Work related with IAU C52: RIFA

M.H. Soffel and W. Han

Main work

- ecliptic in a relativistic framework
- fully documented relativistic VLBI model

Related work

- the post-linear metric (PPN-metric for light rays)
- models for bodies with higher spin-multipole moments

Ecliptic

IAU 2006 Resolution B1: Recognizing 2. the need for a definition of the ecliptic for both astronomical and civil purposes …..

BUT

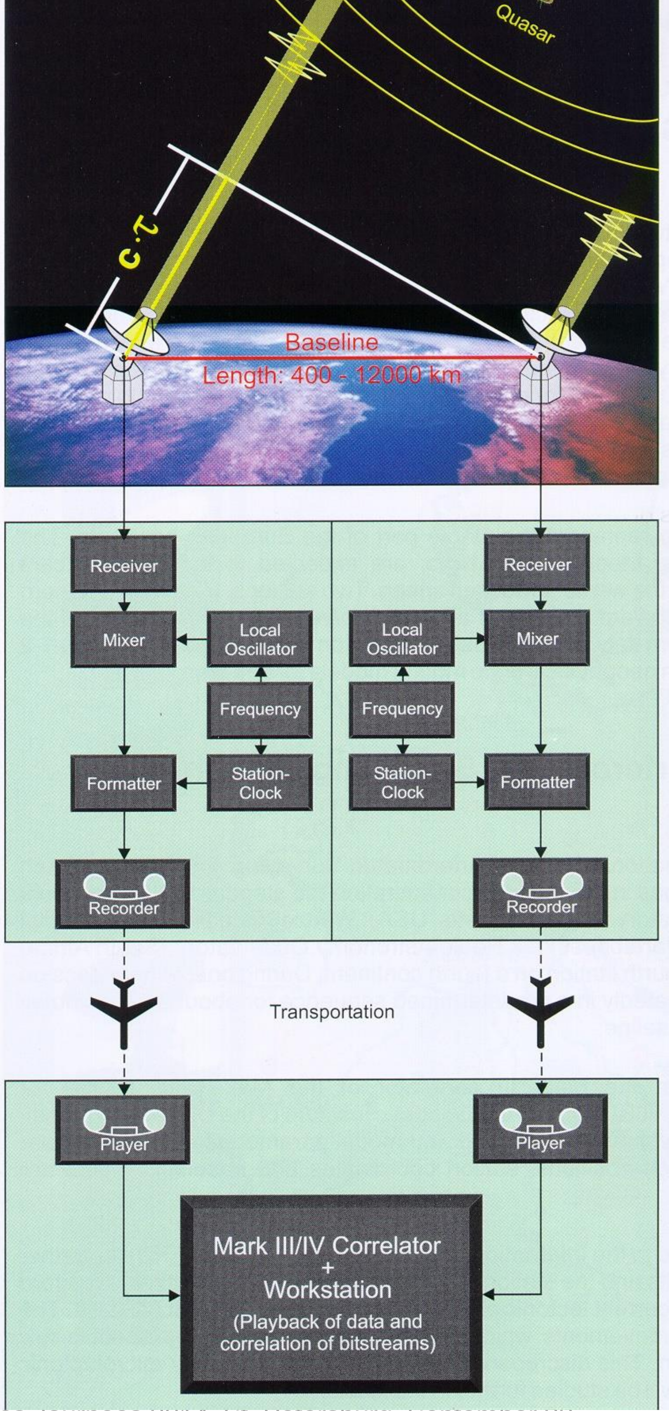
- the ‚ecliptic' basically has to be defined in the BCRS
- the BCRS to GCRS transformation is a 4-dimensional space-time transformation
- one cannot directly transform a BCRS-ecliptic as some BCRS Euclidean spatial coordinate (TCB = const.) plane into the GCRS
- for special purposes an additional convention for some ‚GCRS-ecliptic' might be adopted

more information: talk by Nicole Capitaine on the subject

Relativistic VLBI model

Theoretical model should have an accuracy of at least 1 ps

It must be based upon Einstein's theory of gravity



Standard reference:

IERS Technical Note No. 36
G. Petit, B.Luzum IERS Conventions 2010

Based upon CONSENSUS MODEL

1. Fanselow-Thomas-Treuhaft-Sovers
2. Shapiro
3. Hellings-Shadid-Saless
4. Soffel-Müller-Wu-Xu
5. Zhu-Groten

Eubanks, T.M. (ed.) Proc. of the U. S. Naval Obs. Workshop on Relativistic Models for use in Space Geodesy, USNO, Washington, D. C.

Of importance:
Klioner, S.A., 1991, in: Proc. AGU Chapman Conf. On Geodetic VLBI
W.E.Carter (ed.), NOAA Technical Report NOS 137 NGS 49

The consensus model:

- designed for very remote radio sources (quasars) only (no parallax, no proper motion)

- designed for Earth-bound baselines only

- 1 ps accuracy for the group delay

- not a coherent model; needs detailed documentation

Goals:

To improve the consensus model:

- accuracy 0.1 ps or less

- much larger baselines (e.g., interplanetary)

- valid also for near sources (e.g., on the Moon)

- coherent and well documented

Some kind of working group was established

Neil Ashby
Marshall Eubanks
Toshio Fukushima
Sergei Klioner
Michael Soffel
Slava Turyshev

Wenbiao Han (guest)

We first concentrated on the following papers:

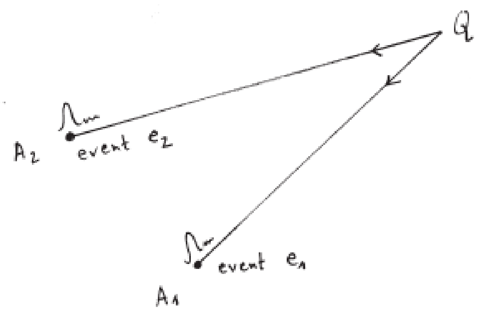
Damour, T., Soffel, M., Xu, C., 1991, Phys.Rev., D 43, 3273

Klioner, S.A., 1991, in: Proc. AGU Chapman Conf. On Geodetic VLBI
W.E.Carter (ed.), NOAA Technical Report NOS 137 NGS 49

Klioner, S.A., Kopeikin, S., 1992, Astron.J., 104, 897

Sekido, M., Fukushima, T., 2006, J.Geod., 80, 137

checked all calculations, tried to find simpler derivations and started with an exhaustive documentation



Any theoretical model is based upon two reference systems:

- BCRS $(t,\mathbf{x})$ $t =$TCB
- GCRS $(T,\mathbf{X})$ $T =$TCG

any event has coordinates $(t,\mathbf{x}) \leftrightarrow (T,\mathbf{X})$

So far the following issues are treated in the

Document on relativistic VLBI:

- **definition of baselines, b:BCRS & B:GCRS**
- **expressions for observable group delay (TT) and B**
- **Relations with BCRS quantities (TCB, b)**
- **expression for geometrical and gravitational delays**

$$\Delta t \equiv t_2 - t_1 = (\Delta t)_{\rm geom} + (\Delta t)_{\rm grav}$$

BCRS metric

$$g_{00} = -1 + \frac{2w}{c^2} - \frac{2w^2}{c^4} + O_6 \quad ,$$

$$g_{0i} = \frac{4w_i}{c^3} + O_5 \quad ,$$

$$g_{ij} = \left(1 + \frac{2w}{c^2}\right)\delta_{ij} + O_4 \quad .$$

For a single body

$$w = G\sum_{l=0} \frac{(-1)^l}{l!}\partial_L \left(\frac{1}{r}M_L^{\pm}\right) + \frac{1}{c^2}\partial_t(\Lambda - \lambda) + O_4 \quad ,$$

$$w_i = -G\sum_{l=1} \frac{(-1)^l}{l!}\left(\partial_{L-1}\left(\frac{1}{r}\right)\frac{d}{dt}M_{iL-1} + \frac{l}{l+1}\epsilon_{ijk}\partial_{jL-1}\left(\frac{1}{r}\right)S_{kL-1}\right)$$
$$- \frac{1}{4}\partial_i(\Lambda - \lambda) + O_2 \quad ,$$

BCRS metric

$$g_{00} = -1 + \frac{2w}{c^2} - \frac{2w^2}{c^4} + O_6 \quad ,$$

$$g_{0i} = \frac{4w_i}{c^3} + O_5 \quad ,$$

$$g_{ij} = \left(1 + \frac{2w}{c^2}\right)\delta_{ij} + O_4 \quad .$$

1PN for light-rays

For a single body

$$w = G \sum_{l=0} \frac{(-1)^l}{l!} \partial_L \left(\frac{1}{r} M_L^{\pm} \right) + \frac{1}{c^2} \partial_t (\Lambda - \lambda) + O_4 \quad ,$$

$$w_i = -G \sum_{l=1} \frac{(-1)^l}{l!} \left(\partial_{L-1} \left(\frac{1}{r} \right) \frac{d}{dt} M_{iL-1} + \frac{l}{l+1} \epsilon_{ijk} \partial_{jL-1} \left(\frac{1}{r} \right) S_{kL-1} \right)$$
$$- \frac{1}{4} \partial_i (\Lambda - \lambda) + O_2 \quad ,$$

BCRS metric

$$g_{00} = -1 + \frac{2w}{c^2} - \frac{2w^2}{c^4} + O_6 \quad ,$$

$$g_{0i} = \frac{4w_i}{c^3} + O_5 \quad ,$$

$$g_{ij} = \left(1 + \frac{2w}{c^2}\right)\delta_{ij} + O_4 \quad .$$

1.5 PN for light-rays

For a single body

$$w = G\sum_{l=0} \frac{(-1)^l}{l!}\partial_L\left(\frac{1}{r}M_L^{\pm}\right) + \frac{1}{c^2}\partial_t(\Lambda - \lambda) + O_4 \quad ,$$

$$w_i = -G\sum_{l=1} \frac{(-1)^l}{l!}\left(\partial_{L-1}\left(\frac{1}{r}\right)\frac{d}{dt}M_{iL-1} + \frac{l}{l+1}\epsilon_{ijk}\partial_{jL-1}\left(\frac{1}{r}\right)S_{kL-1}\right)$$
$$-\frac{1}{4}\partial_i(\Lambda - \lambda) + O_2 \quad ,$$

The gravitational time delay

The PN-terms treated with the Time-Transfer-Function as obtained from $ds^2 = 0$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$dt^2 = \frac{1}{c^2} d\mathbf{x}^2 + \left(h_{00} + 2h_{0i}\frac{dx^i}{dt} + \frac{1}{c^2} h_{ij}\frac{dx^i}{dt}\frac{dx^j}{dt} \right) dt^2$$

$$\mathcal{T}(t_0, \mathbf{x}_0; \mathbf{x}) \equiv t - t_0$$

$$\mathcal{T}(t_0, \mathbf{x}_0; \mathbf{x}) = \frac{R}{c} + \frac{1}{2c}\int_{s_0}^{s} (h_{\mu\nu} n^\mu n^\nu) ds = \frac{R}{c} + \frac{2}{c^3}\int_{s_0}^{s} \left(w - \frac{2}{c}\mathbf{w}\cdot\mathbf{n} \right) ds$$

The Shapiro time delay was derived for a body

a) at rest

b) with a small, constant barycentric velocity

having all (constant) mass- and spin-multipole moments within a few lines

for details see: arXiv: 1409.3743

Corrections for parallax and proper motion of the radio source have been discussed

Related work

- the post-linear metric (PPN-metric for light rays)

- models for bodies with higher spin-multipole moments

PHYSICAL REVIEW D **89**, 104056 (2014)

Post-linear Schwarzschild solution in harmonic coordinates: Elimination of structure-dependent terms

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(Received 28 February 2014; published 27 May 2014)

$$g_{00} = -1 + \frac{2}{c^2} w - \frac{2}{c^4} w^2 + \mathcal{O}(c^{-6}),$$

$$g_{0i} = -\frac{4}{c^3} w^i + \mathcal{O}(c^{-5}),$$

$$g_{ij} = \delta_{ij}\left(1 + \frac{2}{c^2} w + \frac{2}{c^4} w^2\right) + \frac{4}{c^4} q_{ij} + \mathcal{O}(c^{-5}).$$

$$\Delta q_{ij} = -w_{,i} w_{,j} - 4\pi G \sigma^{ij} + \mathcal{O}(c^{-1})$$

$$\sigma^{ij} = T^{ij} - \delta_{ij} T^{ss}$$

we started with the case of a spherically symmetric body;

outside the body: Schwarzschild metric (sole parameter M)

q_ij: during the calculation on faces expressions that depend
upon the internal structure of the body (e.g., radius R)

one has to show that all such ‚bad expressions'

- either cancel because of the local equations of motion
- or can be removed by means of a (harmonic) gauge transformation

-> one gets a new form of the Schwarzschild metric in
harmonic coordinates (not the one in Weinberg, 1972)

further work on the PPN-metric for light-rays:

- we try to extend our calculations to a single body of arbitrary shape and composition (having all PN mass- and spin-moments)

- we are trying to get the (post-linear) Erez-Rosen metric

  an exact solution of Einstein's vacuum equations with two parameters m and q (mass and quadrupole moment)

in harmonic coordinates

Bodies with higher spin-multipole moments

Gravito-magnetism of an extended astronomical body in post-Newtonian approximation

Michel Panhans[1,2], Michael H. Soffel[1]

$$S_L = \int dV \epsilon^{ij<i_l} \hat{x}^{L-1>i} \sigma^j = \frac{4\pi(l-1)!}{(2l+1)!!} \sum_{m=-l}^{l} \hat{Y}_L^{lm} \Xi_{lm}$$

$$\Xi_{lm} = -\int dV r^l \boldsymbol{\sigma} \cdot (\boldsymbol{r} \times \nabla) Y^*_{lm} = -\int dV r^l \boldsymbol{\sigma} \cdot \boldsymbol{\Phi}^*_{lm}$$

σ: mass-energy current density

E.g., for an oblate homogeneous spheroid (a > b) rotating about the symmetry axis with ω

$$M = \frac{4\pi}{3}\rho b a^2 \quad ; \quad \epsilon = \sqrt{1 - \frac{b^2}{a^2}} \quad ; \quad S = \frac{2}{5} M \omega a^2$$

$$\Xi_{2n+10} = S(a\epsilon)^{2n} \cdot \frac{15}{2} \frac{(2n+2)(-1)^n}{(2n+3)(2n+5)} \sqrt{\frac{4n+3}{4\pi}} \quad ,$$

More general body-models, e.g., with a liquid core or a differential rotation, have been studied

Higher spin-multipole moments have been estimated for all solar-system bodies

Spin-octupole precession vector components normalized to the Lense-Thirring precession components due to oblateness at the poles :

$$\delta^{\mathrm{obl}}_{\mathrm{pole}} \leq \frac{6}{7}\epsilon^2$$

	Sun	Mercury	Venus	Earth	Mars	Jupiter	Saturn	Uranus	Neptune
$\delta^{\mathrm{obl}}_{\mathrm{pole}}/\%$	$\approx 10^{-3}$	0	0	0.57	0.99	10.76	15.97	3.88	2.90

FIN