Prediction of the Chandler wobble

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Journees
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Motion of the Earth’s pole

EOP CO1

1846-2010
step 0.05 yr

2D trajectory

\[ m(t) = x - iy \]
Complex Singular Spectrum Analysis (CSSA) of the Polar Motion

SSA-decomposition of X-coordinate of the pole
- Chandler component
- Annual component
- Trend

X-component (Y – similar, with $\pi/2$ phase shift).

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Dynamical model of the rotating Earth

\[
\frac{i}{\sigma_c} \frac{dm(t)}{dt} + m(t) = \Psi(t)
\]

\[
m = m_1 + im_2
\]

\[
\Psi = \Psi_{mass} + \Psi_{motion}
\]

\[
\sigma_c = 2\pi f_c (1 + i/2Q)
\]

\[
f_c = \frac{1}{433} \text{ days}^{-1}
\]

\[
Q = 175
\]

Munk W.H., MacDonald G.J.F., The rotation of the Earth, 1960
Filters’ transfer functions and PM spectrum

\[ L^{-1} = 1 + \frac{i}{2\pi f_c} (i\omega) \]

|L^{-1}| - inverse operator

Chandler PM → annual PM

Panteleev's filter

\[ L_{pant} = \frac{f_0^4}{(f - f_c)^4 + f_0^4} \]

\[ f_0 = 1/30 \text{ YR}^{-1} \]
What is Chandler wobble?

Complex Fourier spectrum represents the signal by the set of constant harmonics, but it incorporates information about changes of instantaneous amplitude and phase.

Filtering in time domain – convolution

Filtering in frequency domain – spectra multiplication

Panteleev filters
- $f_{om}=1/30 \text{ year}^{-1}$
- $f_{om}=1/80 \text{ year}^{-1}$
- $f_{om}=1/10 \text{ year}^{-1}$

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Filtered Chandler wobble

X-component (Y – similar, with $\pi/2$ phase shift).
Chandler wobble and its excitation

X-component (Y – similar, with $\pi/2$ phase shift).

filtered Chandler wobble
prograde Chandler component

$\text{mas}$

year

1840 1860 1880 1900 1920 1940 1960 1980 2000 2020

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Chandler wobble and its excitation

X-component (Y – similar, with \( \pi/2 \) phase shift).

Geodetic excitation in the Chandler band

prograde Chandler excitation
Envelope can be transferred through the dynamical model

\[ \frac{i}{\sigma_c} \frac{dm(t)}{dt} + m(t) = \Psi(t) \]

\[ m(t) = A(t) e^{i \varphi(t)} e^{i \omega_c t} \quad \Psi(t) = E(t) e^{i \theta(t)} e^{i \omega_c t} \]

\[ E(t) e^{i \theta(t)} e^{-i \varphi(t)} = \frac{i}{\sigma_c} (\dot{A}(t) + i \varphi(t) A(t)) + \left(1 - \frac{\omega_c}{\sigma_c}\right) A(t) \]

\[ \omega_c = 2\pi f_c \quad \sigma_c = 2\pi f_c (1 + i/2Q) \quad f_c = \frac{1}{433} \text{ days}^{-1} \quad Q = 175 \]
40-year PM changes will give 20-year oscillations in the excitation envelope. 

\[ A(t) = \sin(\omega t) \] 
\[ E(t) \sim |\cos(\omega t)| \]
Envelope calculation

\[ A(t) \]

\[ E_A(t) \]

\[ \varphi(t) \]

\[ E\varphi(t) \]

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Envelope calculation

Chandler excitation

- Envelope
- Total calculated input

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Amplitude model and forecast

filtered Chandler wobble
- Envelope
- LS model
- LS forecast
- Neural Net forecast

8-layer Neural Network with (7, 7, 1) neurons

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Phase model and forecast

filtered Chandler wobble
- Phase
- LS model
- LS forecast
- Neural Net forecast

3-layer Neural Network with (7, 7, 1) neurons

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Phase and amplitude models

<table>
<thead>
<tr>
<th>~Chandler wobble amplitude NLSM fit</th>
<th>Period, years</th>
<th>Amplitude</th>
<th>Phase (1880)</th>
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</thead>
<tbody>
<tr>
<td>~80-year component</td>
<td>83.44</td>
<td>42.6 mas</td>
<td>40.8°</td>
</tr>
<tr>
<td>~40-year component</td>
<td>42.0</td>
<td>54.6 mas</td>
<td>-101.5°</td>
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<tr>
<td>mean</td>
<td></td>
<td>134.8 mas</td>
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</tbody>
</table>

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<thead>
<tr>
<th>~Chandler wobble phase NLSM fit</th>
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<th>Phase (1880)</th>
</tr>
</thead>
<tbody>
<tr>
<td>~100-year component</td>
<td>117.8</td>
<td>59 dg</td>
<td>-118°</td>
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<tr>
<td>~50-year component</td>
<td>50.9</td>
<td>34 dg</td>
<td>95°</td>
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<tr>
<td>1-order trend</td>
<td></td>
<td>2dg/year</td>
<td></td>
</tr>
</tbody>
</table>
Excitation forecast

Chandler excitation model with forecast
- Real Envelope
- Total modeled input
- LS model forecast
- NNet forecast

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Chandler wobble and its excitation depending on the filter width

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Chandler wobble and its excitation

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Chandler wobble and its excitation

Along the abscissa 18.6-year wave of the Moon orbit inclination

Geodetic excitation in the Chandler band

Chandler excitation difference for $f_{om}$ 1/30-1/80 year$^{-1}$

Moon maximal declination

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18.6 year period of orbital nodes regression

Moon orbit is between ecliptic and equator, It does not have large inclinations

1997, 2015

Moon orbit is above the ecliptic, Its inclinations can be high

1988, 2007

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Moon orbit \( \max \angle 18^\circ \)

Moon orbit \( \max \angle 28^\circ \)
20-year changes in SL rate, LOD, Temperature and Chandler excitation

- LOD inverted
- GMSL PC 2 - 20-year changes in Sea Level Rate (inverted)
- HadCRUT4 PC 2 - 20-year temperature changes
- 20-year changes in Chandler excitation (1/30-1/80) and its envelope
- Moon maximal declination
60-year changes in SL, LOD, MD, Temperature and Chandler excitation

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Conclusions

- CSSA or Panteleev’s filtering allows to extract Chandler wobble component of PM, its amplitude has ~80 and ~40 year modulations.
- From the envelope of the Chandler wobble the excitation envelope can be calculated using Euler-Liouville equation.
- If it’s true, that Chandler wobble has 40-year modulations, then excitation has 20-year amplitude changes.
- Prediction of the Chandler wobble and its excitation can be made, based on the envelope forecast. Now the Chandler wobble has decreased, its phase can jump, our epoch is crucial for understanding.
- Reconstructed Chandler excitation has modulations very similar with the ~20 and ~60-year components of temperature and sea level rate changes.
- The 20-year variations in the Chandler excitation and climate characteristics can be caused by the changes of atmospheric and oceanic circulation under the influence of 18.6-year cycle of the Moon orbital nodes regression.
Thank you