

# Possible improvements of the IAU 2006 precession

## — *The preliminary results*

Jia-Cheng Liu [jcliu@nju.edu.cn]

*School of Astronomy and Space Science,  
Nanjing University, China*



南京大學  
NANJING UNIVERSITY

Nicole Capitaine [n.capitaine@obspm.fr]

*SYRTE, Observatoire de Paris, France*





## *Motivation*

- ❑ More than 10 years after the publication of the IAU 2006 precession
- ❑ Progresses in
  - ❑ New solutions for the EMB motion
  - ❑ Theoretical progresses for the precession of the equator
  - ❑ Longer time span of VLBI observations
  - ❑ Estimations of  $J_2$  variation
- ❑ To investigate the possibility of improving the IAU 2006 precession



## Outline

- The fundamentals of IAU 2006 precession
- Updated expressions for the precession of the ecliptic
- Recent theoretical improvements and new observations of  $J_2$  by SLR
- Updated expressions for the precession of the equator
- Comparison with IAU 2006 model and check against VLBI



## *The IAU 2006 (P03) precession model*

- ◆ Improved polynomial expressions for both the precession of the ecliptic (IAU 1976) and the precession of the equator (IAU 2000)
- ◆ The precession of the equator was derived from the dynamical equation expressing the motion of the mean pole about the ecliptic pole
- ◆ Precession of the ecliptic: VSOP87 fitted to DE406 over 2000 years
- ◆ Precession of the equator is based on
  - ◆ The IAU 2000 precession rates in longitude and obliquity,
  - ◆ The value  $\varepsilon_0 = 84381.406''$  for the mean obliquity of the ecliptic at J2000
  - ◆ Contributions to the precession rates  $r_\psi$ ,  $r_\varepsilon$  of non-rigid Earth model (Williams 1994; Mathews et al. 2002)
  - ◆ “observed” effect resulting from changing  $\varepsilon_0$
  - ◆  $J_2$  rate value of  $dJ_2/dt = -3.0 \times 10^{-9} \text{ cy}^{-1}$

The polynomial coefficients for all the precession angles are in Hilton et al. (2006).



## Improvement of P03 precession in 2005 (P04)

*c.f. Capitaine et al. 2005, A&A 432, 355*

- ◆ Upgrade of the Earth model (non-linear terms) based on Mathews' (2004) work
- ◆ Update of the integration constants (the interpretation of spurious effects should be in opposite sign), which gives
$$r_0 = 5038.481270'' \text{ cy}^{-1} \quad u_0 = -0.024725'' \text{ cy}^{-1}$$

used in this work
- ◆ Check against VLBI data (1980-2004) and modify the precession rates
  - ◆ However, the time span of VLBI data was not sufficiently long to compare the accuracy of P03 and P04.
- ◆ Recommendation to retain P03 as the replacement for IAU 2000 precession
- ◆ Parameterized precession expressions depending on integration constants and  $J_2$  rate are developed
- ◆ The P03 precession has been adopted as the IAU 2006 precession





# Updating precession of the ecliptic

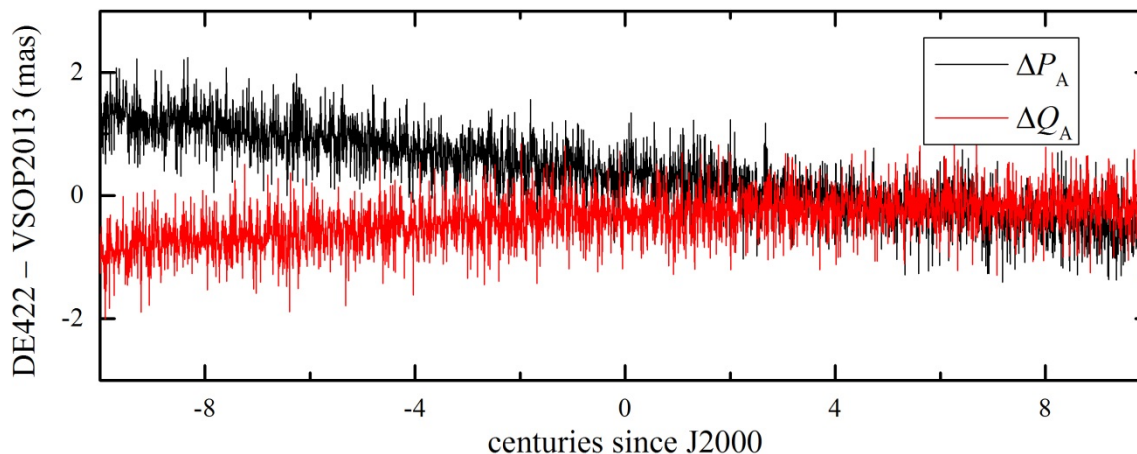
- ◆ New analytical planetary theory VSOP2013 (Simon et al. 2013)
  - ◆ Chebyshev ephemerides – compared with DE422 ephemerides
  - ◆ Poisson series for elliptic elements – fit to DE422 ephemerides
  - ◆ Secular terms of elliptic elements – EMB  $p, q$  for precession of the ecliptic to be adjusted

$$\varepsilon_0 = 84381.41136'', \varphi_0 = -0.05188''$$

angles used for ecliptic-to-equatorial transformation

## ◆ DE422 ephemerides over long time interval

- ◆ observational data
- ◆ used to improve precession of the ecliptic



$P_A$  and  $Q_A$  are the primary quantities for the precession of the ecliptic.

The secular term of  $P_A$  and  $Q_A$  (theoretical) need small adjustments to coincide with observations.



## Fit precession of ecliptic using combined data (VSOP2013 + DE422)

- ◆ The methods are similar to P03
  - ◆ 250-day sampling over J1000 to J3000
  - ◆ Best rotation for DE422 from ecliptic coordinates to equatorial coordinates are found
  - ◆ Small adjustments are applied to VSOP2013 secular solutions for  $p$  and  $q$



Final updated expressions for the precession of the ecliptic

- ◆ The transformation angles:

$$\varepsilon_0 = 84381.411063'', \quad \varphi_0 = -0.051033'', \quad \psi_0 = 0.112513''$$

- ◆ Polynomial expressions for  $P_A$  and  $Q_A$

	$t^1$	$t^2$	$t^3$	$t^4$	$t^5$
$P_A$ (")	4.19903	0.19401	-0.00022353	$-1.04 \times 10^{-6}$	$2.16 \times 10^{-9}$
This work -P03 (μas)	-65	18	1	-0.1	0.01
$Q_A$ (")	-46.81099	0.05102	0.00052137	$-5.58 \times 10^{-7}$	$-1.21 \times 10^{-9}$
This work -P03 (μas)	28	-11	-3	0.09	0.02



## *Recent progress in precession theory*

- ◆ Consideration of relativistic aspect of precession: treat geodetic precession as an additional torque (Gerlach, Klioner, Soffel 2012, arxiv:1202.5870v1)
  - ◆ contribute  $\sim 100 \mu\text{as cy}^{-1}$  to precession rate in longitude
- ◆ Tidal Poisson term contribution to precession rate in obliquity (Folgueira et al. 2007, A&A 469, 1197)
  - ◆ contribute  $88 \mu\text{as cy}^{-1}$  to precession rate in obliquity
- ◆ Second order torque on tidal redistribution (Lambert, Mathews 2008, A&A 481, 883)
  - ◆  $1840 \mu\text{as cy}^{-1}$  to precession rate in obliquity
- ◆ New determination of  $J_2$  variation from SLR observations (\*)
- ◆ Small effect from Galactic aberration (Liu et al. 2012, A&A 548, A50)

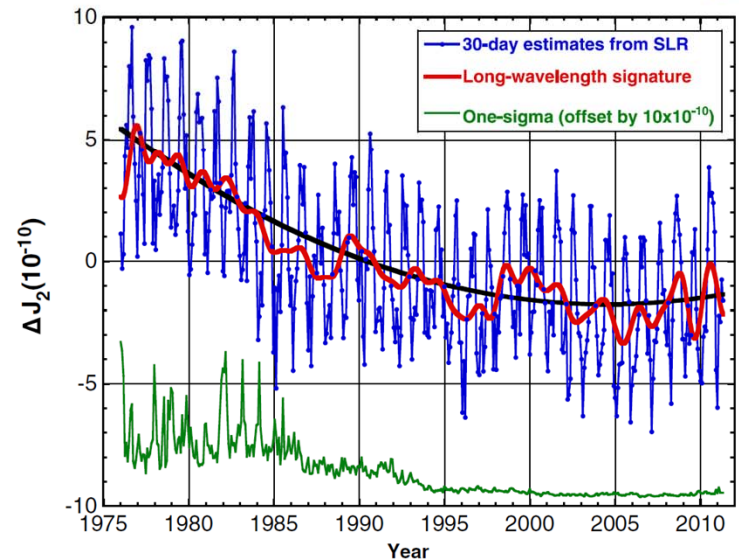




# Deceleration of the Earth's $J_2$

(c.f. Cheng, Tapley, Ries 2013, JGR 118, 740)

- ◆  $J_2$ : gravitational degree-2 harmonic, Earth's dynamical oblateness
- ◆ The long-term trend in  $J_2$  has generally been approximated by a negative linear drift due to postglacial rebound of the Earth's mantle
- ◆ data: time series of 30-day SLR-based estimates of  $J_2$  over 35 years
- ◆ the value of  $dJ_2/dt$  decreases significantly when the time span is increased and more recent SLR data are included, indicating a significant deceleration in  $J_2$
- ◆  $J_2$  appears to be more quadratic than linear in nature



- P03:  $dJ_2 / dt = -3 \times 10^{-9} \text{ cy}^{-1}$
- Fit to parabola:  $J_2 = A_0 + A_1 t + A_2 t^2$
- $t$  in Julian centuries

$$A_0 = 0.001082635820642 \pm 1.64693 \times 10^{-11}$$

$$A_1 = -5.31913 \times 10^{-10} \pm 1.87982 \times 10^{-10} \text{ cy}^{-1}$$

$$A_2 = 1.0894 \times 10^{-8} \pm 1.90071 \times 10^{-9} \text{ cy}^{-2}$$

# Contribution to the precession rates in longitude and obliquity



$$(r_0)_1 / \cos \varepsilon_0 = 5494.040722'' \text{ cy}^{-1}$$

$$(u_0)_1 / \cos \varepsilon_0 = -0.0305096'' \text{ cy}^{-1}$$

**Table 3.** Theoretical contributions (from W94, Williams (1995) and MHB) to the precession rates,  $r_\psi$  and  $r_\epsilon$ , of the equator used in the present paper.

Source of the effect	$\epsilon$ dependence	Contribution in longitude at J2000			Contribution in obliquity at J2000		
		$\mu\text{as/cy}$	$\mu\text{as/cy}^2$	$\mu\text{as/cy}^3$	$\mu\text{as/cy}$	$\mu\text{as/cy}^2$	$\mu\text{as/cy}^3$
Luni-solar & Planetary torque							
Luni-solar 1st order	$\cos \epsilon$	$(r_0)_1$	-3395	-6	0	0	0
Luni-solar 2d order(a)	$6 \cos^2 \epsilon - 1$	-33 100	0	0	0	0	0
Luni-solar 2d order(b)	$3 \cos^2 \epsilon - 1$	-13 680	0	0	0	0	0
Luni-solar $J_4$	$\cos \epsilon (4 - 7 \sin^2 \epsilon)$	+2600	0	0	0	0	0
Planetary 1st order	$\cos \epsilon$	+31 367	0	0	-1400	0	0
$J_2$ and planetary tilts							
$J_2$ and planetary tilt(a)	$\cos 2\epsilon / \sin \epsilon$	-269 430	+1074	0	0	0	0
planetary tilt(b)	$\cos \epsilon$	0	0	0	$(u_0)_1$	-44	+3
Tides							
tides(a)	$\cos^2 \epsilon$	0	-102	0	0	0	0
tides(b)	$\cos^3 \epsilon$	0	-133	0	0	0	0
tides(c)	$\sin \epsilon \cos \epsilon$	0	0	0	+2400	0	0
$J_2$ rate	$\cos \epsilon$	0	-14 000	0	0	0	0
Non-linear effect	1	-21 050	0	0	0	0	0
Geodesic precession	1	-1 919 883	+3	+1	-1	0	+5
Total		$(r_0)_1 - 2 223 176$	-16 553	-5	$(u_0)_1 + 999$	-44	+8

$$-960 \mu\text{as cy}^{-1}$$

$$J_2 \text{ rate contribution: } -2482 \mu\text{as cy}^{-2}$$

$$J_2 \text{ second order contribution: } +50629 \mu\text{as cy}^{-3}$$

$$+340 \mu\text{as cy}^{-1}$$



# Integration of the dynamical equations

$$\sin \omega_A \frac{d\psi_A}{dt} = (r_\psi \sin \varepsilon_A) \cos \chi_A - r_\varepsilon \sin \chi_A; \quad \frac{d\omega_A}{dt} = r_\varepsilon \cos \chi_A + (r_\psi \sin \varepsilon_A) \sin \chi_A$$

RKF-7(8) integrator  
P03: GREGOIRE software

The new precession of the equator is based on:

- updated ecliptic precession derived from VSOP2013 and DE422 ephemerides
- P04 integration constants  $r_0 = 5038.481270'' \text{ cy}^{-1}$   $u_0 = -0.024725'' \text{ cy}^{-1}$
- updated contributions to precession rates including  $J_2$  secular variation from SLR

	$T^1$	$t^2$	$t^3$	$t^4$	$t^5$
$\psi_A$ (")	5038.481270	-1.0732468	0.01573403	0.000127135	$-1.0197 \times 10^{-7}$
This work -P03 (μas)	-237	5760	16875	-6	-0.007
$\omega_A$ (")	-0.024725	0.0512626	-0.0077249	$-2.67 \times 10^{-7}$	$2.67 \times 10^{-7}$
This work -P03 (μas)	1029	0.3	0.1	0.1	-0.07
$p_A$ (")	5028.796129	1.1111242	0.0169552	-0.000020031	$-1.7 \times 10^{-8}$
This work -P03 (μas)	-66	5689	16876	4	0.002
$\varepsilon_A$ (")	-46.835705	-0.0001936	0.00200005	$-5.94 \times 10^{-7}$	$-1.22 \times 10^{-8}$
This work -P03 (μas)	1064	-10	-3	-0.02	0.03
$\chi_A$ (")	10.556240	-2.3813876	-0.00121311	0.000160286	$-8.60 \times 10^{-8}$
This work -P03 (μas)	-163	54	-1	-10	-0.03



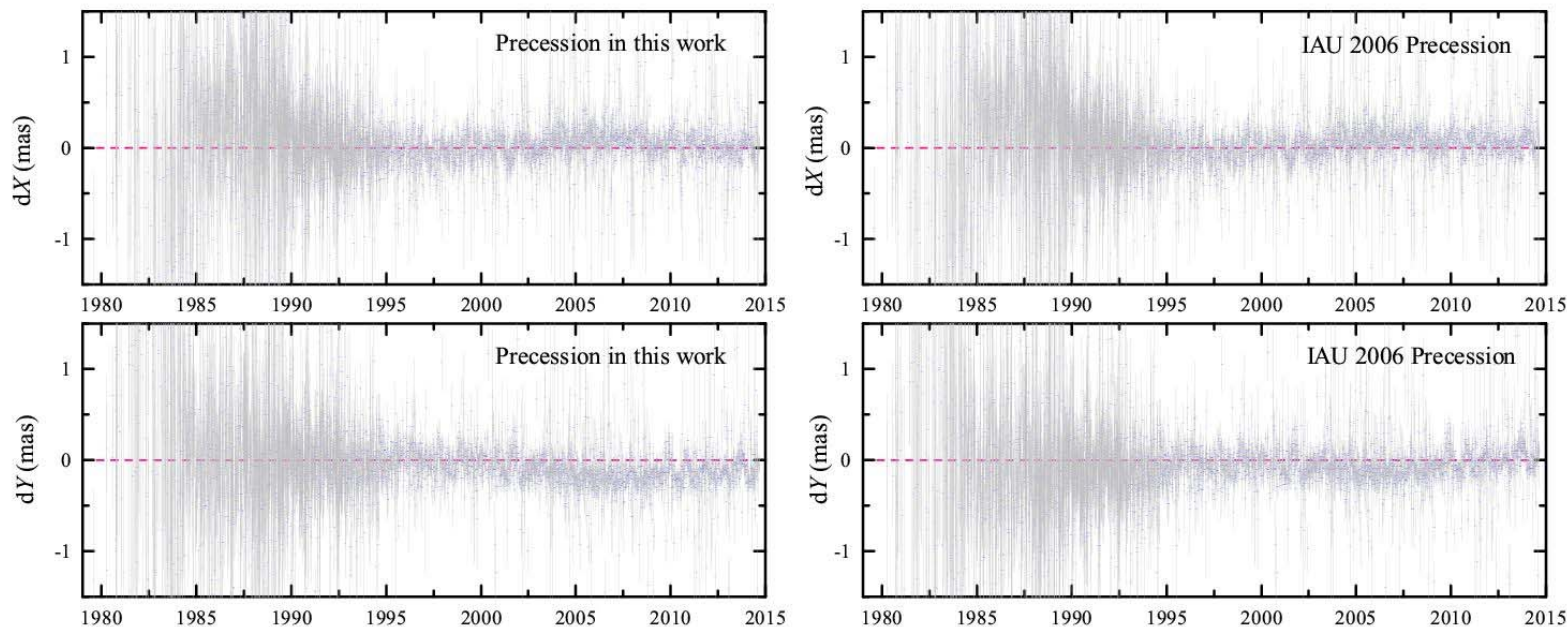
# Comparison with VLBI



SOFA board are gratefully acknowledged!

## Two series of celestial pole offsets

1. based on frame bias + [precession in this work](#) + IAU 2000A nutation
2. based on frame bias + [IAU 2006 precession](#) + IAU2000A nutation



		Weighted Mean (mas)	WRMS (mas)
This work	dX	0.0304	0.1292
IAU	dX	0.0467	0.1349
This work	dY	-0.1167	0.1794
IAU	dY	-0.0565	0.1442



## *Discussion and concluding remarks*

- ◆ Precession of the ecliptic has been updated based on recent new ephemerides
- ◆ More recent model for the  $J_2$  variation has been used in the integration
- ◆ The P04 integration constants have been used
- Changes in the 2<sup>nd</sup> and 3<sup>rd</sup> degree coefficients for the precession in longitude are significant
- Whether a new model is necessary to replace the IAU 2006 precession after 10 years of its publication?
- The IAU 2006 precession is still trustworthy

## *For the future precession model*

- Solve precession and nutation simultaneously
- Precession model in the Gaia reference frame / ICRF3