# Resonances in the Solar and exoplanetary systems

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# Abstract

Dynamical problems on the orbital resonances, including mean motion resonances (both two-body and three-body ones) and secular resonances, are considered in application to the dynamics of the Solar and exoplanetary systems. The analyzed systems include multiplanetary (those with two or more than two planets) systems and planetary systems of double stars. Theoretical methods and criteria for revealing stability or instability of various planetary configurations are described.

# Resonances in the dynamics of planetary systems

**Resonances**, **interaction** of resonances, and the **chaotic behaviour**, caused by this interaction, play an essential role in the dynamics of bodies of the Solar and exoplanetary systems, in many respects determining the observed architecture of the planetary systems.

The orbital resonances in celestial-mechanical systems subdivide in **mean motion resonances** and **secular resonances**. The first class represents commensurabilities between mean orbital frequencies (of two or even a greater number of orbiting bodies), and the second one represents commensurabilities between orbital precession frequencies. Both kinds often manifest itself in the dynamics of the Solar and exoplanetary systems.

The planetary resonances are believed to be the natural outcome of the primordial **migration** of the planets due to their gravitational interaction with the protoplanetary disk. The presence of mean motion resonances and their interaction implies the possibility for **dynamical chaos** in the dynamics of exoplanets.

# Interaction of resonances
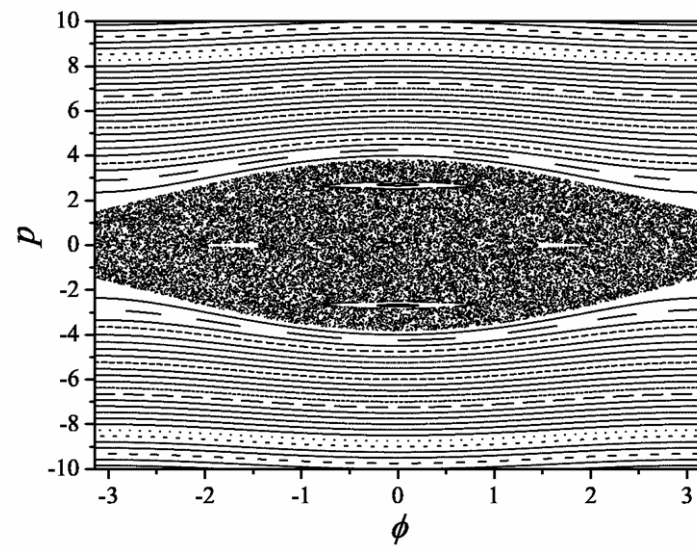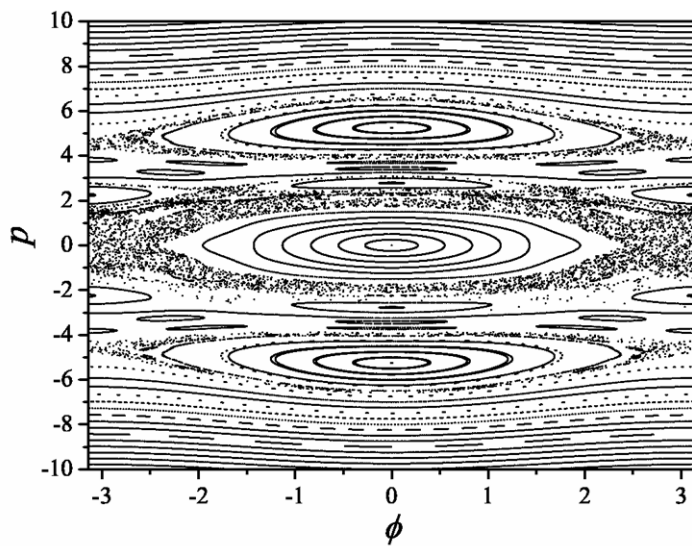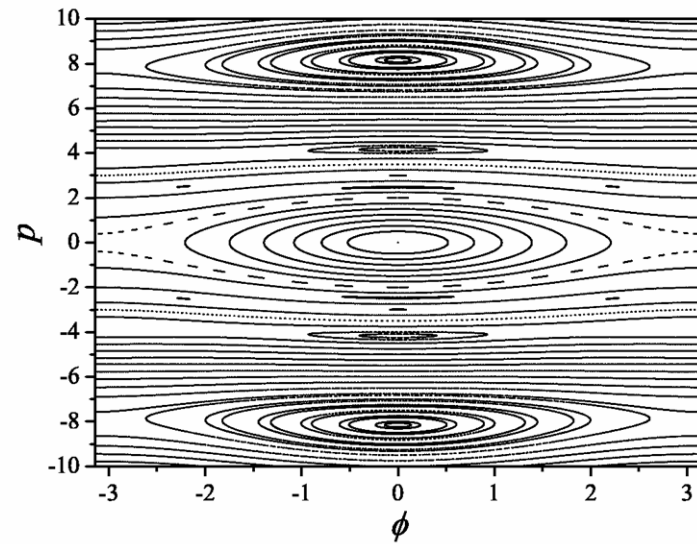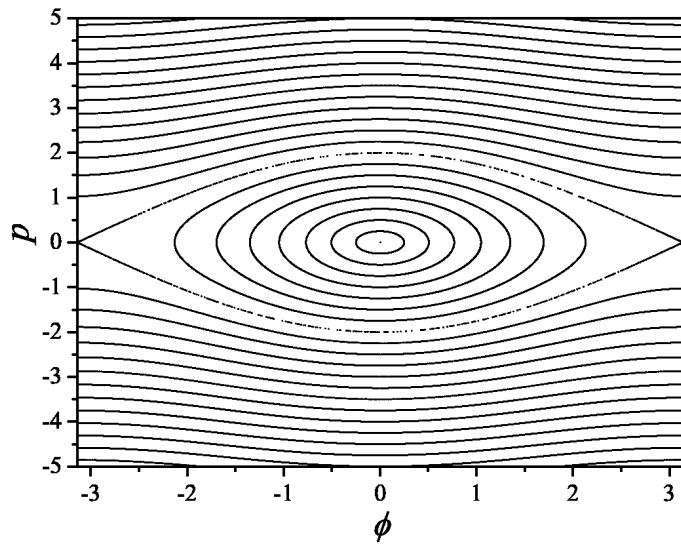
# Stability criteria

Up to now, analytical, as well as numerical-experimental, *criteria of stability* of planetary systems have been developed. In the first case, they are based on adaptation of the *Hill criterion* (Gladman, 1993) and *Chirikov's resonance overlap criterion* (Duncan et al., 1989; Mudryk, Wu, 2006).

In the second case, they are based on computations of *MEGNO* (Cincotta et al., 2003; Goździewski, 2003; Goździewski et al., 2013), *Lyapunov exponents* (Popova, Shevchenko, 2013), fundamental frequencies of motion (the *frequency analysis*, Correia et al., 2009; Laskar, Correia, 2009), as well as on numerical assessment of the escape/encounter conditions (Holman, Wiegert; 1999; Pilat-Lohinger et al., 2003; Kholshevnikov, Kuznetsov, 2011).

## The Hill criterion

The radius of the stability inner zone is directly proportional to the radius of the Hill sphere calculated at the secondary's pericenter:

$$r_\mathrm{H} \approx (\mu/3)^{1/3} a (1 - e),$$

where $\mu = M_\mathrm{sec}/M_\mathrm{prim}$ is the secondary-primary mass ratio. This formula renders the so-called "Hill sphere at pericenter scaling". The Hill sphere ordinary radius is $r_\mathrm{H} = (\mu/3)^{1/3} a$.

Hamilton, D.P., & Burns, J.A. 1992, Icarus 96, 43

# Stability criteria

## The Wisdom criterion

In the planar circular restricted three-body problem, the radial half-width of the instability neighborhood of the pertuber's orbit is given by

$$\Delta a_{\text{overlap}} \approx 1.3 \mu^{2/7} a',$$

where $\mu = m_2/(m_1 + m_2)$ is the mass parameter, $a'$ is the semimajor axis of the perturber; $e \lesssim 0.15$. The particles with $a$ within the interval $a' \pm \Delta a_{\text{overlap}}$ move chaotically.

The value of $p$, critical for the overlap of the first order resonances $(p + 1) : p$, is

$$p_{\text{overlap}} \approx 0.51 \mu^{-2/7}.$$

Wisdom, J. 1980, Astron. J., 85, 1122
Duncan, M., Quinn, T., & Tremaine, S. 1989, Icarus, 82, 402

# Resonances in the Solar system

In the Solar system dynamics, some ***approximate commensurabilities*** of the orbital periods of planets are well-known: Jupiter-Saturn (the ratio of orbital frequencies ≈ **5/2**), Saturn-Uranus (≈ **3/1**), Uranus-Neptune (≈ **2/1**); not to mention the Neptune-Pluto resonance (**3/2**).

At the end of eighties, Sussman and Wisdom (1988, 1992) and Laskar (1989) made first estimates of the ***Lyapunov time*** of the Solar planetary system in numerical experiments. It turned out to be not at all infinite, i.e., the motion of the Solar system is not regular; moreover, its Lyapunov time is three orders less than its age. Murray and Holman (1999) conjectured that the revealed chaos is due to interaction of ***subresonances*** in a ***multiplet*** corresponding to a particular ***three-body resonance*** Jupiter-Saturn-Uranus.

# Resonances in the Solar system

In the Solar system, many planetary satellites form resonant or close-to-resonant configurations. In the Jovian satellite system, the Galilean satellites Io and Europa, as well as Europa and Ganymede, are in the 2/1 mean motion resonance; thus the system of these three satellites is involved in the ***three-body resonance 4:2:1*** (called the ***Laplace resonance***). In the Saturnian system, Mimas and Tethys, as well as Enceladus and Dione, are in the 2/1 mean motion resonance, Dione and Rhea are close to resonance 5/3, Titan and Hyperion are in the 4/3 resonance. In the Uranian system, all resonances are approximate: Miranda and Umbriel are close to resonance 3/1, Ariel and Umbriel — 5/3, Umbriel and Titania — 2/1, Titania and Oberon — 3/2.

Captures of satellite systems in orbital resonances are natural stages of tidal evolution of these celestial-mechanical systems. Of particular interest is that the motion of the Prometheus-Pandora system (the 16th and 17th Saturnian satellites — the shepherd satellites of the ring F) is chaotic, as follows from both observation and theory. The Lyapunov time of this system, which resides in the mean motion resonance 121/118, was estimated both in numerical experiments and analytically; it turned out to be only ≈ 3 yr (Goldreich and Rappaport, 2003; Shevchenko, 2008).

# Resonant trans-Neptunian objects



Sheppard, S. 2006, in: New Horizons in Astronomy, ASP Conf. Series, 352, p. 3
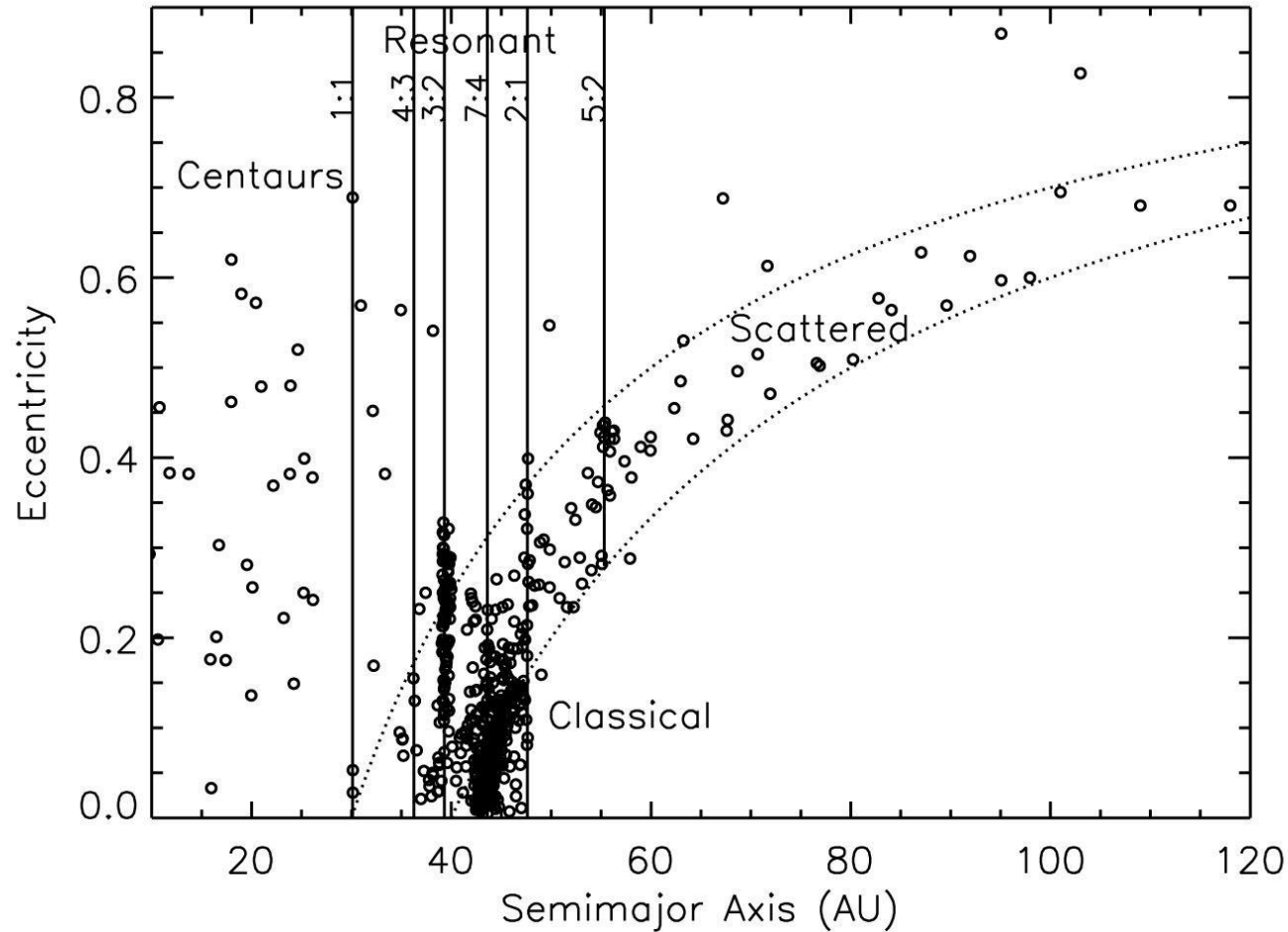
# Multiplanet systems

# Multiplanet systems

About one third of the discovered exoplanets reside in **multiplanet systems**, i.e., the systems with two or more planets (H.Rein, 2012, MNRAS, 427, L21).

Orbital resonances are ubiquitous in planetary systems, as confirmed in computations of the behaviour of resonant arguments. For many systems, the observational data on planetary orbital elements still suffer from uncertainties; however, the occurrence of low-order resonances (such as **2/1** and **3/2**) is statistically significant (Wright et al., 2011; Fabrycky et al., 2012), especially in pairs of planets with similar masses (Ferraz-Mello et al., 2006).

The well-known systems with planets in the **2/1 resonance** are Gliese 876 and HD 82943, in the **3/1 resonance** is the 55 Cnc system,  while Gliese 876 is an example of a system where three planets are involved in the **Laplace resonance 4:2:1** (Martí et al., 2013) like the innermost three Galilean satellites of Jupiter. Moreover, a closely packed multi-planet resonant system, KOI-730, exhibiting a **8:6:4:3 mean motion resonance**, was reported (Lissauer et al., 2011).

# KOI-730



**KOI-730** is a yellow dwarf (G5V) with 4 planets, discovered by the transit method.
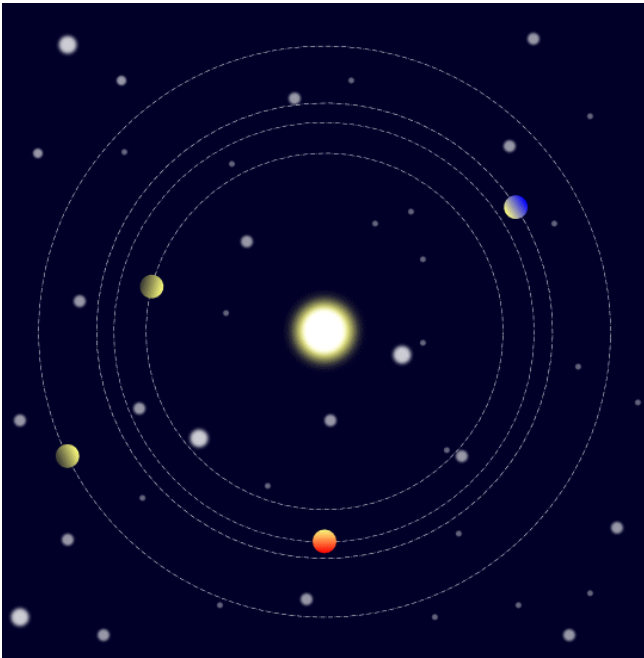
The planetary system KOI-730 is the most remarkable example of a ***closely packed resonant system***.

Radii of planets: b, c, d, e: 1.8, 2.1, 2.8, 2.4 $R_{Earth}$.
Orbital periods: 7.4, 9.8, 14.8, 19.7 d.

The orbital frequencies satisfy the ratios **8:6:4:3**.

"This resonant chain is potentially the missing link that explains how planets that are subject to migration in a gas or planetesimal disk can avoid close encounters with each other, being brought to a very closely packed, yet stable, configuration."
(J.J.Lissauer et al., 2011, ApJ Suppl. Ser. 197, 8).
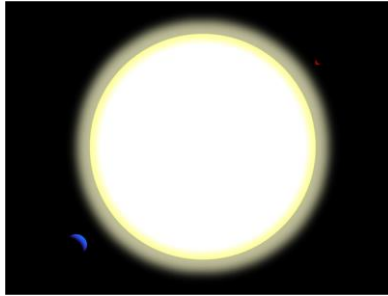
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# Upsilon And



Image Wikimedia Commons

**Upsilon And** is a double star consisting of a yellow (F8) dwarf and a red dwarf; the size of the binary is 750 AU. Three giant planets orbit around the first one (R.P.Butler et al.,1999, ApJ 526, 916).

Masses of the planets b, c, d: >0.687, >1.97, >3.93 $M_J$.
Orbital periods: 4.6, 241, 1290 d.

Planets c and d are close to **resonance 11/2**. The system as a whole is stable (T.A.Michtchenko, R.Malhotra, 2004, Icarus, 168, 237).

# Planetary dynamics
# in double star systems

## The Holman–Wiegert criterion

In the planar problem, the radius $a_{cr}$ of the instability inner zone for the initially circular prograde outer particle orbits is given by the numerical-experimental fitting relation

$$a_{cr}/a_b = 1.60 + 5.10e_b - 2.22e_b^2 + 4.12\mu - 4.27e_b\mu - 5.09\mu^2 + 4.61e_b^2\mu^2,$$

where $\mu = m_2/(m_1 + m_2)$ is the binary's mass parameter, $a_b$ and $e_b$ are the binary's semimajor axis and eccentricity.

Holman, M.J., & Wiegert, P.A. 1999, Astron. J., 117, 621

# Stability criteria

## The "Kepler map" criterion

The energy width of a one-sided chaotic band in the vicinity of the perturbed parabolic orbit scales as the power 2/5 of the system mass parameter (Petrosky, 1986):

$$\Delta E_{\mathrm{cr}} \propto \mu^{2/5},$$

where $\mu = m_2/(m_1 + m_2)$ is the mass parameter. The particles with $E$ within the interval $-\Delta E_{\mathrm{cr}} < E < 0$ move chaotically.

Using the formulas for the parameters $\lambda$ and $W$ of the Kepler map (Shevchenko, 2011), one has

$$\Delta E_{\mathrm{cr}} \simeq A\mu^{2/5} q^{-1/10} \exp\left(-Bq^{3/2}\right),$$

where $A = 3^{2/5}\pi^{3/5}2^{-1/2}K_G^{-2/5} = 2.2061\ldots$, $B = 2^{5/2}/15 = 0.3771\ldots$, $K_G = 0.9716\ldots$.

The critical eccentricity is given by

$$e_{\mathrm{cr}} = 1 - 2q\Delta E_{\mathrm{cr}}.$$

The orbits with $e > e_{\mathrm{cr}}$ for a given value of $q$ are chaotic.

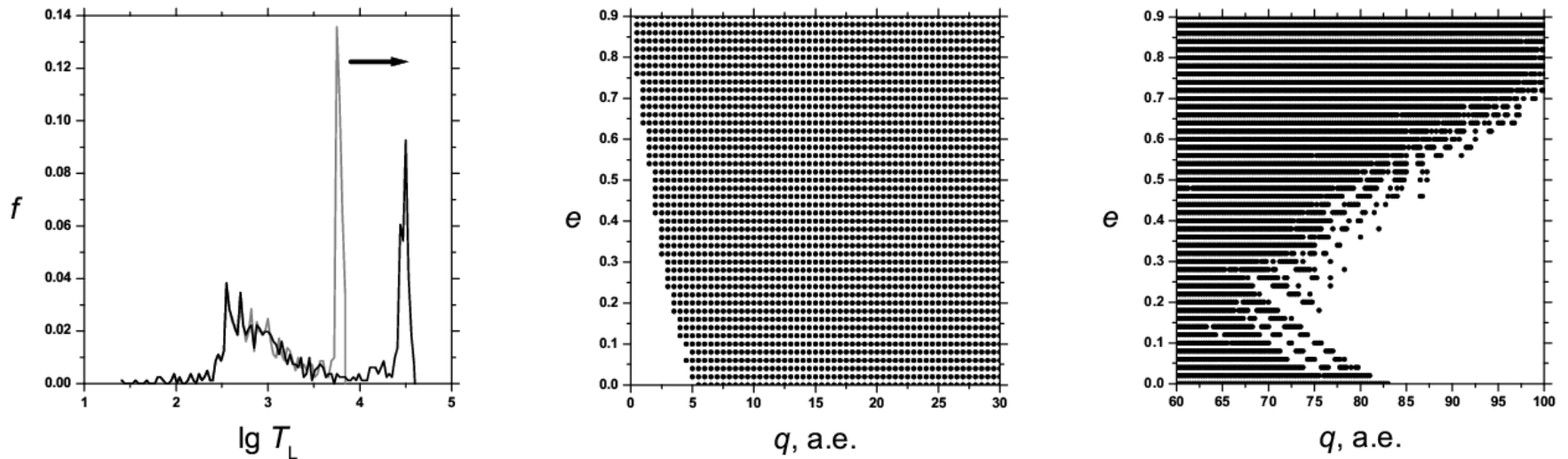# Planetary systems of double stars

| System | $m_1$ ($m_S$) | $m_2$ ($m_S$) | $m_p$ ($m_J$) | $a_b$ (AU) | $e_b$ | $a_p$ (AU) | $e_p$ |
|---|---|---|---|---|---|---|---|
| *α* Centauri | 1.11 | 0.93 | – | 23.4 | 0.52 | – | – |
| *Kepler*-16 | 0.69 | 0.20 | 0.33 | 0.22 | 0.16 | 0.71 | 0.007 |
| *Kepler*-34 | 1.05 | 1.02 | 0.22 | 0.23 | 0.52 | 1.09 | 0.18 |
| *Kepler*-35 | 0.89 | 0.81 | 0.13 | 0.18 | 0.14 | 0.60 | 0.042 |

D.Pourbaix et al., 1999, Astron. Astrophys., 344, 172.
L. Doyle et al., 2011, Science, 333, 1602.
W.F.Welsh et al., 2012, Nature, 481, 475.
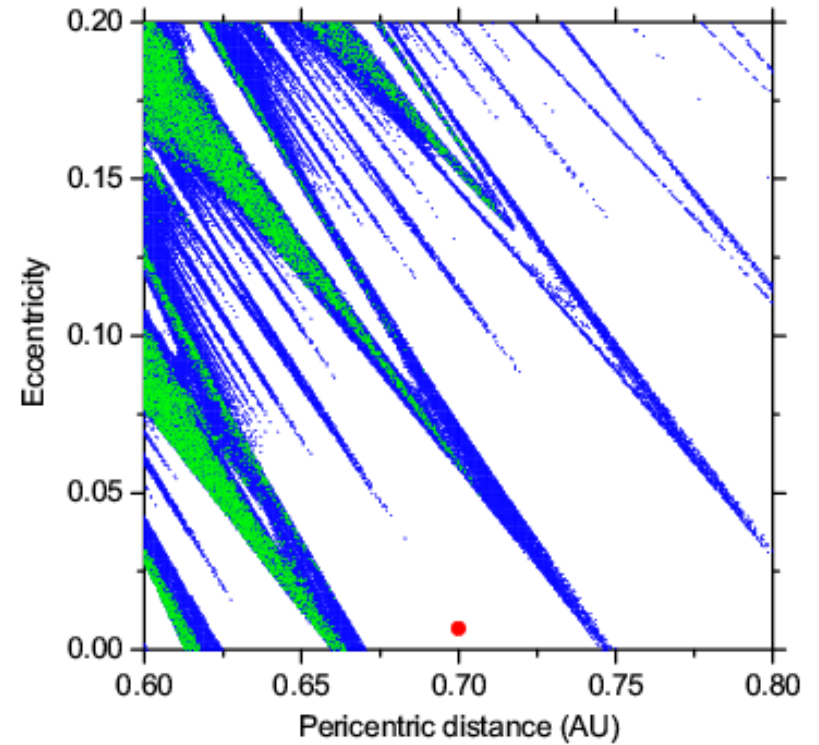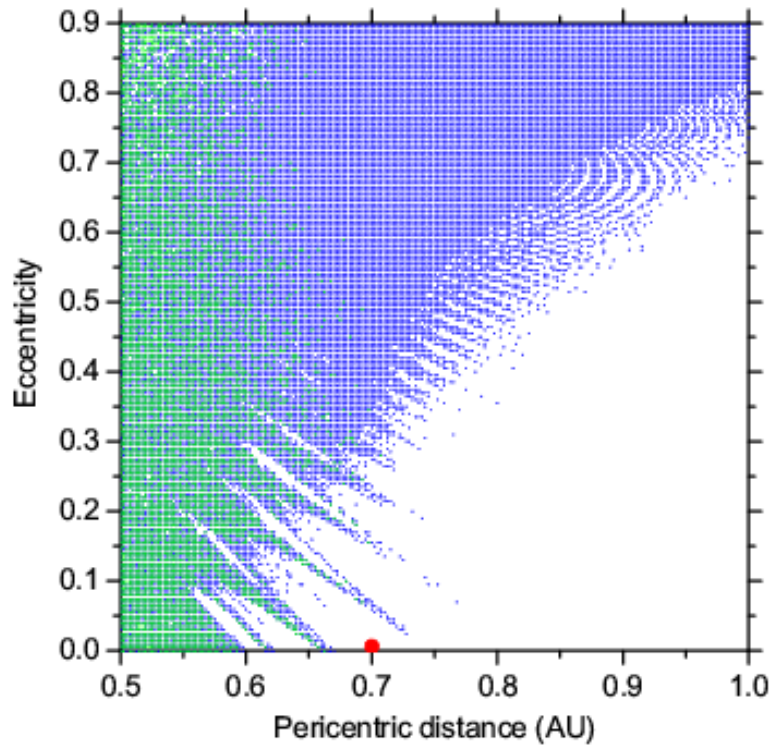
# Planetary dynamics in the α Centauri system
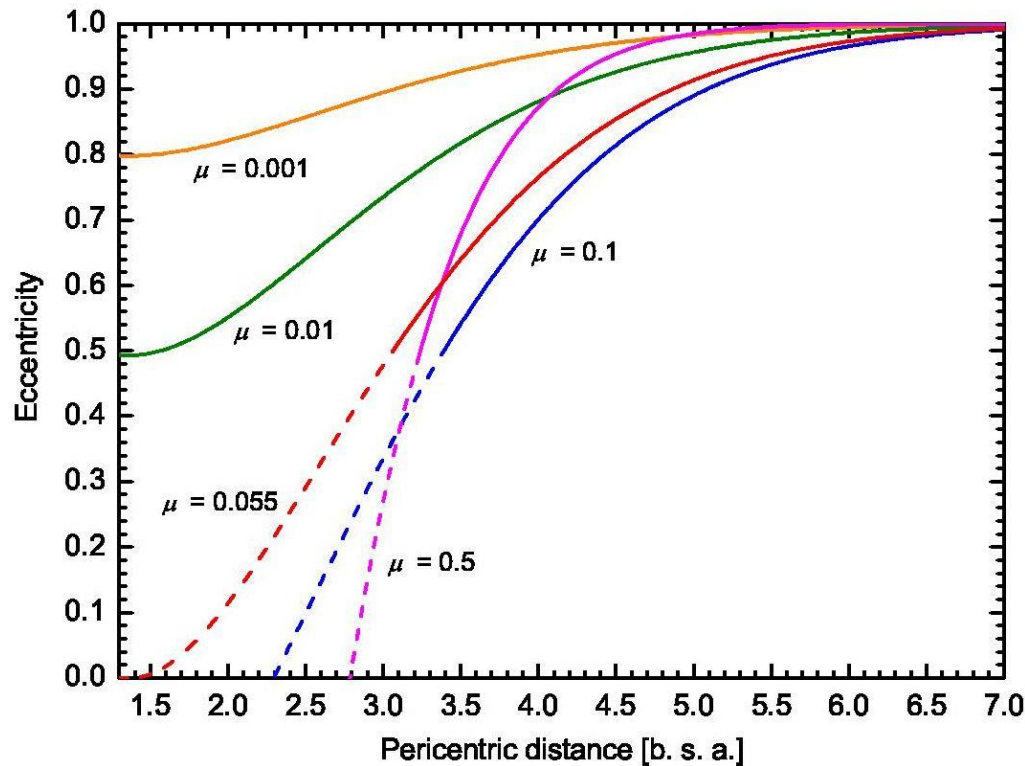
# Kepler-16



L.Doyle et al., 2011, Science, 333, 1602.

# Planetary dynamics in the Kepler-16 system
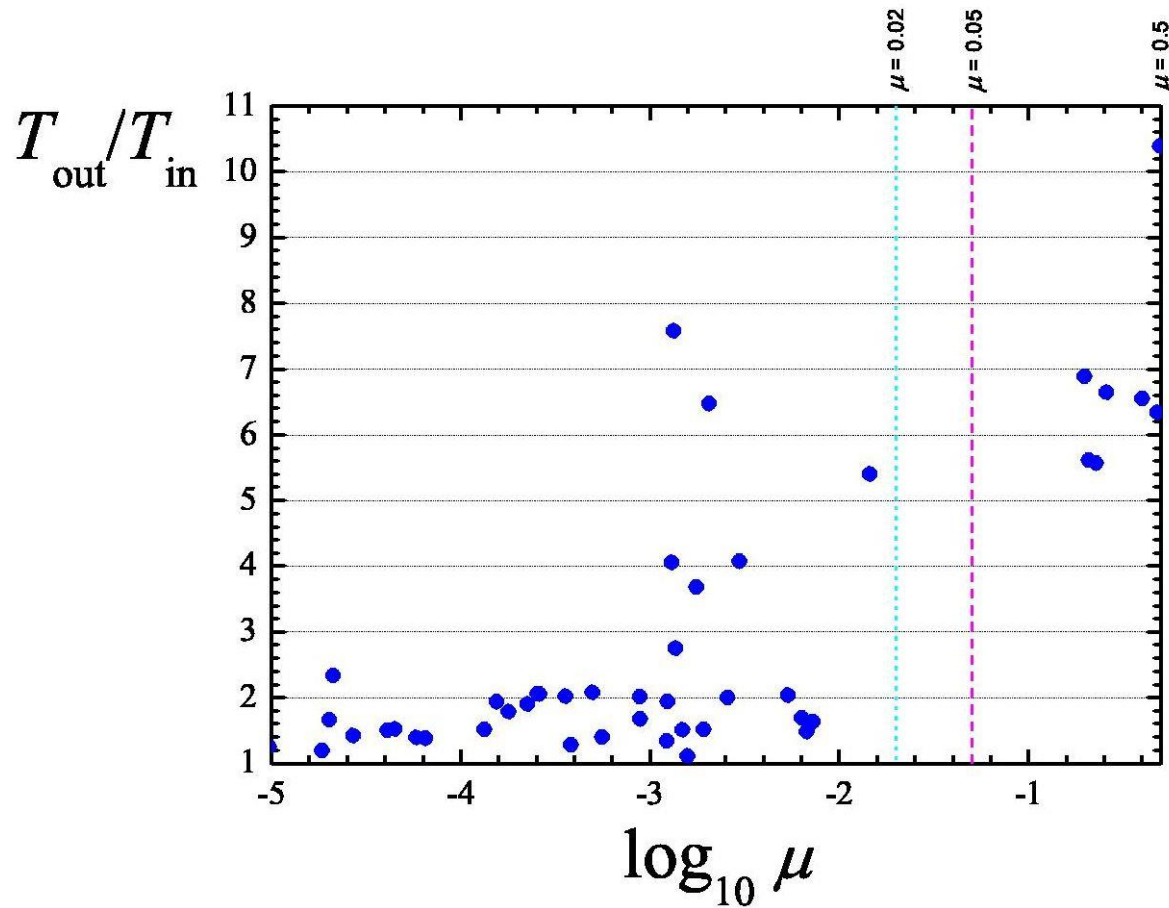


E.A.Popova, I.I.Shevchenko, 2013, ApJ, 769, 152.

# Extent of chaotic domains: general



I.I.Shevchenko, *Chaotic zones around gravitating binaries*.
ArXiv: http://arxiv.org/abs/1405.3788 (2014).

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# Challenges and prospects

- Stability criteria development.
- Prevalence of particular resonances in exoplanetary systems.
- Resonant architectures of closely packed exosystems.
- Resonant and near-resonant architectures of circumbinary systems (TNO analogies).
- Circumbinary planets at the "edge of chaos".