Method of Determining the Orbits of the Small Bodies in the Solar System Based on an Exhaustive Search of Orbital Planes

Dmitrii Vavilov, Yurii Medvedev

Introduction to the problem:

Let we have n ≥ 3 positional observations of a body: points in time t_j $(j = \overline{1,n})$, right ascensions α_j, and declinations δ_j. Then, unit vectors $\boldsymbol{L}_j$ pointing to the body in the topocentric equatorial coordinate system have the following form:

$$\boldsymbol{L}_j = \left(\cos\alpha_j\cos\delta_j, \sin\alpha_j\cos\delta_j, \sin\delta_j\right), \qquad j = \overline{1,n}.$$

The relationship between the heliocentric and topocentric vectors of the celestial body positions is determined by the equations:

$$\boldsymbol{X}_j = \rho_j(i,\Omega)\,\boldsymbol{L}_j + \boldsymbol{E}_j, \qquad j = \overline{1,n},$$

where $\boldsymbol{X}_j$ are the heliocentric vectors of the celestial body positions, ρ_j are the topocentric distances, and $\boldsymbol{E}_j$ are the heliocentric vectors of the observer's position, i and Ω are inclination and longitude of the ascending node of the asteroid's orbit, correspondingly. Note that ρ_j is a function of i and Ω.

Therefore, it is necessary to know ρ_j in order to determinate topocentric distances $\boldsymbol{X}_j$. Normally, these vectors are determined by solving a system of nonlinear equations. These equations are generally solved using iterations where a type of orbit is assigned beforehand since different formulas are used to calculate different types of orbits, which is not always convenient. Moreover, iterations can diverge in some unfavorable cases, such as the nonuniform distribution or the presence of erroneous observations, making it impossible to determine orbital elements of a small body.

Description of the method:

Exhaustive search for orbital planes (inclinations and longitudes of the ascending node of the heliocentric orbit) is fulfiled. For each plane we make the following operations:

- Calculation ρ_j according to:

$$\rho_j = \left|\frac{(\boldsymbol{N},\boldsymbol{E}_j)}{(\boldsymbol{N},\boldsymbol{L}_j)}\right|,$$

where $\boldsymbol{N} = (\sin i\,\sin\Omega, -\sin i\,\cos\Omega, \cos i)$ is the normal vector to the orbital plane.

- Aberration correlations are taken into account:

$$t_j' = t_j - \frac{1}{c}\rho_j,$$

where c is the velocity of light.

- Two reference observations are chosen (generally the first and the last observations).
- Determination of the orbit using the Gauss method of determining orbital elements based on two heliocentric positions and points in time.
- The differences between the observed and calculated positions of the body (O–C) are calculated and rms of the observation fit is determined:

$$\sigma = \sqrt{\frac{1}{2n}\sum_{j=0}^{n}\left(\alpha_j - \alpha_j^c\right)^2\cos^2\delta_j + \left(\sigma_j - \sigma_j^c\right)^2},$$

where α_j^c, δ_j^c are the calculated equatorial coordinates of the celestial body.

We choose that plane, which has the least σ, and improve the orbit, corresponding to the plane, using the differential method.

Designation	$R_{\mathrm{MPC}} - R_{\mathrm{IAA}}$	R_{IAA}	σ_{IAA}	σ_{MPC}	ΔT_1	ΔT	Gauss
2010 SJ	226	673	651	895	0.97	20.1	
2010 SX11	118	101	300	715	1.24	21.4	-
2011 KO17	114	52	354	1124	1.32	188.8	
2010 SS3	59	15	31	78	1.96	22.9	
2011 KR12	57	157	54	75	1.93	17.8	
2011 KJ15	34	366	231	251	1.74	12.6	
2011 KD11	13	59	50	61	1.09	27.8	
2011 KH4	9	29	122	199	1.68	59.9	
2010 SY3	9	28	74	96	2.77	43.9	-
2010 SA12	6	6	213	410	2.98	144.8	
2011 KC15	4	21	48	55	1.99	85.7	
2010 SG13	3	27	4042	3242	2.12	340.0	-
2010 SL13	2	5	3140	5362	10.15	2360.1	-
2011 JA8	2	5	102	231	19.74	227.8	
2011 KF9	1	21	13	14	1.83	34.0	
2010 SP3	1	99	28	28	0.58	13.9	
2010 OC127	1	1	2	2	59.31	104.3	-
2011 KW15	1	117	1187	534	1.87	361.5	
2010 PT66	0	1	1	2	33.13	57.0	-
2011 KK15	0	16	4	8	1.63	36.9	-
2010 RG137	0	19	246	269	14.66	147.9	-
2011 KQ12	0	16	210	209	4.71	60.8	
2011 KP17	0	10	61	57	19.58	244.9	-
2011 JP29	0	1	43	22	23.78	230.9	-
2010 BK118	-1	3	132	126	235.22	877.3	-
2010 SV3	-1	6	23486	12344	1.66	543.9	
2011 KW19	-1	27	106	96	3.08	53.8	
2011 KN17	-1	8	4	3	1.72	23.0	
2011 KQ19	-2	15	25	23	3.00	30.8	
2010 SE12	-12	111	83	74	2.03	19.1	
2010 SR3	-15	73	1064	757	1.96	167.7	
2010 SD13	-16	34	748	385	1.40	111.7	
2011 KP16	-27	43	44	8	1.78	16.2	
2010 SH13	-41	97	105	63	2.11	33.1	

References:

Yu.S. Bondarenko, D.E. Vavilov, Yu.D. Medvedev *Method of Determining the Orbits of the Small Bodies in the Solar System Based on an Exhaustive Search of Orbital Planes*, Solar System Research, 2014, Vol. 48, No. 3, pp. 212–216.