New high-precision Earth and Moon rotation series at long time intervals

Vladimir V. Pashkevich

Central (Pulkovo) Astronomical Observatory
of the Russian Academy of Science
St. Petersburg

Journees 2014
INTRODUCTION

- **In the previous investigation** (Pashkevich, 2013), (Pashkevich and Eroshkin, 2011) the high-precision Rigid Earth Rotation Series (designated RERS2013) and the high-precision Moon Rotation Series (designated MRS2011) were constructed at a long time intervals.

- **RERS2013** is dynamically adequate to the JPL DE422/LE422 ephemeris over 2000 and 6000 years, and include about 4113 periodical terms (without attempt to estimate new sub-diurnal and diurnal periodical terms).

- **MRS2011** is dynamically adequate to the JPL DE406/LE406 ephemeris over 418, 2000 and 6000 years, and include about 1520 periodical terms.
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Aims of the present research:

1) **Construction of the improved high-precision Rigid Earth Rotation Series RERS2014** (with including new sub-diurnal and diurnal periodical terms) and the **high-precision Moon Rotation Series MRS2014** dynamically adequate to the JPL DE422/LE422 ephemeris, over 2000 and 6000 years, respectively.

2) **Investigation of discrepancies** between the high-precision numerical solutions and the semi-analytical solutions for the rigid Earth and the Moon rotation problems with respect to the fixed ecliptic of epoch J2000, by the least-squares method and by the spectral analysis methods.

3) **Comparison of the new Series RERS2014 and MRS2014** with the previous solution RERS2013 (Pashkevich, 2013) and MRS2011 (Pashkevich and Eroshkin, 2011), respectively.

The rigid Earth rotation problem is solved for the relativistic (kinematical) case in which the geodetic perturbations (the most essential relativistic perturbations) in the Earth rotation are taken into account.
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Mathematical model of the body rotation problem

LAGRANGE DIFFERENTIAL EQUATIONS OF THE SECOND KIND:

\[\frac{d}{dt} \frac{\partial L}{\partial \dot{\lambda}_i} - \frac{\partial L}{\partial \lambda_i} = 0, \quad i = 0, 1, 2, 3.\]

where \(L = T + U\),

Problem expressed in the Rodrigues – Hamilton parameters:

\[\lambda_0 = \cos \frac{\theta}{2} \cos \frac{\psi + \phi}{2}, \quad \lambda_1 = \sin \frac{\theta}{2} \cos \frac{\psi - \phi}{2}, \quad \lambda_2 = \sin \frac{\theta}{2} \sin \frac{\psi - \phi}{2}, \quad \lambda_3 = \cos \frac{\theta}{2} \sin \frac{\psi + \phi}{2},\]

which are functions of the Euler angles \(\psi, \theta\) and \(\phi\).
Mathematical model of the Earth rotation problem

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where \( L = T + U, \quad T = \frac{1}{2} (A\omega^2_1 + B\omega^2_2 + C\omega^2_3), \quad \vec{\omega}_\ast = \vec{\omega} + \vec{\sigma}, \)

\( \vec{\omega}_\ast \) is the relativistic angular velocity vector.

VECTOR OF THE GEODETIC ROTATION OF THE EARTH:

\[
\vec{\sigma} = \sum_{j \neq \oplus} \frac{G m_j}{c^2 |\vec{R}_j - \vec{R}_\oplus|^3} \left\{ (2 \dot{\vec{R}}_j - \frac{3}{2} \ddot{\vec{R}}_\oplus) \times (\vec{R}_j - \vec{R}_\oplus) \right\}.
\]

This investigation is carried out for the relativistic (kinematical) case.
Mathematical model of the Earth rotation problem

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\( U \) is the force function of the gravitational interaction of the Earth with the disturbing bodies (the Moon, the Sun and major planets).

The force function \( U \) is expanded in the spherical harmonics and only the terms with the coefficients \( C_{j0} \) for \( j=2, \ldots, 5 \), \( C_{22}, C_{3k}, S_{3k} \) for \( k=1, 2, 3 \) and \( C_{41}, S_{41} \) are used.

The orbital motions of the disturbing celestial bodies are defined by the JPL DE422/LE422 ephemeris.

This investigation is carried out for the relativistic (kinematical) case.
Mathematical model of the Moon rotation problem

LAGRANGE DIFFERENTIAL EQUATIONS OF THE SECOND KIND:

In the previous investigation (Pashkevich and Eroshkin, 2011)

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\frac{d}{dt} \frac{\partial L}{\partial \dot{\lambda}_i} - \frac{\partial L}{\partial \lambda_i} = 0, \quad i = 0,1,2,3.
\]

where \( L = T + U, \quad T = \frac{1}{2} (A\omega_1^2 + B\omega_2^2 + C\omega_3^2) \),

\( U \) is the force function of the gravitational interaction of the Moon with the disturbing bodies (the Earth, the Sun and major planets).

The force function \( U \) is expanded in the spherical harmonics and only the terms with the coefficients \( C_{j0} \) for \( j = 2, \ldots, 4 \), \( C_{22}, C_{3k}, S_{3k} \) for \( k = 1,2,3 \) and \( C_{4k}, S_{4k} \) for \( k = 1,\ldots,4 \) are used.

The orbital motions of the disturbing bodies was defined by the DE406/LE406 ephemeris.

This investigation is carried out only for the Newtonian (dynamical) case.
Mathematical model of the Moon rotation problem

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\frac{d}{dt} \frac{\partial L}{\partial \dot{\lambda}_i} - \frac{\partial L}{\partial \lambda_i} = 0, \quad i = 0, 1, 2, 3.
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The orbital motions of the disturbing bodies are defined by the DE422/LE422 ephemeris.

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The physical librations of the Moon

Cassini’s relations:

a) **The Moon rotates** with a constant angular velocity around its polar axis. **The rotation period** is equal to the mean sidereal period of its orbital motion around the Earth.

b) **The inclination of the lunar equator to the ecliptic** is a constant angle (near $1^\circ 32'$).

c) **The ascending node** of the lunar orbit on the ecliptic coincides with the **descending node** of the lunar equator on the ecliptic.

OR

\begin{align*}
\text{a)}\quad \varphi + \psi &= 180^\circ + L_\zeta \\
\text{b)}\quad \theta &= I \\
\text{b)}\quad \psi &= \Omega
\end{align*}

Here $L_\zeta$ is the mean longitude of the Moon and $\Omega$ is the mean longitude of the ascending node of its orbit.
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a) **The Moon rotates** with a constant angular velocity around its polar axis. **The rotation period** is equal to the mean sidereal period of its orbital motion around the Earth.

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c) **The ascending node** of the lunar orbit on the ecliptic coincides with **the descending node** of the lunar equator on the ecliptic.

Since Cassini’s relations are not exact, one must consider the perturbed Euler angles:

\[ \varphi + \psi = 180^\circ + L_\varphi + \tau \]

\[ \theta = I + \rho \]

\[ \psi = \Omega + \sigma \]

Here \( L_\varphi \) is the mean longitude of the Moon and \( \Omega \) is the mean longitude of the ascending node of its orbit; \( \tau, \rho \) and \( \sigma \) are the perturbing terms of the physical librations in the longitude, in the inclination and in the node longitude, respectively.
Expressions for the perturbing terms of the physical librations for the fixed ecliptic of epoch J2000:

a) \( \tau = \varphi + \psi - 180^\circ - L_\zeta \)

b) \( \rho = \theta - I \)

c) \( \sigma = \psi - \Omega \)

where \( \psi \) is the longitude of the descending node of epoch J2000 of the lunar equator,
\( I \) is a constant angle of the inclination of the lunar equator to the fixed ecliptic J2000 (\( I \sim 1^\circ 32' \)),
\( \theta \) is the inclination of the lunar equator to the fixed ecliptic J2000,
\( \varphi \) is the proper rotation angle between the descending node of epoch J2000 and the principal axis A (with the minimum moment of inertia);
\( L_\zeta \) is the mean longitude of the Moon and
\( \Omega \) is the mean longitude of the ascending node of its orbit;
\( \tau, \rho \) and \( \sigma \) are the perturbing terms of the physical librations in the longitude, in the inclination and in the node longitude, respectively.
ITERATIVE ALGORITHM:

1. **Numerical solution of the studied body rotation (the rigid Earth or Moon)** is implemented with the quadruple precision of calculations. **The initial conditions** are computed by the semi-analytical solution of the studied body rotation (RERS2013 for the Earth or MRS2011 for the Moon). **Discrepancies between the numerical solution and the semi-analytical solution** are obtained in Euler angles over all investigation time interval with 0.1 day spacing (for the Earth) or 1 day spacing (for the Moon).

2. **Investigation of the discrepancies** is carried out by the least squares method (LSQ) and by the spectral analysis (SA) method (Pashkevich and Eroshkin, 2005, 2010). The sets of the frequencies of the semi-analytical solutions are used without change. **Only the coefficients** of the systematical terms, **the coefficients** of the periodical terms and **the coefficients** of the Poisson terms are improved. **The systematic, periodic and Poisson terms** representing the new high-precision studied body rotation series (RERS2014i for the Earth or MRS2014i for the Moon) (where \( i \) is the number of iteration) are determined.

3. **Numerical solution of the studied body rotation is constructed anew** with **the new initial conditions**, which are calculated by RERS2014i (for the Earth) or MRS2014i (for the Moon).

4. **Steps 2 and 3 are repeated** till the assumed convergence level of the discrepancies between the new numerical solution and the new semi-analytical solution (RERS2014i for the Earth or MRS2014i for the Moon) has been achieved.
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ITERATIVE ALGORITHM:

1. \[
\Delta \psi = \sum_{k=0}^{8} \psi_k t^k + \sum_{j} \sum_{k=0}^{4} [\psi_{S,jk} \sin(\nu_{j0} + \nu_{j1}t) + \psi_{C,jk} \cos(\nu_{j0} + \nu_{j1}t)] t^k \quad \text{(for the Earth)}
\]

\[
\Delta \theta = \sum_{k=0}^{8} \theta_k t^k + \sum_{j} \sum_{k=0}^{4} [\theta_{S,jk} \sin(\nu_{j0} + \nu_{j1}t) + \theta_{C,jk} \cos(\nu_{j0} + \nu_{j1}t)] t^k
\]

\[
\Delta \varphi = \sum_{k=0}^{8} \varphi_k t^k + \sum_{j} \sum_{k=0}^{4} [\varphi_{S,jk} \sin(\nu_{j0} + \nu_{j1}t) + \varphi_{C,jk} \cos(\nu_{j0} + \nu_{j1}t)] t^k
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3. \[
\Delta \tau = \sum_{j} \sum_{k=0}^{3} [\tau_{S,jk} \sin(\nu_{j0} + \nu_{j1}t) + \tau_{C,jk} \cos(\nu_{j0} + \nu_{j1}t)] t^k \quad \text{(for the Moon)}
\]

\[
\Delta \rho = \sum_{j} \sum_{k=0}^{3} [\rho_{S,jk} \sin(\nu_{j0} + \nu_{j1}t) + \rho_{C,jk} \cos(\nu_{j0} + \nu_{j1}t)] t^k
\]

4. \[
\Delta I\sigma = I \sum_{j} \sum_{k=0}^{3} [\sigma_{S,jk} \sin(\nu_{j0} + \nu_{j1}t) + \sigma_{C,jk} \cos(\nu_{j0} + \nu_{j1}t)] t^k
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\begin{align*}
\psi_{\text{RERS2014i}} &= \Delta \psi_{i-1} + \psi_{\text{RERS2014i-1}} \\
\theta_{\text{RERS2014i}} &= \Delta \theta_{i-1} + \theta_{\text{RERS2014i-1}} \\
\varphi_{\text{RERS2014i}} &= \Delta \varphi_{i-1} + \varphi_{\text{RERS2014i-1}} \\
\tau_{\text{MRS2014i}} &= \Delta \tau_{i-1} + \tau_{\text{MRS2014i-1}} \\
\rho_{\text{MRS2014i}} &= \Delta \rho_{i-1} + \rho_{\text{MRS2014i-1}} \\
\sigma_{\text{MRS2014i}} &= \Delta \sigma_{i-1} + \sigma_{\text{MRS2014i-1}}
\end{align*}

3. Numerical solution of the studied body rotation is constructed anew with the new initial conditions, which are calculated by RERS2014i (for the Earth) or MRS2014i (for the Moon).

4. Steps 2 and 3 are repeated till the assumed convergence level of the discrepancies between the new numerical solution and the new semi-analytical solution (RERS2014i for the Earth or MRS2014i for the Moon) has been achieved.
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3. Numerical solution of the studied body rotation is constructed anew with the new initial conditions, which are calculated by RERS2014i (for the Earth) or MRS2014i (for the Moon).

4. Steps 2 and 3 are repeated till the assumed convergence level of the discrepancies between the new numerical solution and the new semi-analytical solution (RERS2014i for the Earth or MRS2014i for the Moon) has been achieved.
Results:

Investigation of the Earth rotation over 2000 years
The new improved high-precision Rigid Earth Rotation Series RERS2014 dynamically adequate to the DE422/LE422 have been constructed (after formal removal of the secular trends in the proper rotation angle) (over 2000 years)

Fig. 1. The numerical solutions minus RERS2013 (Pashkevich, 2013) and the numerical solutions 3 minus RERS2014-2 (2nd iteration)
Results:

Investigation of the Moon rotation over 6000 years
Fig. 2. The numerical solutions minus MRS2011 (dynamically adequate to the JPL DE406/LE406 ephemeris) (Pashkevich, Eroshkin, 2011) and the numerical solutions 2 minus MRS2014-1 (1st iteration) (over 6000 years).
CONCLUSIONS

• **The new improved high-precision Rigid Earth Rotation Series RERS2014** (with including new sub-diurnal and diurnal periodical terms) dynamically adequate to the DE422/LE422 ephemeris over 2000 years have been constructed.

• **RERS2014 include** about 4113 periodical terms. **Discrepancies between the numerical solution and RERS2014** do not surpass 3 µas over 2000 years.

• **The new high-precision Moon Rotation Series MRS2014** dynamically adequate to the DE422/LE422 ephemeris over 6000 years have been constructed.

• **MRS2014 include** about 1520 periodical terms. **Discrepancies between the numerical solution and MRS2014** do not surpass 8 arc seconds over 6000 years,

=> it means a good consistency of the **MRS2014** series with the DE422/LE422 ephemeris.
CONCLUSIONS

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• The new high-precision Moon Rotation Series MRS2014 dynamically adequate to the DE422/LE422 ephemeris over 6000 years have been constructed.

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• \textbf{RERS2014 include} about 4113 periodical terms. Discrepancies between the numerical solution and \textbf{RERS2014} do not surpass 3 \textmu as over 2000 years.

• The new high-precision Moon Rotation Series \textbf{MRS2014} dynamically adequate to the DE422/LE422 ephemeris over 6000 years have been constructed.

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ACKNOWLEDGMENTS

The investigation was carried out at the Central (Pulkovo) Astronomical Observatory of the Russian Academy of Sciences and the Space Research Centre of the Polish Academy of Sciences, under a financial support of the Cooperation between the Polish and Russian Academies of Sciences, Theme No 34.