On the systematics in apparent proper motions of radio sources observed by VLBI

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Motivation

- With the improvement of the VLBI system and the increase of number of VLBI observations, it is possible to identify systematic effects on radio source positions.

- The current celestial reference frame only takes into account the positions of the radio sources. However, some radio sources show apparent proper motions, thus it is not possible to consider their positions stable in time.

  ➔ The estimation and analysis of the systematic effects can provide us the physical reasons of these variations and an alternative way to correct the positions.

  ➔ Such a study can be used to improve the next realization of the celestial reference frame: ICRF3.
The study

- Estimate the systematics in apparent proper motions with VieVS.

- Compare results obtained with VieVS against results of Calc/Solve → comparison using same approach used by Titov and Lambert (2013)

- Approach applied by Titov and Lambert (2013) based on a three-step procedure. The steps are:
  1. Estimation of source coordinate time series from analysis of VLBI sessions.
  2. Least square method to fit proper motions to the source coordinate time series.
  3. Fit of spherical harmonics to the proper motions of the sources

| T&L (2013) – Calc/Solve | Present study – VieVS |
1. Radio source time series estimation

<table>
<thead>
<tr>
<th>VieVS</th>
<th>Calc/Solve</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>➢ Number of sessions: 4677</td>
<td>➢ Number of sessions: 5632</td>
</tr>
</tbody>
</table>

- With VieVS, sessions which are not suitable for reliable EOP determination were excluded: [http://lupus.gsfc.nasa.gov/files_IVS-AC/eop_exclusion.txt](http://lupus.gsfc.nasa.gov/files_IVS-AC/eop_exclusion.txt)
- Sources with less than three observations were excluded from the analysis.
- Models following the IERS 2010 Conventions
- Cut-off elevation angle: 5°
- Terrestrial datum: NNR/NNT constraints to VTRF2008 stations without breaks (in total)
For the sources:

<table>
<thead>
<tr>
<th></th>
<th>NNR</th>
<th>σ [mas]</th>
<th>Dip. ampl. [μas/yr]</th>
<th>α (°)</th>
<th>δ (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yes</td>
<td>2000</td>
<td>6.4 ±1.5</td>
<td></td>
<td>263 ±11</td>
<td>-20 ±12</td>
</tr>
<tr>
<td>No</td>
<td>2000</td>
<td>6.5 ±1.5</td>
<td></td>
<td>263 ±11</td>
<td>-23 ±12</td>
</tr>
</tbody>
</table>

Constraining each source to the ICRF2 position using very loose constraints (σ=2mas) is equivalent to apply loose NNR constraint with the same weight (1/σ^2)

4 solutions are computed with VieVS: each source is tied to ICRF2 positions using:

- σ=10^{-5} rad (~ 2000 mas)
- σ=10^{-6} rad (~ 200 mas)
- σ=10^{-7} rad (~20 mas)
- σ=10^{-8} rad (~2 mas)

and compared with the T&L (2013) solution (loose NNR constraint with σ=10^{-5} rad)
2. Proper motions fitted to the time series

<table>
<thead>
<tr>
<th>STEPS</th>
<th>$10^{-5}$ rad</th>
<th>$10^{-6}$ rad</th>
<th>$10^{-7}$ rad</th>
<th>$10^{-8}$ rad</th>
<th>T&amp;L(2013)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. Sessions with RMS ≤ 100ps</td>
<td>4556</td>
<td>4557</td>
<td>4562</td>
<td>4555</td>
<td>5583</td>
</tr>
<tr>
<td>b. Sources (spec. handling excl.)</td>
<td>3288</td>
<td>3288</td>
<td>3187</td>
<td>3137</td>
<td>3596</td>
</tr>
<tr>
<td>c. Sources for veloc. (after first iterative process*) with &gt;10 sessions</td>
<td>545</td>
<td>545</td>
<td>545</td>
<td>543</td>
<td>707</td>
</tr>
</tbody>
</table>

*1st Iterative process to remove outliers

- source $i \rightarrow$ if $|d\alpha \cos \delta_{ij} - \text{mean}_{d\alpha \cos \delta}| > T_1 * \sigma_{d\alpha \cos \delta_{ij}}$
- $|d\delta_{ij} - \text{mean}_{d\delta}| > T_1 * \sigma_{d\delta_{ij}}$

- session $j$ excluded for the source $i$

$T_1 = 90 \rightarrow \chi_i^2 = \Sigma \left(\frac{(x_{ij} - \text{mean}_x)^2}{\sigma_{ij}^2}\right) \sim 1$ (T&L, 2013)
Comparison of proper motions depending on the constraints

Velocities

- Constraint to the sources ($\sigma=10^{-5}$ rad)
- Loose NNR constraint ($\sigma=10^{-5}$ rad)

Number of sessions

- Constraint to the sources ($\sigma=10^{-6}$ rad)
- Loose NNR constraint ($\sigma=10^{-5}$ rad)
Comparison of proper motions depending on the constraints
3. Spherical harmonics fitted to the proper motion field

\[
\Delta \mu_\alpha \cos \delta = -d_1 \sin \alpha + d_2 \cos \alpha + r_1 \cos \alpha \sin \delta + r_2 \sin \alpha \sin \delta - r_3 \cos \delta \\
\Delta \mu_\delta = -d_1 \cos \alpha \sin \delta - d_2 \sin \alpha \sin \delta + d_3 \cos \delta - r_1 \sin \alpha + r_2 \cos \alpha
\]

\[(\Delta \mu_\alpha \cos \delta, \Delta \mu_\delta) \rightarrow \text{systematic part of the proper motion field} \]

\[(d_1, d_2, d_3) \rightarrow \text{electric harmonics of degree 1} \rightarrow \text{acceleration of the SSB} \]

\[(r_1, r_2, r_3) \rightarrow \text{magnetic harmonics of degree 1} \rightarrow \text{global rotation} \]

<table>
<thead>
<tr>
<th>STEPS</th>
<th>10^{-5} \text{ rad}</th>
<th>10^{-6} \text{ rad}</th>
<th>10^{-7} \text{ rad}</th>
<th>10^{-8} \text{ rad}</th>
<th>T&amp;L(2013)</th>
</tr>
</thead>
<tbody>
<tr>
<td>c. Sources for VSH* (after second iterative process**)</td>
<td>320</td>
<td>407</td>
<td>388</td>
<td>425</td>
<td>427</td>
</tr>
</tbody>
</table>

*VSH= vector of spherical harmonics

**2^{nd} Iterative process to exclude unstable sources

proper motion \( i \rightarrow \text{if residual}_\text{veloc}_i > T_2 \times \text{RMS(residuals)} \rightarrow \text{source } i \text{ excluded from the VSH estimation} \]

\[T_2 = 7 \ (T&L, 2013)\]
Vector of spherical harmonics for the approaches developed and the reference solution T&L 2013

<table>
<thead>
<tr>
<th>VSH</th>
<th>$10^{-5}$ rad</th>
<th>$10^{-6}$ rad</th>
<th>$10^{-7}$ rad</th>
<th>$10^{-8}$ rad</th>
<th>T&amp;L (2013)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_1$ [μas/yr]</td>
<td>58.6 ± 18.8</td>
<td>-0.2 ± 1.9</td>
<td>-0.6 ± 0.6</td>
<td>0.0 ± 0.4</td>
<td>-0.4 ± 0.7</td>
</tr>
<tr>
<td>$d_2$ [μas/yr]</td>
<td>-63.4 ± 18.6</td>
<td>-5.8 ± 1.6</td>
<td>-5.7 ± 0.7</td>
<td>-4.5 ± 0.4</td>
<td>-5.7 ± 0.8</td>
</tr>
<tr>
<td>$d_3$ [μas/yr]</td>
<td>-20.8 ± 10.0</td>
<td>0.8 ± 1.3</td>
<td>1.1 ± 0.7</td>
<td>0.8 ± 0.4</td>
<td>-2.8 ± 0.9</td>
</tr>
<tr>
<td>$r_1$ [μas/yr]</td>
<td>77.3 ± 11.9</td>
<td>0.31 ± 1.5</td>
<td>2.8 ± 0.7</td>
<td>2.5 ± 0.4</td>
<td>-1.1 ± 0.9</td>
</tr>
<tr>
<td>$r_2$ [μas/yr]</td>
<td>214.1 ± 13.3</td>
<td>-2.4 ± 1.8</td>
<td>0.6 ± 0.7</td>
<td>0.4 ± 0.4</td>
<td>1.4 ± 0.8</td>
</tr>
<tr>
<td>$r_3$ [μas/yr]</td>
<td>-1151.5 ± 37.1</td>
<td>-20.9 ± 1.5</td>
<td>-2.0 ± 0.5</td>
<td>0.6 ± 0.3</td>
<td>0.7 ± 0.6</td>
</tr>
</tbody>
</table>

loose

tight
Source distribution and amplitude of the dipole depending on the constraints

- Distribution of radio sources w.r.t. the declination
- Dipole amplitude w.r.t. the constraint
Summary and conclusions

- The number of suitable sources to estimate velocities was ~ 550 (out of ~3200 sources) → in most of the cases the sources were observed in less than 11 sessions, most of them, VCS sources.

- Systematics in apparent proper motions were estimated with VieVS using different weights. This preliminary study aims to be the first step to create the final configuration for systematics estimation.

- The best agreement w.r.t T&L (2013) is when VieVS applies a constraint of $10^{-7}$ rad ~ 20mas. However, there are significant differences for the third component of the dipole ($d_3$). These should be study in depth.

- A comparison between different software and approaches for the estimation of very small effects, such as the galactic aberration effect, from VLBI observations are essential.
Future goals

- Determine the most reliable results for galactic aberration and for gravitational wave systematics on the proper motions.

- Develop other approaches to estimate the systematic effects and compare them.
  - Estimating the radio source velocities directly in a global solution with VieVS, and the estimating the vector spherical harmonics by a fit to the velocities.
  - Estimating the vector spherical harmonics directly in the global solution.
Thank you for your attention

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