

# Estimation of nutation rates from combination of ring laser and VLBI data

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# Motivation

- Ring laser gyroscopes (RLG) are instruments which present a dynamical approach to the determination of Earth rotation parameters. They enable measuring Earth rotation on the surface of the Earth, as they are sensitive to variations in the instantaneous rotation vector. They are considered as a potential complement to space geodetic techniques in studying Earth rotation.
- To date many experiments have been conducted in order to investigate possible advantages of combining ring laser observations and data from space techniques, especially from Very Long Baseline Interferometry (VLBI). The majority of those experiments concern polar motion and UT1 variations. In this work we examine the potential usage of ring laser observations for estimation of nutation rates.

# Investigation

- checking possibility of nutation rates estimation based on RLG observations - simulation level
- combination of simulated RLG data with real VLBI data
- comparison of estimates based on real RLG data (Wettzell) with results of simulation

# Simulation level

Based on equation for relative Sagnac frequency:

$$\Delta S = \cot \phi (m_x \cos \lambda + m_y \sin \lambda) + m_z + \Delta S_{tilt} + \Delta S_{instr}$$

and taking into account relations between CIP and IRP:

$$m_x = \frac{1}{\Omega_0} (\Omega_0 x_p - \dot{y}_p + \dot{d}X \sin \theta - \dot{d}Y \cos \theta)$$

$$m_y = \frac{1}{\Omega_0} (-\Omega_0 y_p - \dot{x}_p + \dot{d}X \cos \theta + \dot{d}Y \sin \theta)$$

$$m_z = 1.0027 \cdot d\dot{UT}1$$

# Simulation level

We constructed the design matrix  $A$ , assuming one value of the nutation offset rates  $dX$  and  $dY$  and the instrumental error per day as unknowns,

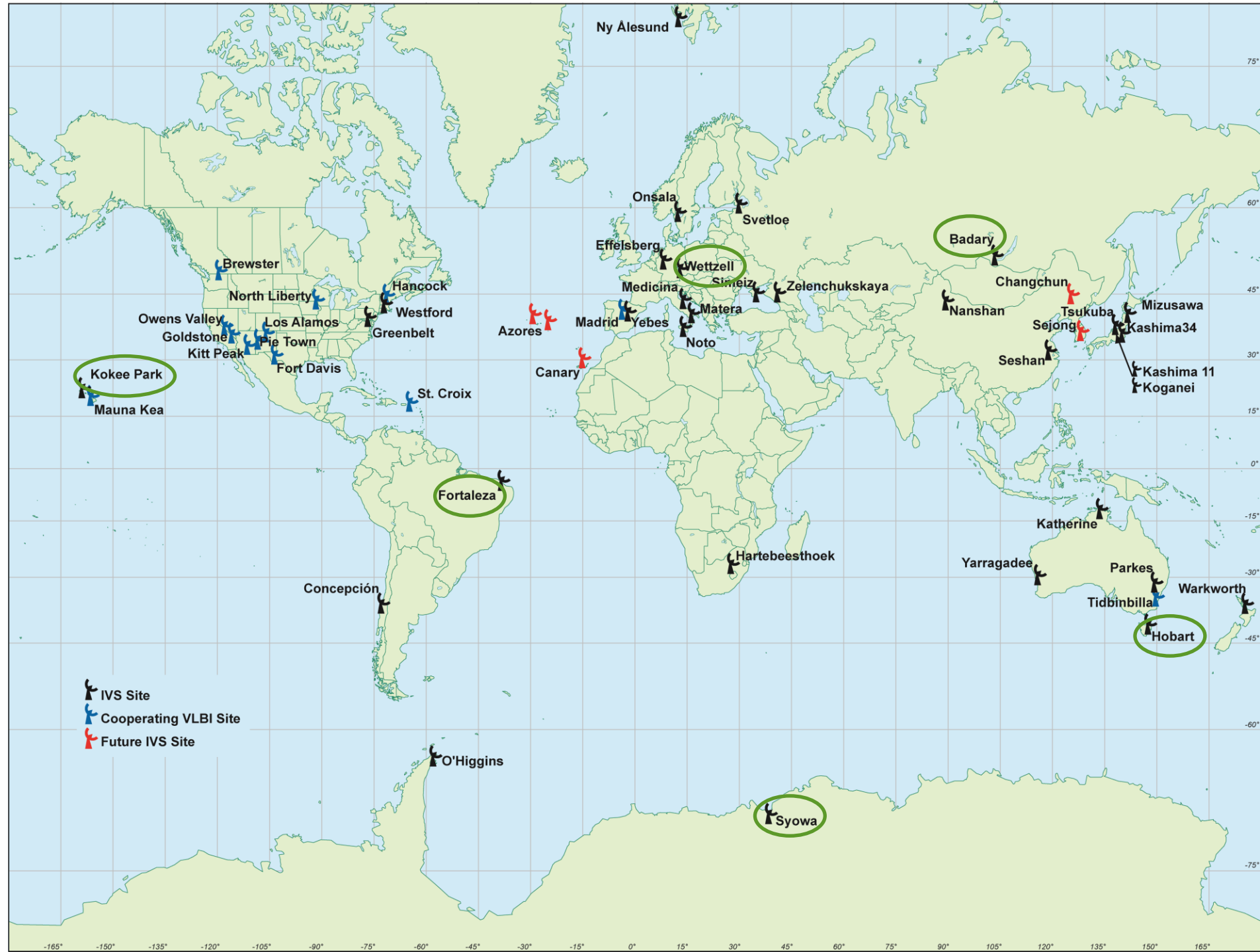
the right-hand side vector is  $L = \Delta S_{\text{obs}} - \Delta S_{\text{comp}} + \text{random noise}$ , where:

	polar motion (xp, yp)	dUT1	nutation (dX, dY)
$\Delta S_{\text{obs}}$	C04 08 series + ocean tides	C04 08 series + ocean tides	The IAU 2000/2006 model + offsets from C04 08 series
$\Delta S_{\text{comp}}$			The IAU 2000/2006 model

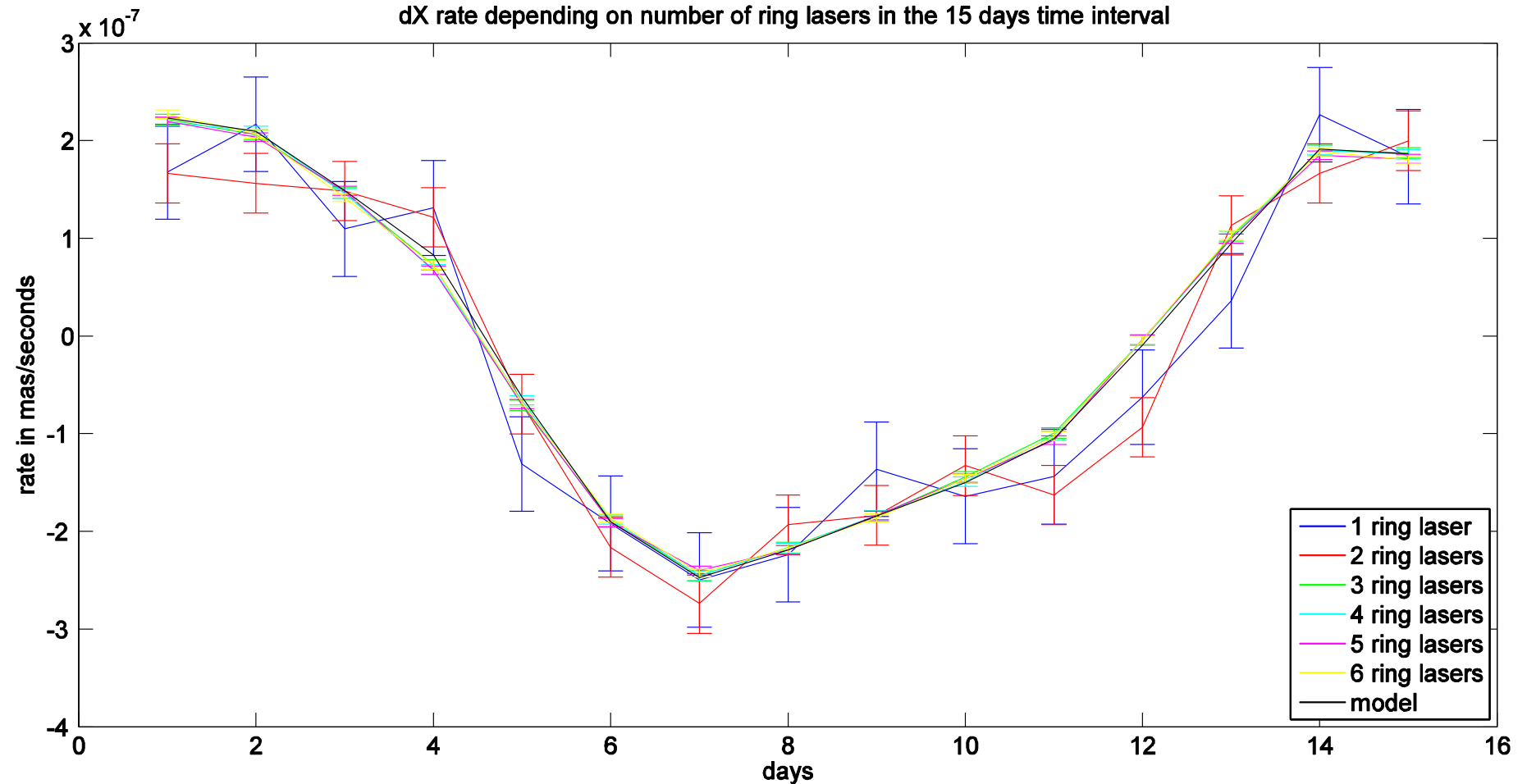
and the weighing matrix:

$$P = \text{diag}((\sigma^2)^{-1}), \text{ with } \sigma = 10^{-11},$$

the first day of observations – 15 Sep. 2011



# Simulation level

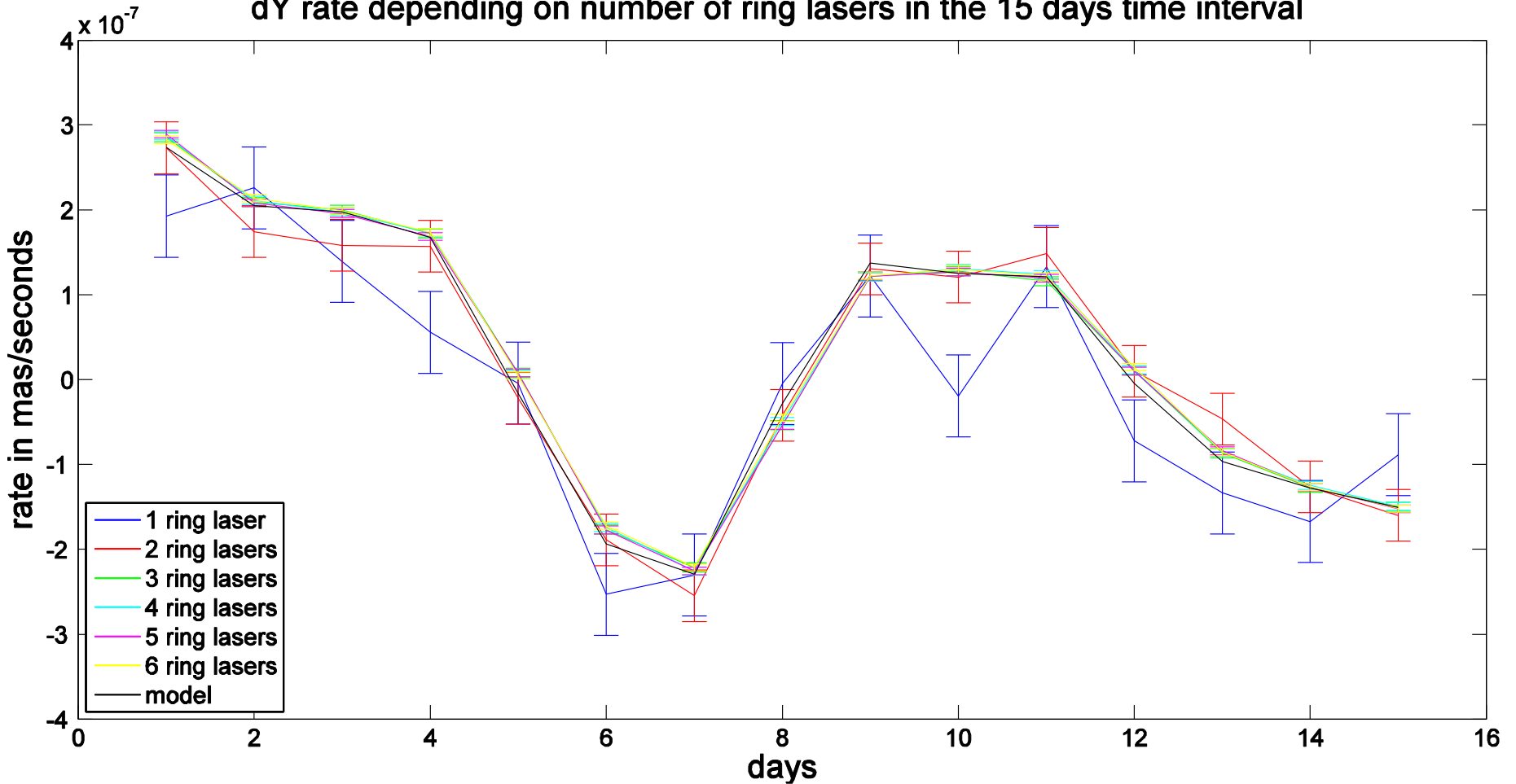


RMS of the residuals for 1 to 6 ring laser respectively ( $\times 10^{-7}$  mas/s):

dX	0.3944	0.3781	0.0495	0.0468	0.0603	0.0464
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# Simulation level

dY rate depending on number of ring lasers in the 15 days time interval



RMS of the residuals for 1 to 6 ring laser respectively ( $\times 10^{-7}$  mas/s):

dY	0.6307	0.2179	0.1267	0.1235	0.1274	0.1227
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# Simulated RLG data + VLBI

- Cont11 session, processed with VieVS (Vienna VLBI Software) according to Nilsson et al. (2012)
- Combination on the normal equation level:

$$N = \begin{bmatrix} N_{VLBI} & 0 \\ 0 & N_{RLG} \end{bmatrix}, L = \begin{bmatrix} L_{VLBI} \\ L_{RLG} \end{bmatrix}$$

$$N_C = \begin{bmatrix} N & C^T \\ C & 0 \end{bmatrix}; L_C = \begin{bmatrix} L \\ 0 \end{bmatrix}$$

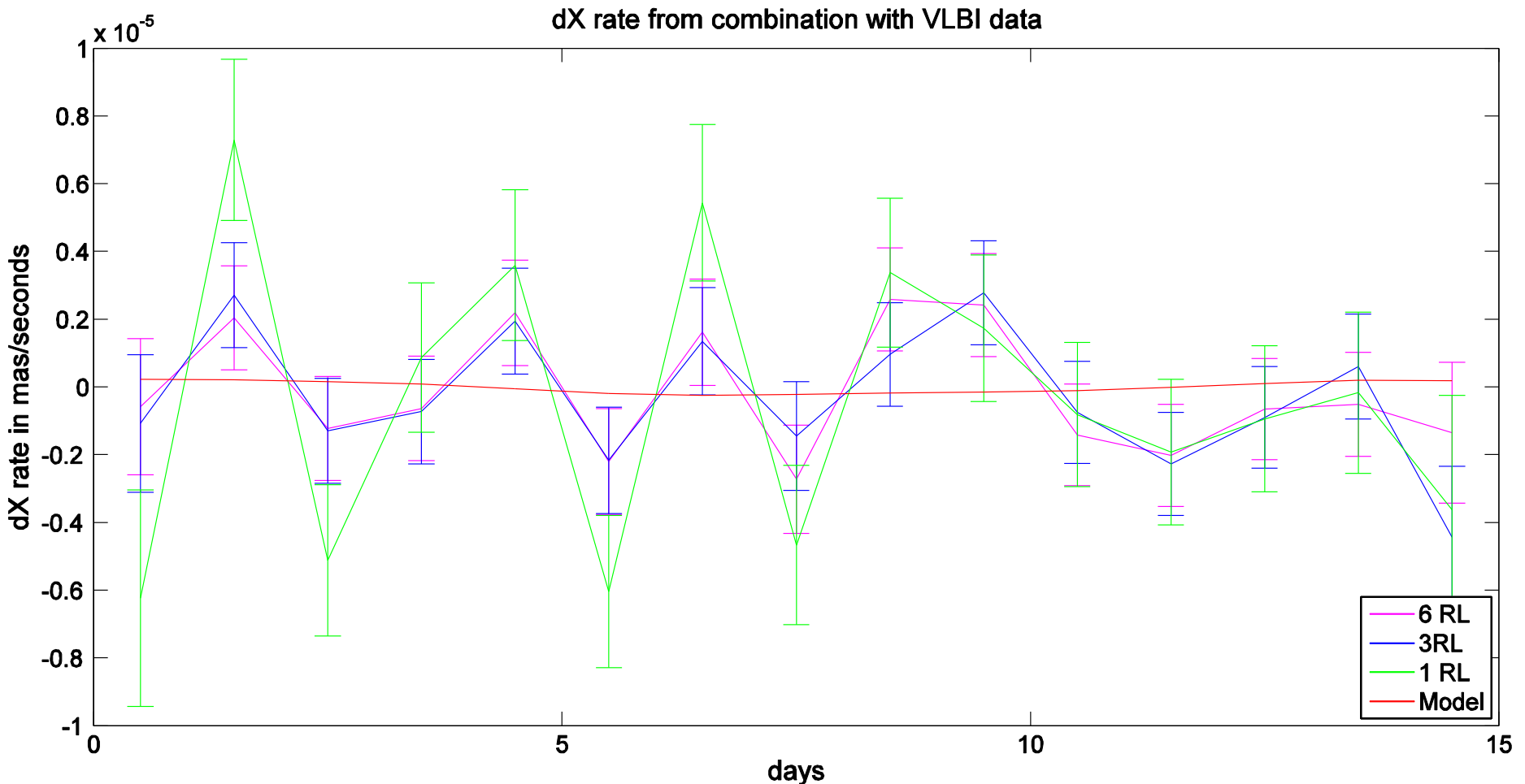
$C$  ... constraints

Nutation rates = nutation finite differences

# Simulated RLG data + VLBI

- Computation for one, three and six RLG instruments
- Starting with present-day accuracy level  $10^{-8}$  followed by refinement:  $10^{-9}$ ,  $10^{-10}$ ,  $10^{-11}$

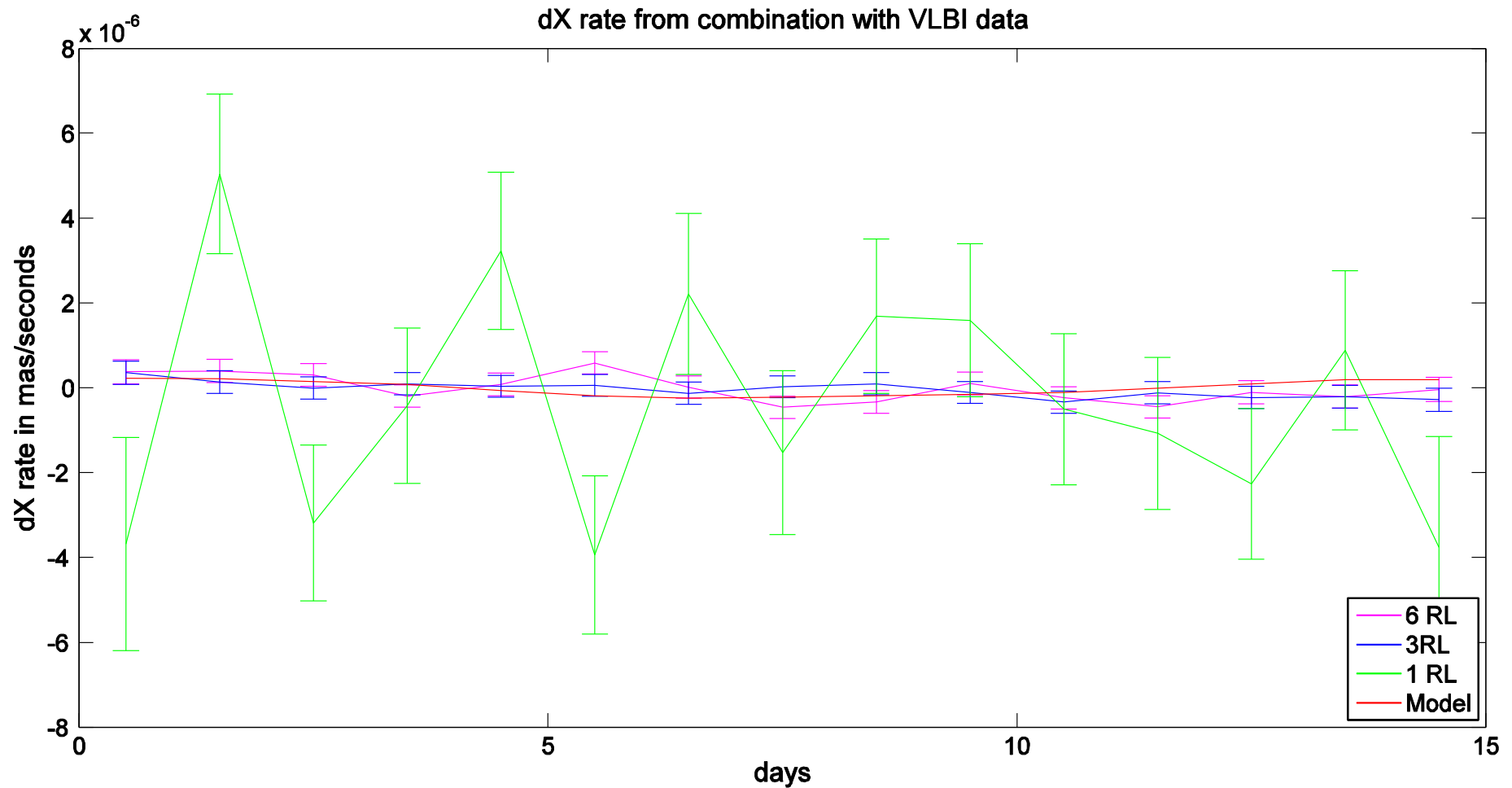
# Simulated data + VLBI (ac. lev. $10^{-8}$ )



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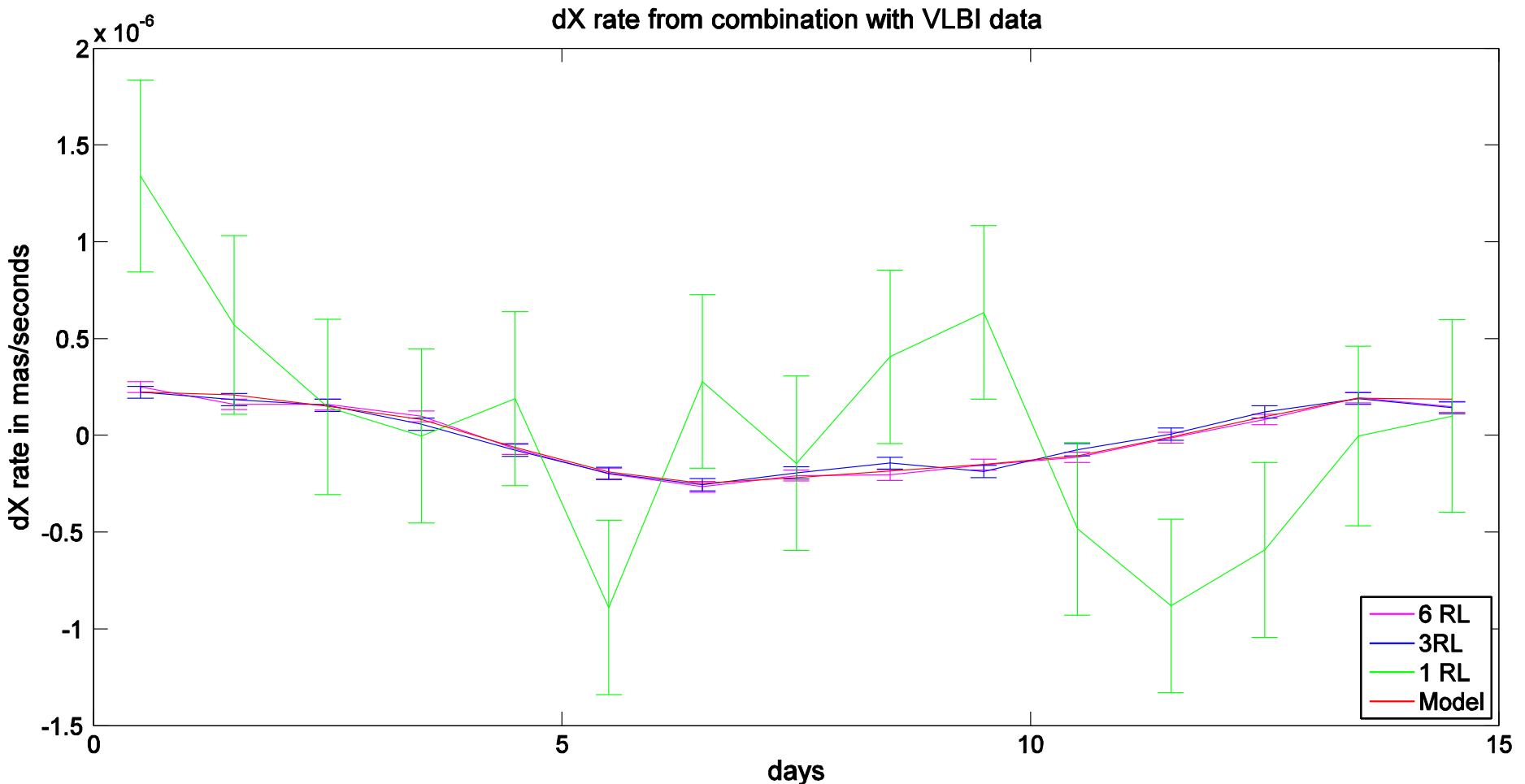
# Simulated data + VLBI (ac. lev. $10^{-9}$ )



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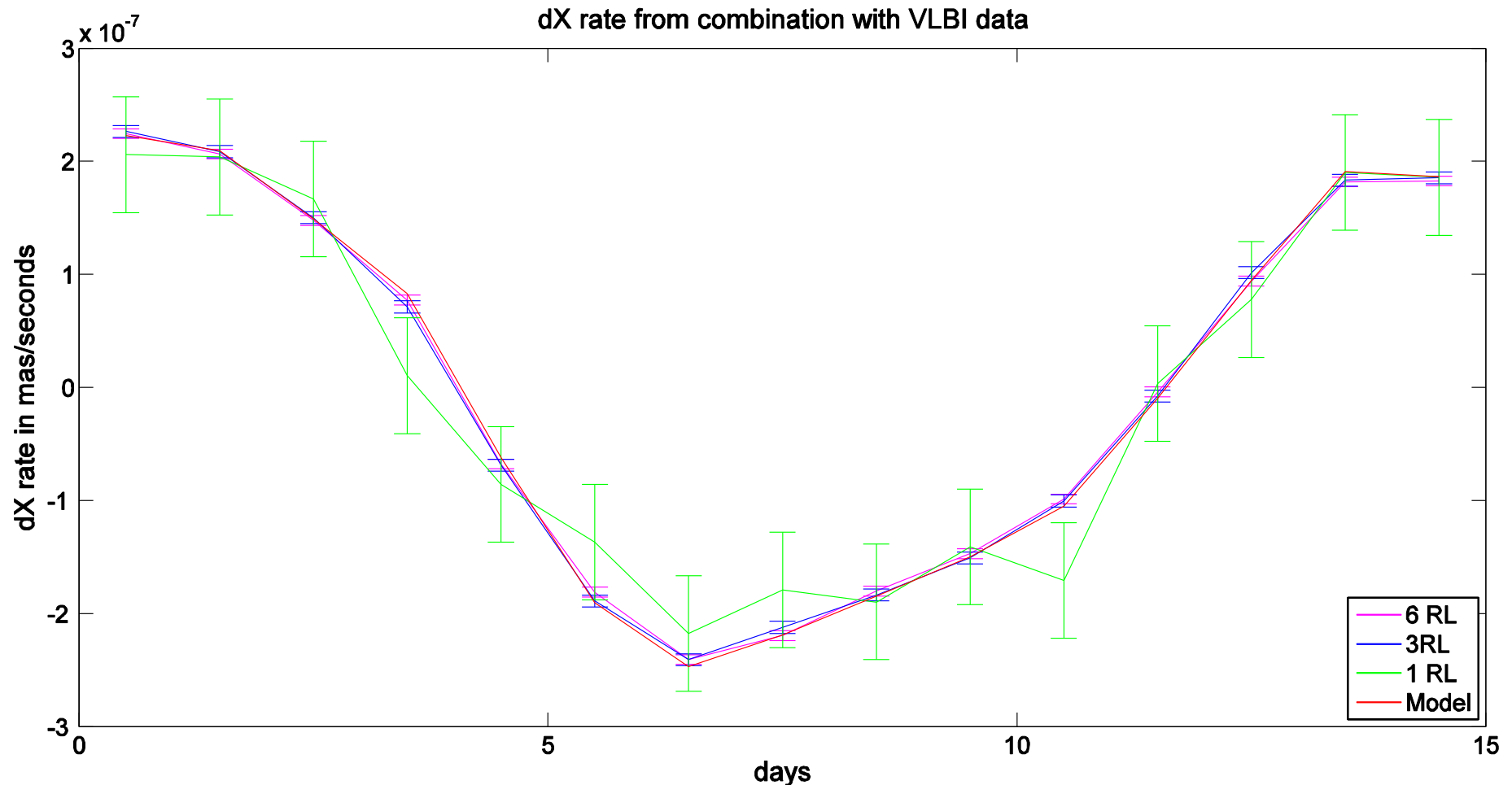
# Simulated data + VLBI (ac. lev. $10^{-10}$ )



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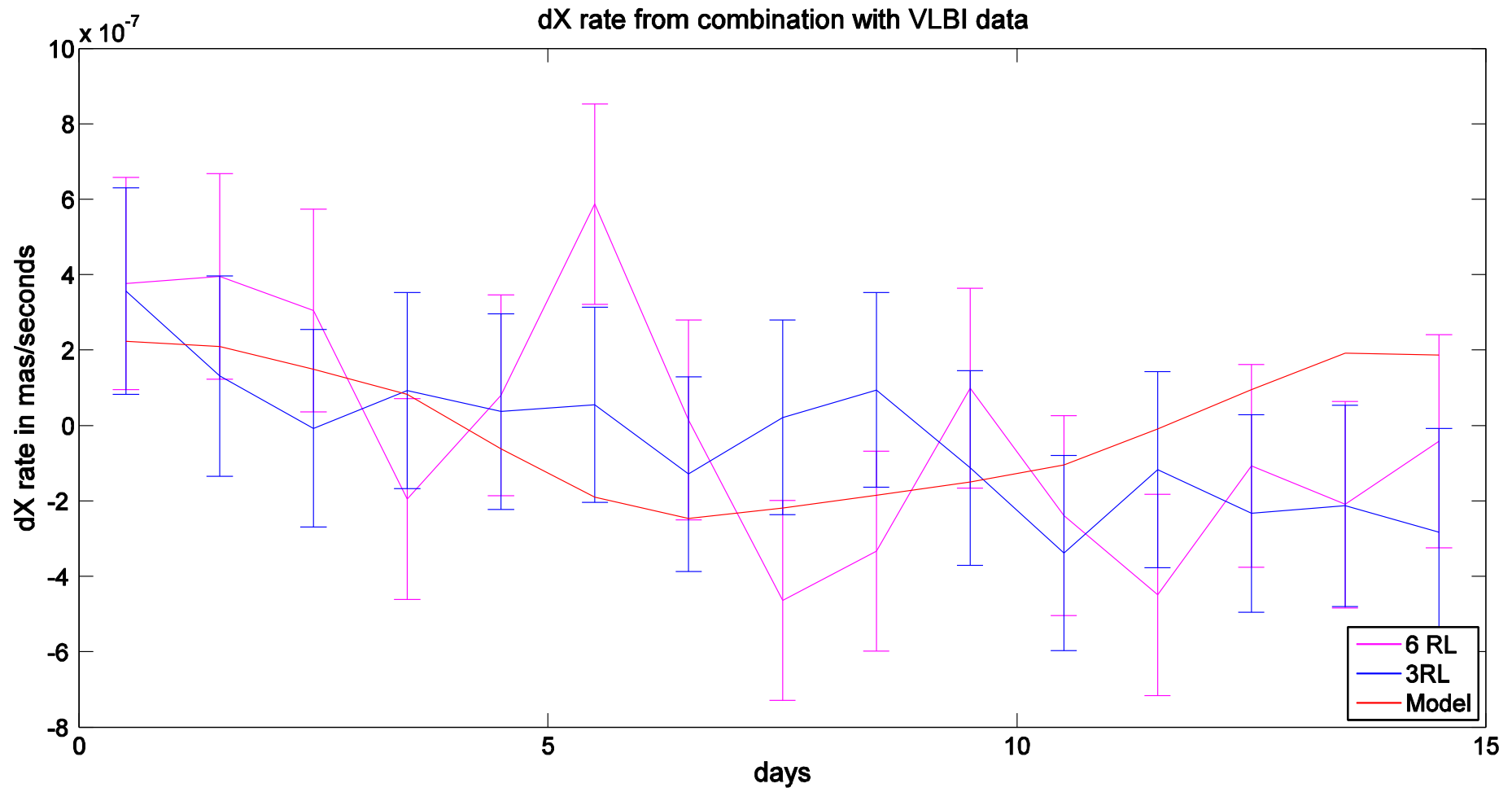
# Simulated data + VLBI (ac. lev. $10^{-11}$ )



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# Simulated data + VLBI (ac. lev. $10^{-9}$ )



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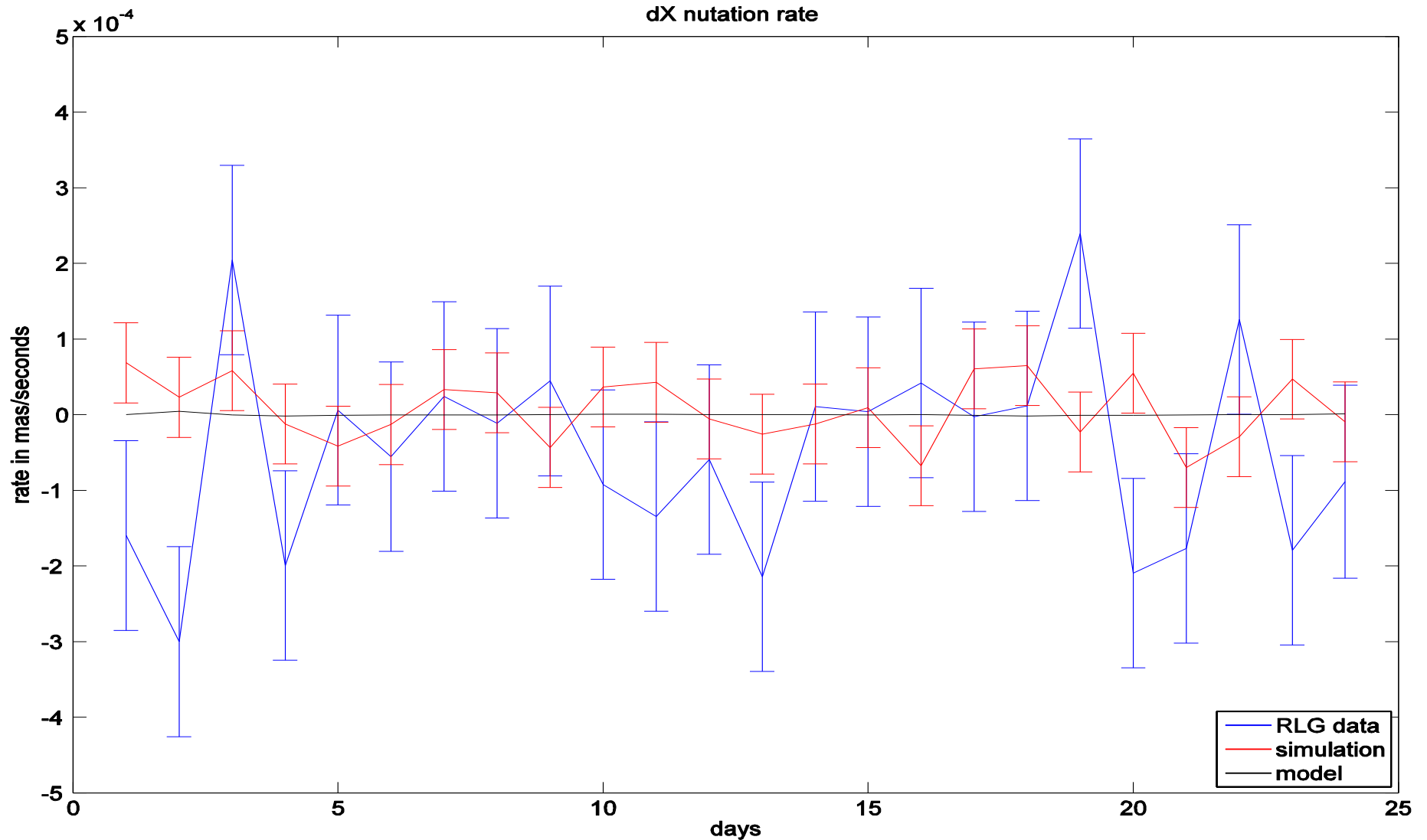
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# Simulated and real RLG data

- Data from the Wettzell Observatory
- Time span 24 days (01 may to 25 may 2010)
- Present-day accuracy level  $10^{-8}$

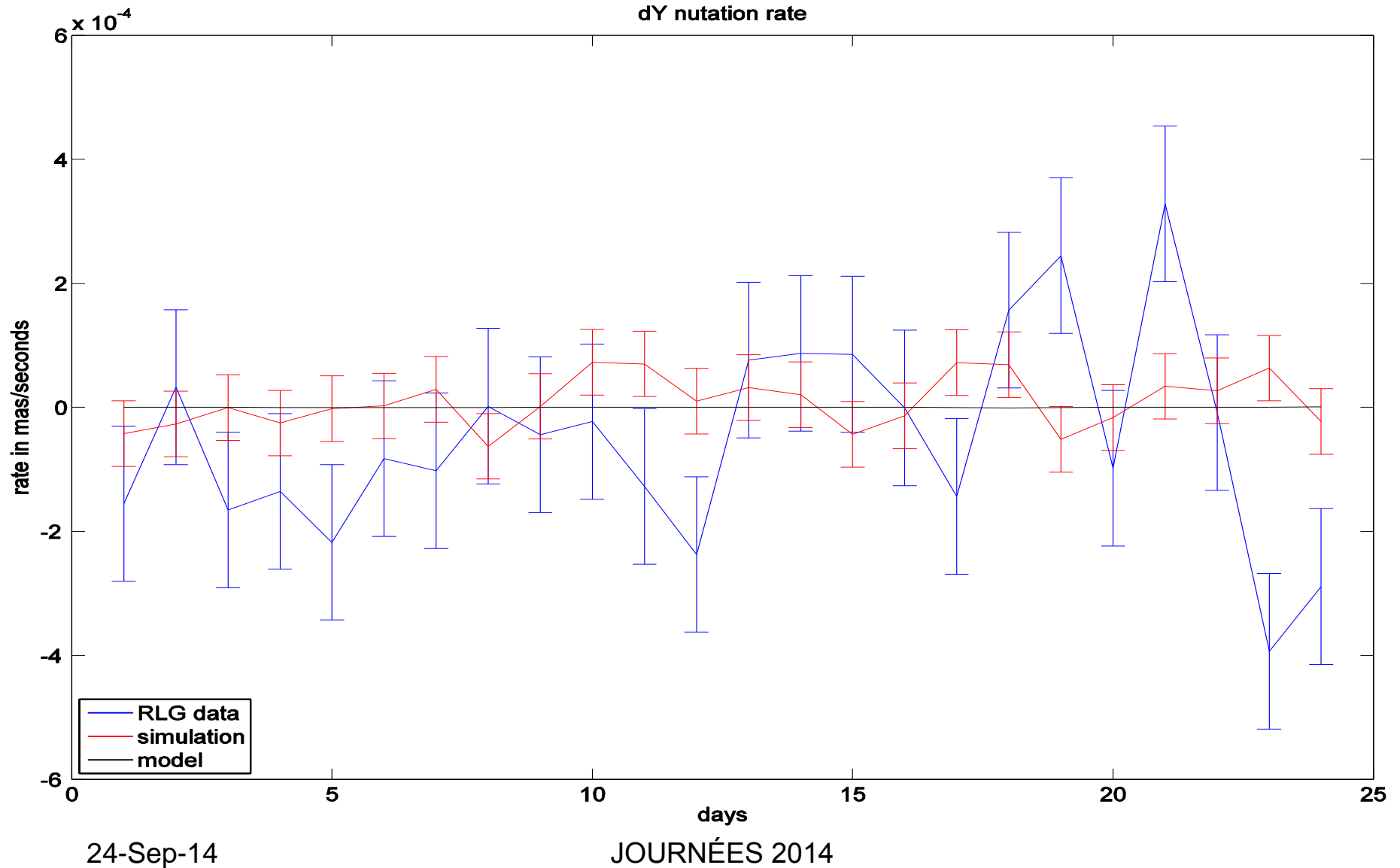
# Simulated and real RLG data



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# Simulated and real RLG data



# Conclusions:

On a first, very positive note, the estimation of nutation rates from ring laser observations is possible, but...

- there should be at least three RLG instruments
- the accuracy level should be three orders higher than it is nowadays
- even combination with VLBI observation requires at least  $10^{-10}$  accuracy level and three instruments
- our simulations appeared to be more optimistic than real data.

# Our goals for future:

- Combination with GNSS data
- Investigation of a more sophisticated model of the relative Sagnac frequency

## *Acknowledgements:*

*Local Organization Committee for free  
accommodation and waiving the registration fee,  
IAG for Young Scientists Travel Award*