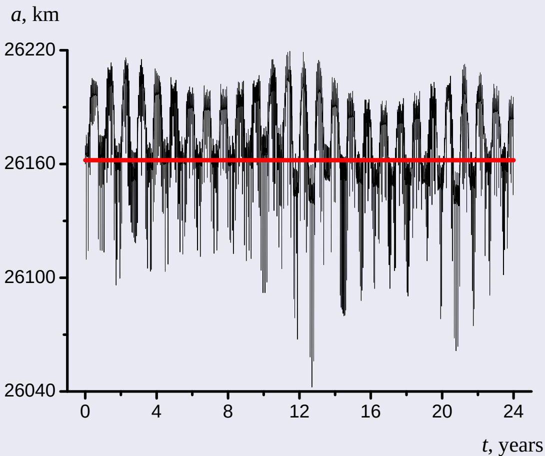
Evolution of the critical argument Φ_3 near the 22:45 resonance region

$a_0 = 26162$ km, $\varphi_0 = 0^\circ$, AMR is 0.02 m^2/kg



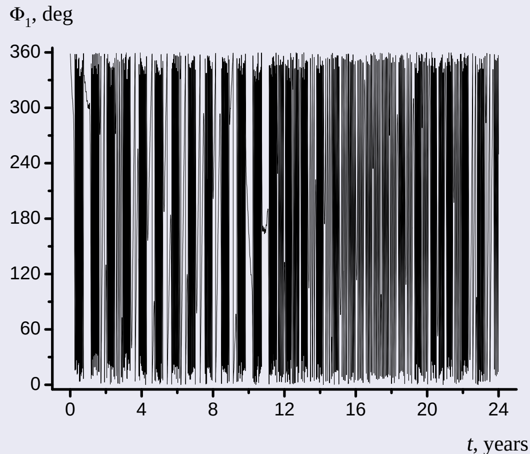
Evolution of the semi-major axis a near the 22:45 resonance region

$a_0 = 26162$ km, $\varphi_0 = 0°$, AMR is 2 m^2/kg



Evolution of the critical argument Φ_1 near the 22:45 resonance region

$a_0 = 26162$ km, $\varphi_0 = 0°$, AMR is 2 m^2/kg



Outline

Formation of the stochastic trajectories

The influences of the Poynting–Robertson effect

- Secular decrease in the semi-major axis, which, for a spherically symmetrical satellite with AMR $= 2$ m^2/kg near the 22:45 resonance region, equals approximately 0.5 km/year
- The effect weakens slightly, in resonance regions
- Objects pass through the regions of high-order resonances

The integrated autocorrelation function $\mathcal{A}$

$\mathcal{A} \rightarrow 1$

- constant time series

$\mathcal{A} \rightarrow 0.5$

- time series representing a uniformly sampled sine wave

$\mathcal{A}$ tends to a finite value not far from 0.5

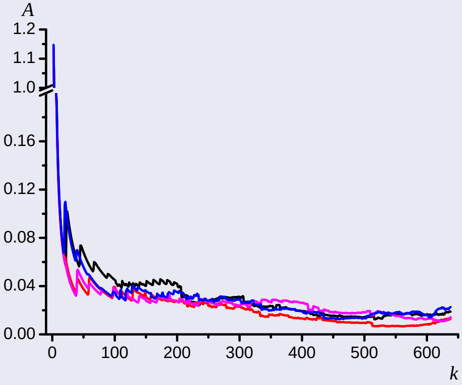
- other periodic and quasi-periodic time series

$\mathcal{A} \rightarrow 0$ with a speed proportional to the inverse of the exponential decay time

- chaotic orbits

The integrated autocorrelation function $\mathcal{A}$ for the semi-major axis a near the 22:45 resonance region

$a_0 = 26162$ km, AMR is 0.02 m^2/kg



$\varphi_0 = 0°$
$\varphi_0 = 90°$
$\varphi_0 = 180°$
$\varphi_0 = 270°$

Conclusion

- The Poynting–Robertson effect
- and secular perturbations of semi-major axis
- lead to the formation of weak stochastic trajectories in HEO region.

Thank you
for your attention!