# Triaxial Earth's rotation: Chandler wobble, free core nutation and diurnal polar motion

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## 1 Introduction

For a biaxial elastic Earth, the free motions CW and FCN are circular motions.

When non-rotational-symmetric effects are taken into consideration, these free motion would become elliptical ones rather than circular ones.

An ellpitical motion can be decomposed into two circular motions with opposite frequencies. So in the frequency domain, for every free motion, CW or FCN, there are two resonant frequencies. If the excitation function is very near to these two resonant frequencies, resonance will happen.

In this study, we used a matrix based method to study the influence of triaxility on the frequencies of CW and FCN and on the diurnal polar motion.

## 2 Influence of triaxiltiy on frequnecies of CW and FCN

To estimate the frequencies of CW and FCN, we treat the Euler-Liouville's equation as a set of linear differential equations with constant coefficients. Then the frequencies can be derived by using the theory of solution of a set of linear differential equations with constant coefficients.

We find that the influence of triaixlity on CW is very small, which is in accordance with the previous stuides.

When it comes to the influence of triaixlity on FCN, we find that our results suggest that considering trixailty of the core would make the FCN period shorter, which is in contradiction with the conclusion given by van Hoolst and Dehant (2002).

However, the theoretical system used by van Hoolst and Dehant(2002) is different from the theoretical system used in this study, which is the same as that used by Mathews et al., (2002) and that used by Chen and Shen (2010). The lateral inhomogeneity of the Earth is modelled as triaxial buldge in the study of van Hoolst and Dehant(2002), which means that the expression of FCN contains contribution of lateral inhomogeneity.

After we consider the contribution of lateral inhomogeneity, the period of FCN is also increased as shown in table 1.

Table 1 Influence of triaxilaity on the period of FCN

| | Biaxial Core | Triaxial Core |
|---|---|---|
| van Hoolst and Dehant (2002)[b] | 429.613213 | 0.018699 |
| This study (triaxility alone) | 430.941420 | -0.007421 |
| This study (traixility+lateral inhomogeneity) | 430.941420 | 0.004864 |

## 3 Resonance at negative FCN

We solve the polar motion equaiton for a two-layer triaxial Earth model. The wobble can be decomposed into two parts: the same frequency response $m^s$ that is excited by exciation process $\psi^s$ of same frequency and opposite frequency response $m^o$ that is excited by excitation process $\psi^o$ of opposite frequency. The resonance can be shown in Figure 1.

To find out how large the resonaces shown in figure 1 would be, we use the ocean tide model C of Chao(1996) as the excitation function. We focus on the resonance strength at negative FCN the resonance at negative FCN(1d) is only slightly samller than resonance at FCN (1a). The results are shown in Table 2.

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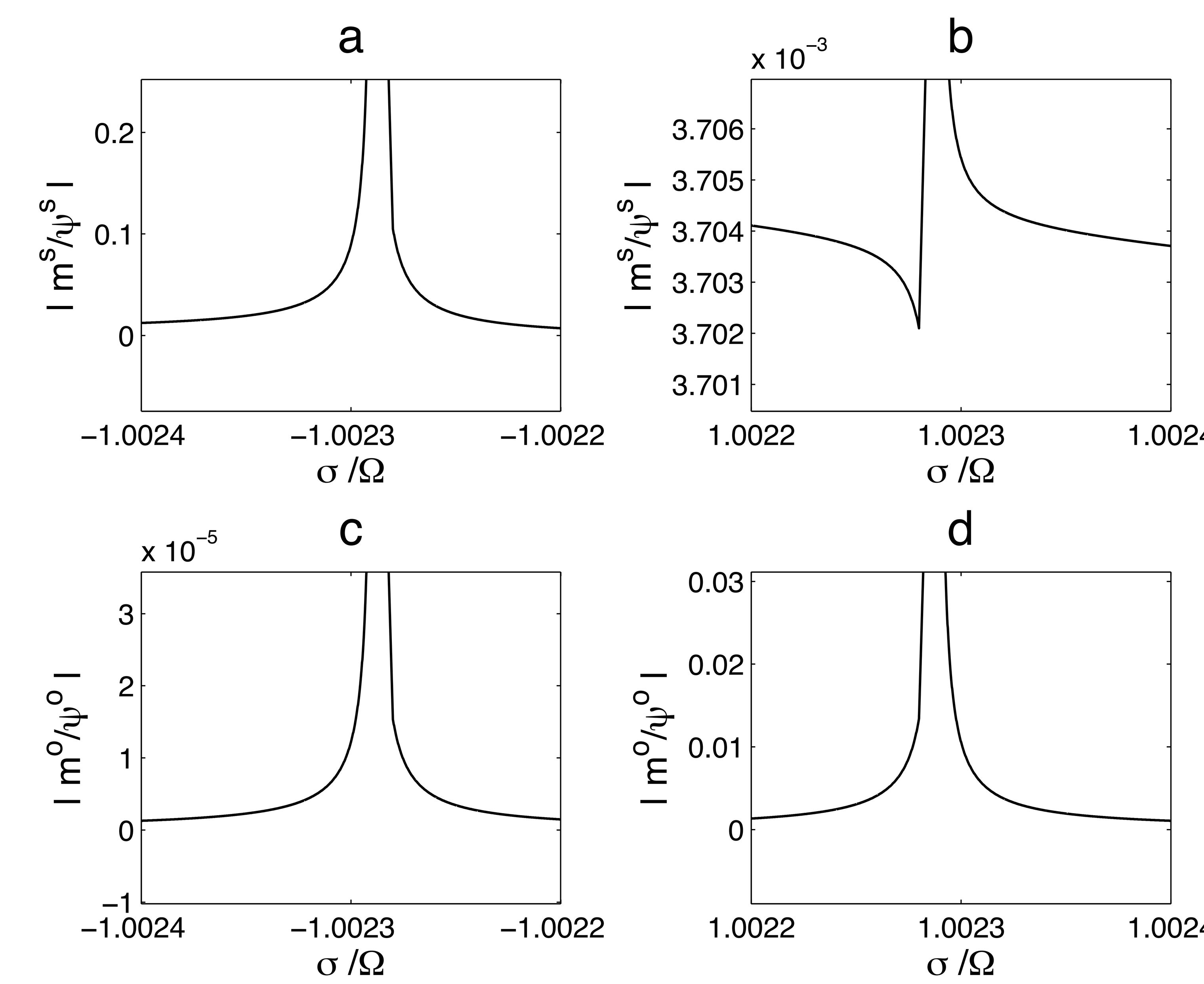


Figure 1 Resonances near FCN (a,c) and negative FCN(b,d). The resonances in (b,c,d) won't exist if the Earth mode is biaxial.

Table 2 Difference of polar motion amplitude between biaxial case and triaxial case. The closer the exciation is to the negative FCN, the larger the difference between the biaxial case and triaxial case.

| Tide | Frequency (cpsd) | Triaxial-biaxial(μas) |
|---|---|---|
| Q1 | 0.891 | -0.002 |
| O1 | 0.927 | -0.018 |
| P1 | 0.995 | -0.089 |
| K1 | 1.000 | -0.954 |
| Negavtive FCN | 1.002288790 | |

## 4 Conclusion

Trixaility would alter the frequencies of CW and FCN by a small amount. Triaxility also makes the free motion of CW and FCN become elliptical motions rather than circular motions as suggested by biaxial Earth model.

Since an elliptical motion can be decomposed into two circular motion, there is an extra resonance frequency at complex Fourier's spectrum.

At negative FCN frequency, we find that this resonance would bring a difference of the order of 1μas.

**References**

Chao, B. F., R. D. Ray, J. M. Gipson, G. D. Egbert, and C. Ma (1996), Diurnal/semidiurnal polar motion excited by oceanic tidal angular momentum, Journal of Geophysical Research, 101(B9), 20151-20163, doi:10.1029/96JB01649.

Chen, W., and W. Shen (2010), New estimates of the inertia tensor and rotation of the triaxial nonrigid Earth, Journal of Geophysical Research, 115(B12), B12419, doi: 10.1029/2009JB007094.

Mathews, P. M., T. A. Herring, and B. A. Buffett (2002), Modeling of nutation and precession: New nutation series for nonrigid Earth and insights into the Earth's interior, Journal of Geophysical Research, 107(B4), 2068, doi:10.1029/2001JB000390.

van Hoolst, T., and V. Dehant (2002), Influence of triaxiality and second-order terms in flattenings on the rotation of terrestrial planets I. Formalism and rotational normal modes, Physics of the Earth and Planetary Interiors, 134(1-2), 17-33, doi:10.1016/S0031-9201(02)00068-7.