# New high-precision Earth and Moon rotation series at long time intervals

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# INTRODUCTION

- **In the previous investigation** (Pashkevich, 2013), (Pashkevich and Eroshkin, 2011) **the high-precision Rigid Earth Rotation Series (designated RERS2013)** and **the high-precision Moon Rotation Series (designated MRS2011)** were constructed **at a long time intervals**.

- **RERS2013** is dynamically adequate to the JPL DE422/LE422 ephemeris over 2000 and 6000 years, and include about 4113 periodical terms (without attempt to estimate new sub-diurnal and diurnal periodical terms).

- **MRS2011** is dynamically adequate to the JPL DE406/LE406 ephemeris over 418, 2000 and 6000 years, and include about 1520 periodical terms.

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1) **Construction of the improved high-precision Rigid Earth Rotation Series RERS2014** (with including new sub-diurnal and diurnal periodical terms) and **the high-precision Moon Rotation Series MRS2014** dynamically adequate to the JPL DE422/LE422 ephemeris, over 2000 and 6000 years, respectively.

2) **Investigation of discrepancies** between the high-precision numerical solutions and the semi-analytical solutions **for the rigid Earth and the Moon rotation problems with respect to the fixed ecliptic of epoch J2000,** by the least-squares method and by the spectral analysis methods.

3) **Comparison of the new Series RERS2014 and MRS2014** with the previous solution **RERS2013 (Pashkevich, 2013) and MRS2011 (Pashkevich and Eroshkin, 2011**), **respectively.**

**The rigid Earth rotation problem is solved for the relativistic (kinematical) case** in which the geodetic perturbations (the most essential relativistic perturbations) in the Earth rotation are taken into account.

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# Mathematical model of the body rotation problem

## LAGRANGE DIFFERENTIAL EQUATIONS OF THE SECOND KIND:

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\lambda}_i} - \frac{\partial L}{\partial \lambda_i} = 0, \quad i = 0, 1, 2, 3.$$

where $L = T + U$,

## Problem expressed in the Rodrigues – Hamilton parameters:

$$\lambda_0 = \cos\frac{\theta}{2}\cos\frac{\psi+\phi}{2}, \lambda_1 = \sin\frac{\theta}{2}\cos\frac{\psi-\phi}{2}, \lambda_2 = \sin\frac{\theta}{2}\sin\frac{\psi-\phi}{2}, \lambda_3 = \cos\frac{\theta}{2}\sin\frac{\psi+\phi}{2},$$

which are functions of the Euler angles $\psi, \theta$ and $\phi$.

# Mathematical model of the Earth rotation problem

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where $L = T + U$, $T = \frac{1}{2}(A\omega_{*1}^2 + B\omega_{*2}^2 + C\omega_{*3}^2)$, $\bar{\omega}_* = \bar{\omega} + \bar{\sigma}$,

$\bar{\omega}_*$ is the relativistic angular velocity vector.

## VECTOR OF THE GEODETIC ROTATION OF THE EARTH:

$$\bar{\sigma} = \sum_{j \neq \oplus} \frac{Gm_j}{c^2 \left|\bar{R}_j - \bar{R}_\oplus\right|^3} \left\{ (2\dot{\bar{R}}_j - \frac{3}{2}\dot{\bar{R}}_\oplus) \times (\bar{R}_j - \bar{R}_\oplus) \right\}.$$

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$U$ **is the force function of the gravitational interaction of the Earth with the disturbing bodies (the Moon, the Sun and major planets).**

**The force function** $U$ **is expanded in the spherical harmonics** and only the terms with the coefficients **$C_{j0}$ for j=2,…,5, $C_{22}$, $C_{3k}$, $S_{3k}$ for k=1,2,3 and $C_{41}$, $S_{41}$** are used.

**The orbital motions of the disturbing celestial bodies are defined by the JPL DE422/LE422 ephemeris.**

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## Mathematical model of the Moon rotation problem

**LAGRANGE DIFFERENTIAL EQUATIONS OF THE SECOND KIND:**

In the previous investigation (Pashkevich and Eroshkin, 2011)

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where $L = T + U, \;\; T = \frac{1}{2}(A\omega_1^2 + B\omega_2^2 + C\omega_3^2),$

$U$ **is the force function of the gravitational interaction of the Moon with the disturbing bodies (the Earth, the Sun and major planets).**

**The force function $U$ is expanded in the spherical harmonics** and only the terms with the coefficients **$C_{j0}$ for j=2,…,4, $C_{22}$, $C_{3k}$, $S_{3k}$ for k=1,2,3 and $C_{4k}$, $S_{4k}$ for k=1,…,4** are used.

**The orbital motions of the disturbing bodies was defined by the DE406/LE406 ephemeris.**

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# The physical librations of the Moon

## Cassini's relations:

a) **The Moon rotates** with a constant angular velocity around its polar axis. **The rotation period** is equal to the mean sidereal period of its orbital motion around the Earth.

b) **The inclination of the lunar equator to the ecliptic** is a constant angle (near 1° 32').

c) **The ascending node** of the lunar orbit on the ecliptic coincides with **the descending node** of the lunar equator on the ecliptic.

OR

а) $\varphi + \psi = 180^\circ + L_☾$

б) $\theta = I$

в) $\psi = \Omega$

Here $L_☾$ is the mean longitude of the Moon and $\Omega$ is the mean longitude of the ascending node of its orbit.

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c) **The ascending node** of the lunar orbit on the ecliptic coincides with **the descending node** of the lunar equator on the ecliptic.

Since Cassini's relations are not exact, one must consider the perturbed Euler angles:

$$\text{a)}\ \varphi + \psi = 180^\circ + L_{☾} + \tau$$

$$\text{б)}\ \theta = I + \rho$$

$$\text{в)}\ \psi = \Omega + \sigma$$

Here $L_{☾}$ is the mean longitude of the Moon and $\Omega$ is the mean longitude of the ascending node of its orbit; $\tau$, $\rho$ and $\sigma$ are the perturbing terms of the physical librations in the longitude, in the inclination and in the node longitude, respectively.

## Expressions for the perturbing terms of the physical librations for the fixed ecliptic of epoch J2000:

a) $\tau = \varphi + \psi - 180^\circ - L_☾$

b) $\rho = \theta - I$

c) $\sigma = \psi - \Omega$

where $\psi$ is the longitude of the descending node of epoch J2000 of the lunar equator,
$I$ is a constant angle of the inclination of the lunar equator to the fixed ecliptic J2000 ($I$ ~ $1^{o}$ 32'),
$\theta$ is the inclination of the lunar equator to the fixed ecliptic J2000,
$\varphi$ is the proper rotation angle between the descending node of epoch J2000 and the principal axis A (with the minimum moment of inertia);
$L_☾$ is the mean longitude of the Moon and
$\Omega$ is the mean longitude of the ascending node of its orbit;
$\tau$, $\rho$ and $\sigma$ are the perturbing terms of the physical librations in the longitude, in the inclination and in the node longitude, respectively.

# ITERATIVE ALGORITHM:

1. **Numerical solution of the studied body rotation (the rigid Earth or Moon)** is implemented with the quadruple precision of calculations. **The initial conditions** are computed by the semi-analytical solution of the studied body rotation (RERS2013 for the Earth or MRS2011 for the Moon). **Discrepancies between the numerical solution and the semi-analytical solution** are obtained in Euler angles over all investigation time interval with 0.1 day spacing (for the Earth) or 1 day spacing (for the Moon).

2. **Investigation of the discrepancies** is carried out by the least squares method (LSQ) and by the spectral analysis (SA) method (**Pashkevich and Eroshkin, 2005, 2010**). **The sets of the frequencies of the semi-analytical solutions** are used without change. **Only the coefficients** of the systematical terms, **the coefficients** of the periodical terms and **the coefficients** of the Poisson terms are improved. **The systematic, periodic and Poisson terms** representing the new high-precision studied body rotation series (**RERS2014i for the Earth or MRS2014i for the Moon**) (where **i** is the number of iteration) are determined.

3. **Numerical solution of the studied body rotation is constructed anew** with **the new initial conditions,** which are calculated by **RERS2014i** (for the Earth) or **MRS2014i** (for the Moon).

4. **Steps 2 and 3 are repeated** till the assumed convergence level of the discrepancies between the new numerical solution and the new semi-analytical solution (RERS2014i for the Earth or MRS2014i for the Moon) has been achieved.

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1.

$$\left.\begin{aligned}
\Delta\psi &= \sum_{k=0}^{8}\psi_k\ t^k + \sum_j\sum_{k=0}^{4}[\psi_{S\,jk}\sin(\nu_{j0}+\nu_{j1}t)+\psi_{C\,jk}\cos(\nu_{j0}+\nu_{j1}t)]\ t^k \\
\Delta\theta &= \sum_{k=0}^{8}\theta_k\ t^k + \sum_j\sum_{k=0}^{4}[\theta_{S\,jk}\sin(\nu_{j0}+\nu_{j1}t)+\theta_{C\,jk}\cos(\nu_{j0}+\nu_{j1}t)]\ t^k \\
\Delta\varphi &= \sum_{k=0}^{8}\varphi_k\ t^k + \sum_j\sum_{k=0}^{4}[\varphi_{S\,jk}\sin(\nu_{j0}+\nu_{j1}t)+\varphi_{C\,jk}\cos(\nu_{j0}+\nu_{j1}t)]\ t^k
\end{aligned}\right\}\ \text{(for the Earth)}$$

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$$\left.\begin{aligned}
\Delta\tau &= \sum_j\sum_{k=0}^{3}[\tau_{S\,jk}\sin(\nu_{j0}+\nu_{j1}t)+\tau_{C\,jk}\cos(\nu_{j0}+\nu_{j1}t)]\ t^k \\
\Delta\rho &= \sum_j\sum_{k=0}^{3}[\rho_{S\,jk}\sin(\nu_{j0}+\nu_{j1}t)+\rho_{C\,jk}\cos(\nu_{j0}+\nu_{j1}t)]\ t^k \\
\Delta I\sigma &= I\sum_j\sum_{k=0}^{3}[\sigma_{S\,jk}\sin(\nu_{j0}+\nu_{j1}t)+\sigma_{C\,jk}\cos(\nu_{j0}+\nu_{j1}t)]\ t^k
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2. $$\left.\begin{aligned}\psi_{\mathrm{RERS2014i}} &= \Delta\psi_{\mathrm{i-1}} + \psi_{\mathrm{RERS2014i-1}}\\ \theta_{\mathrm{RERS2014i}} &= \Delta\theta_{\mathrm{i-1}} + \theta_{\mathrm{RERS2014i-1}}\\ \varphi_{\mathrm{RERS2014i}} &= \Delta\varphi_{\mathrm{i-1}} + \varphi_{\mathrm{RERS2014i-1}}\end{aligned}\right\};\quad \left.\begin{aligned}\tau_{\mathrm{MRS2014i}} &= \Delta\tau_{\mathrm{i-1}} + \tau_{\mathrm{MRS2014i-1}}\\ \rho_{\mathrm{MRS2014i}} &= \Delta\rho_{\mathrm{i-1}} + \rho_{\mathrm{MRS2014i-1}}\\ \sigma_{\mathrm{MRS2014i}} &= \Delta\sigma_{\mathrm{i-1}} + \sigma_{\mathrm{MRS2014i-1}}\end{aligned}\right\}$$

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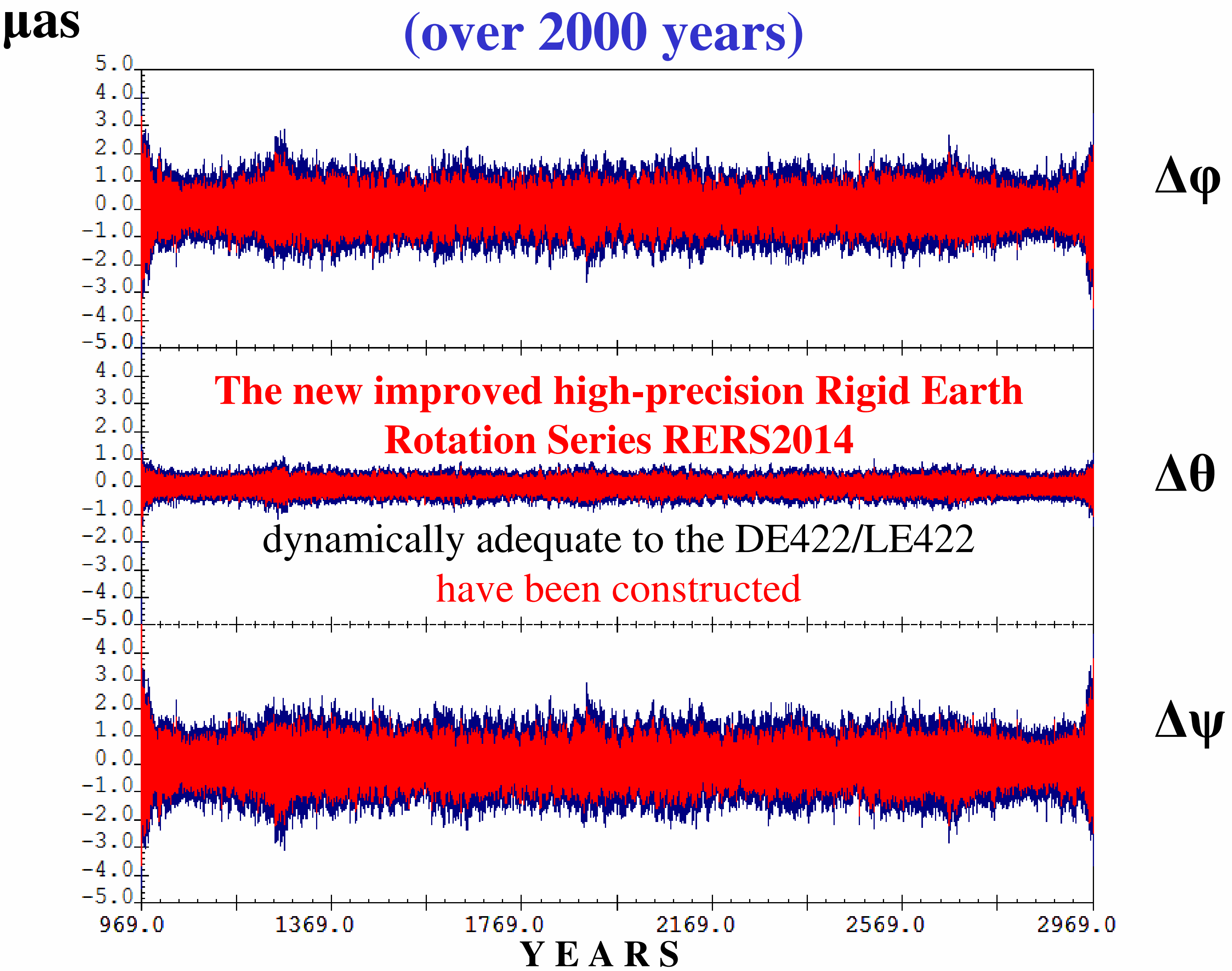
# Results:

Investigation of
the Earth rotation **over 2000 years**

Fig.1. **The numerical solutions minus RERS2013** (Pashkevich, 2013) and **the numerical solutions 3 minus RERS2014-2** (2nd iteration)

(after formal removal of the secular trends in the proper rotation angle)

**(over 2000 years)**



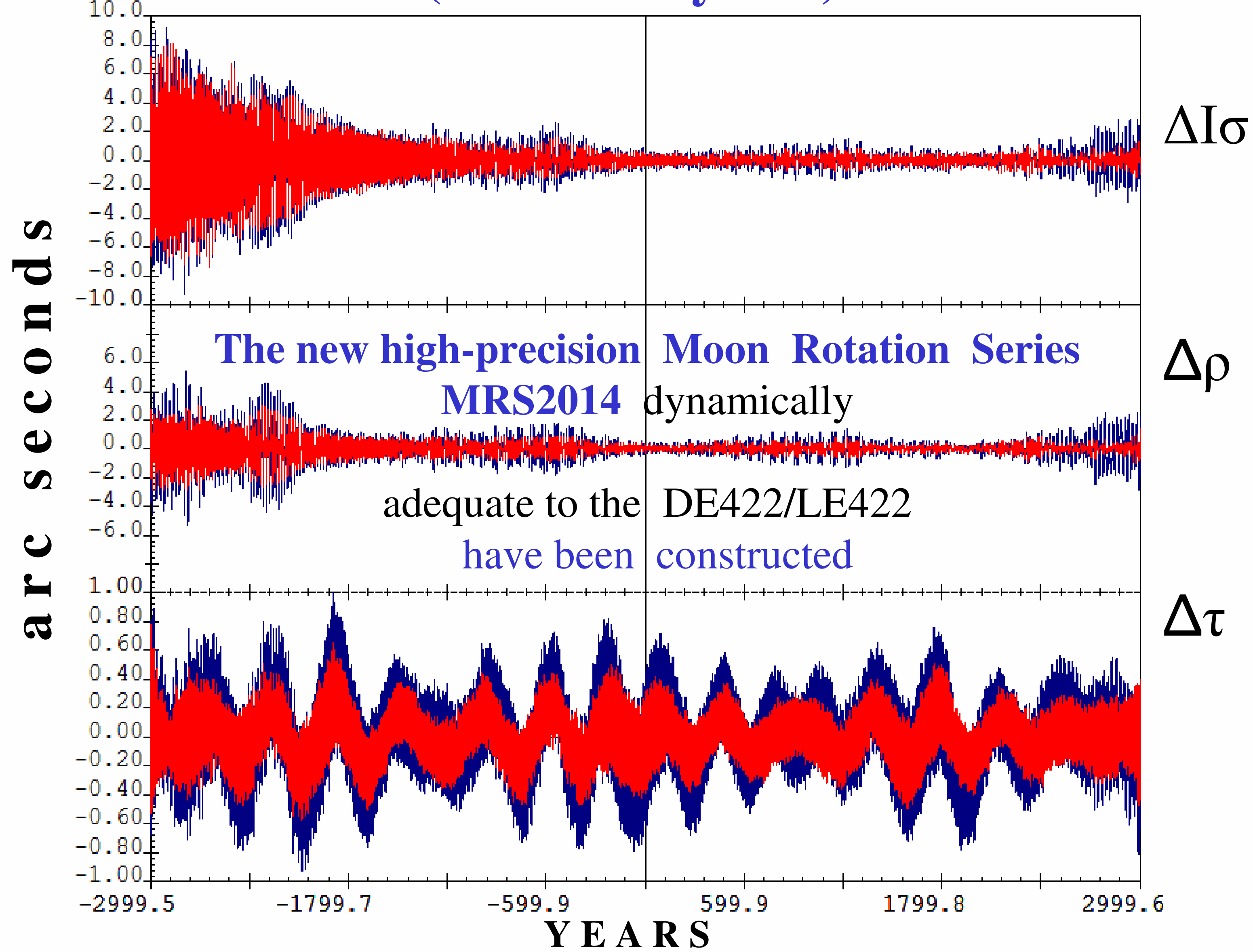
# Results:

Investigation of
the Moon rotation **over 6000 years**

Fig.2. **The numerical solutions minus MRS2011** (dynamically adequate to the JPL DE406/LE406 ephemeris) (Pashkevich, Eroshkin, 2011) and

**the numerical solutions 2 minus MRS2014-1** (1st iteration)

**(over 6000 years)**



# CONCLUSIONS

- **The new improved high-precision Rigid Earth Rotation Series RERS2014** (with including new sub-diurnal and diurnal periodical terms) dynamically adequate to the DE422/LE422 ephemeris over 2000 years have been constructed.

- **RERS2014 include** about 4113 periodical terms. **Discrepancies between the numerical solution and RERS2014** do not surpass 3 μas over 2000 years.

- **The new high-precision Moon Rotation Series MRS2014** dynamically adequate to the DE422/LE422 ephemeris over 6000 years have been constructed.

- **MRS2014 include** about 1520 periodical terms. **Discrepancies between the numerical solution and MRS2014** do not surpass 8 arc seconds over 6000 years,

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- **The new high-precision Moon Rotation Series MRS2014** dynamically adequate to the DE422/LE422 ephemeris over 6000 years have been constructed.

- **MRS2014 include** about 1520 periodical terms. **Discrepancies between the numerical solution and MRS2014** do not surpass 8 arc seconds over 6000 years,

**=>** it means a good consistency of the **MRS2014** series with the DE422/LE422 ephemeris.

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