Determination of the figure of the dwarf planet Haumea from observations of a stellar occultation and photometry data

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ОПРЕДЕЛЕНИЕ ФИГУРЫ КАРЛИКОВОЙ ПЛАНЕТЫ ХАУМЕА

ИЗ НАБЛЮДЕНИЙ ПОКРЫТИЯ ЗВЕЗДЫ ФОНА И ФОТОМЕТРИЧЕСКИХ ДАННЫХ

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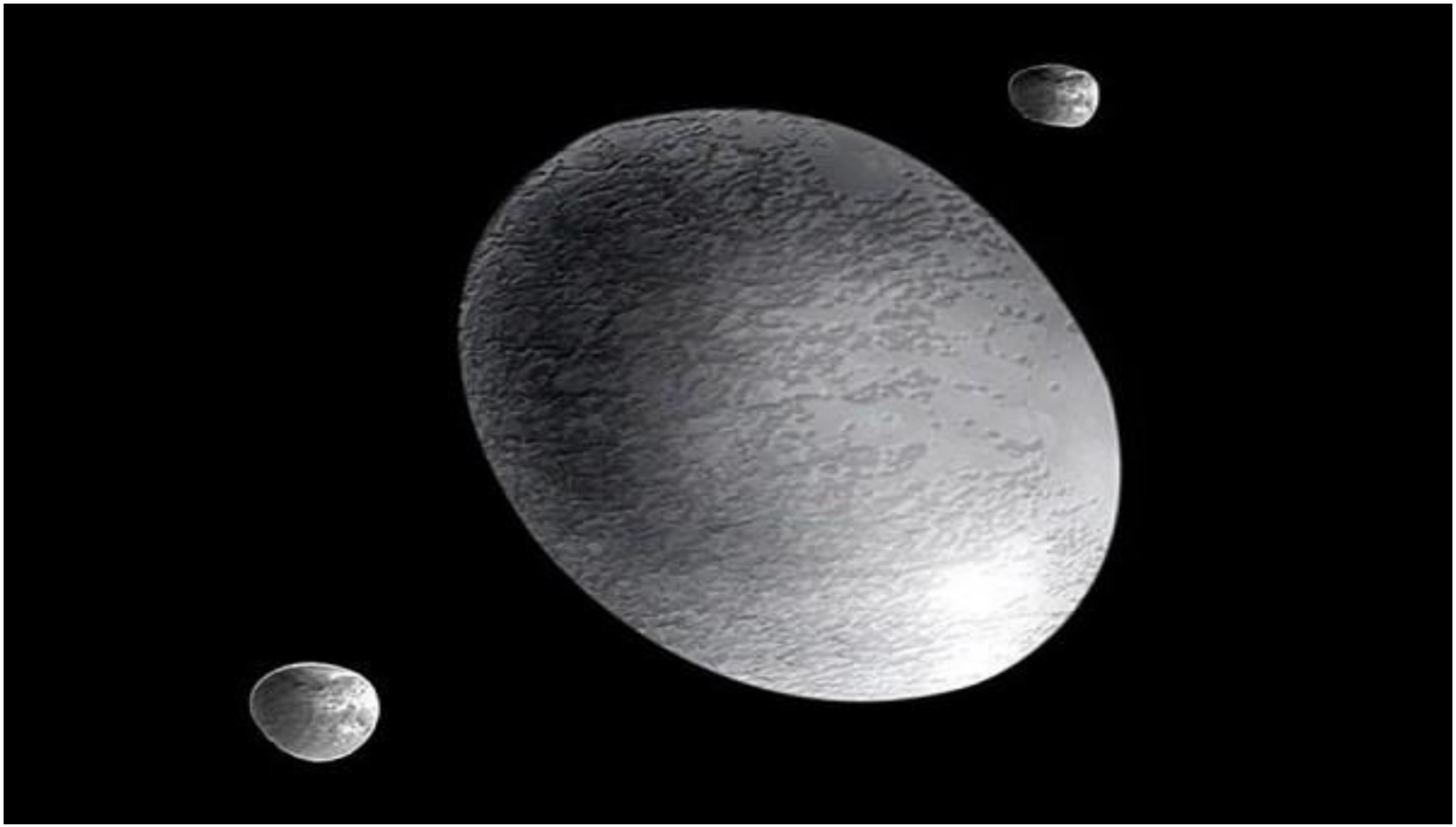
Карликовая планета Haumea была открыта в поясе Койпера в 2005 году и является одним из самых крупных занептунных объектов.

Kuiper Belt Objects
Dwarf planets

Now there are many dwarf planets in the solar system. Among them the dwarf planet Haumea

Haumea and two satellites: Hi'iaka and Namaka



A recognized hypothesis in America is the hypothesis of the impact of the origin of satellit
we proposed another mechanism for the formation of satellites

Хаумеа

В поперечнике составляет примерно 2350 км

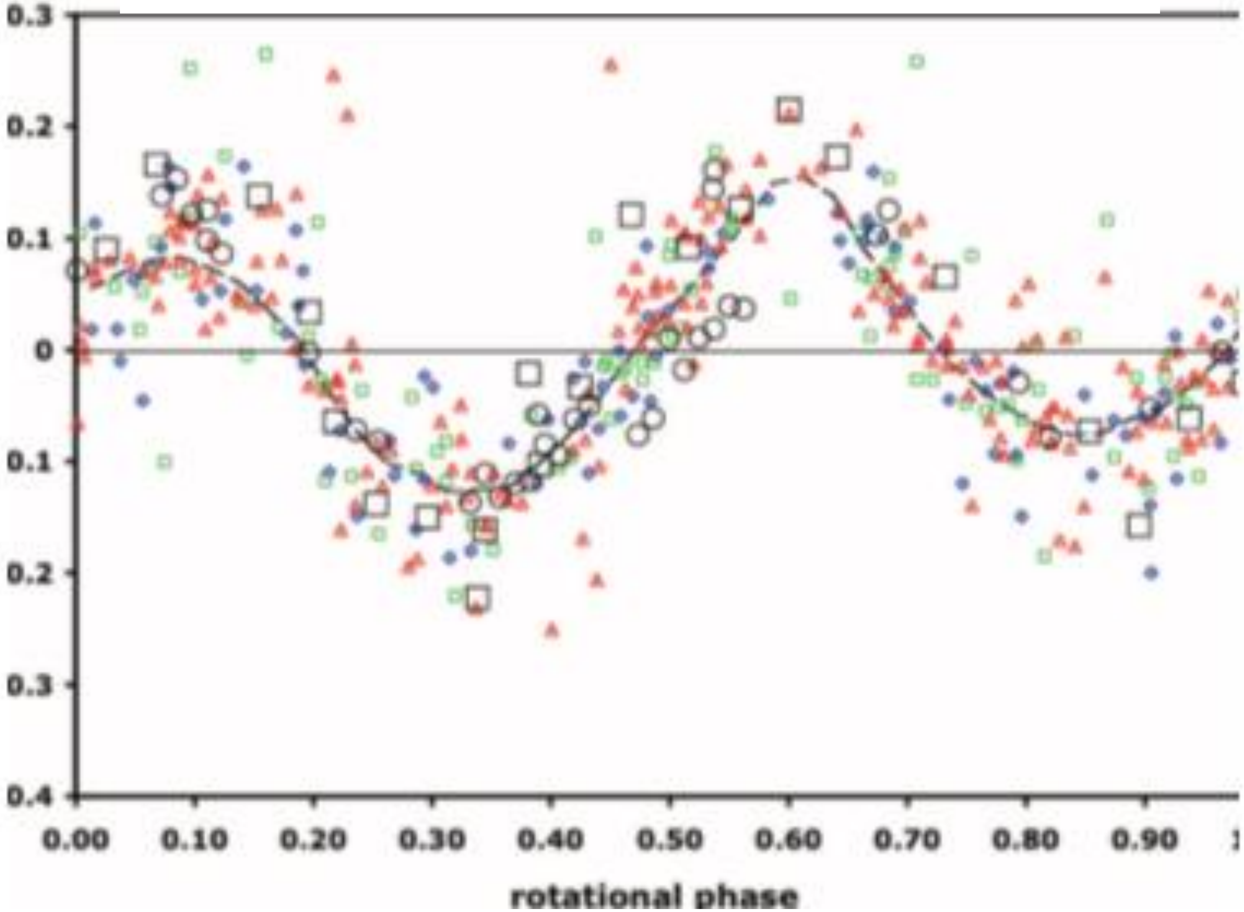
Масса (по спутникам) $M = 4.006 \cdot 10^{24}$ g

Период вращения вокруг Солнца 281.83 years

Период суточного вращения

$T = 3.915483113\ h.$

Light curve: two unequal maximum-
- hint of a triaxial form Xaumea



$\Delta m = 0.32\ mag$ - difference max and min of the light curve

Сразу же стало ясно: Хаумеа обрадает удивительными физическими свойствами

(Трехосная форма + необычайно быстрое спиновое вращение), и её пример расширил наши

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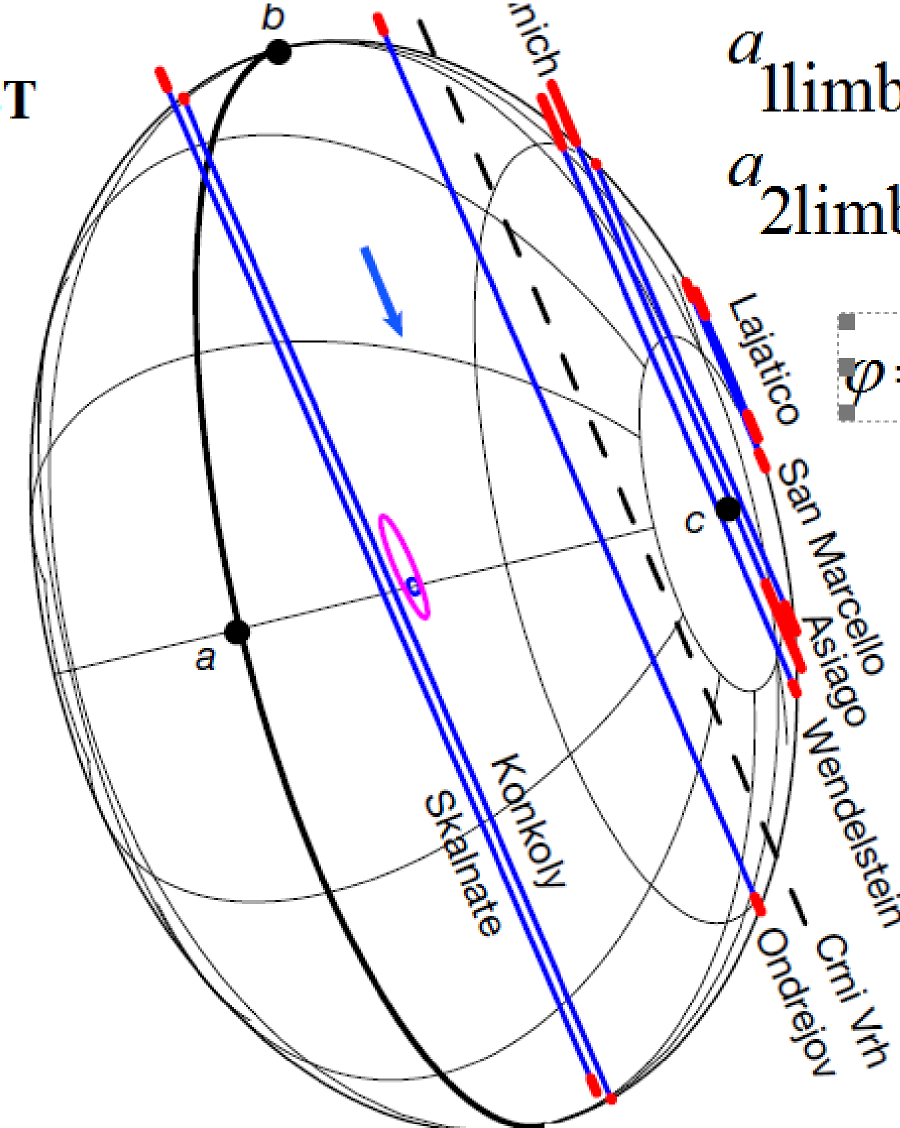
НОВАЯ ИНФОРМАЦИЯ О ХАУМЕА ИЗ НАБЛЮДЕНИЙ ЗАТМЕНИЯ ЗВЕЗДЫ ФОНА

21 ЯНВАРЯ КАРЛИКОВАЯ ПЛАНЕТА ХАУМЕА ПОКРЫЛА ОТДАЛЕННУЮ ЗВЕЗДУ.

Это очень редкое событие раблюдали 12 обсерваторий на Земле (Ortiz et al. 2017)

FIRST



$$a_{1limb} = 852 \pm 2\ km,$$

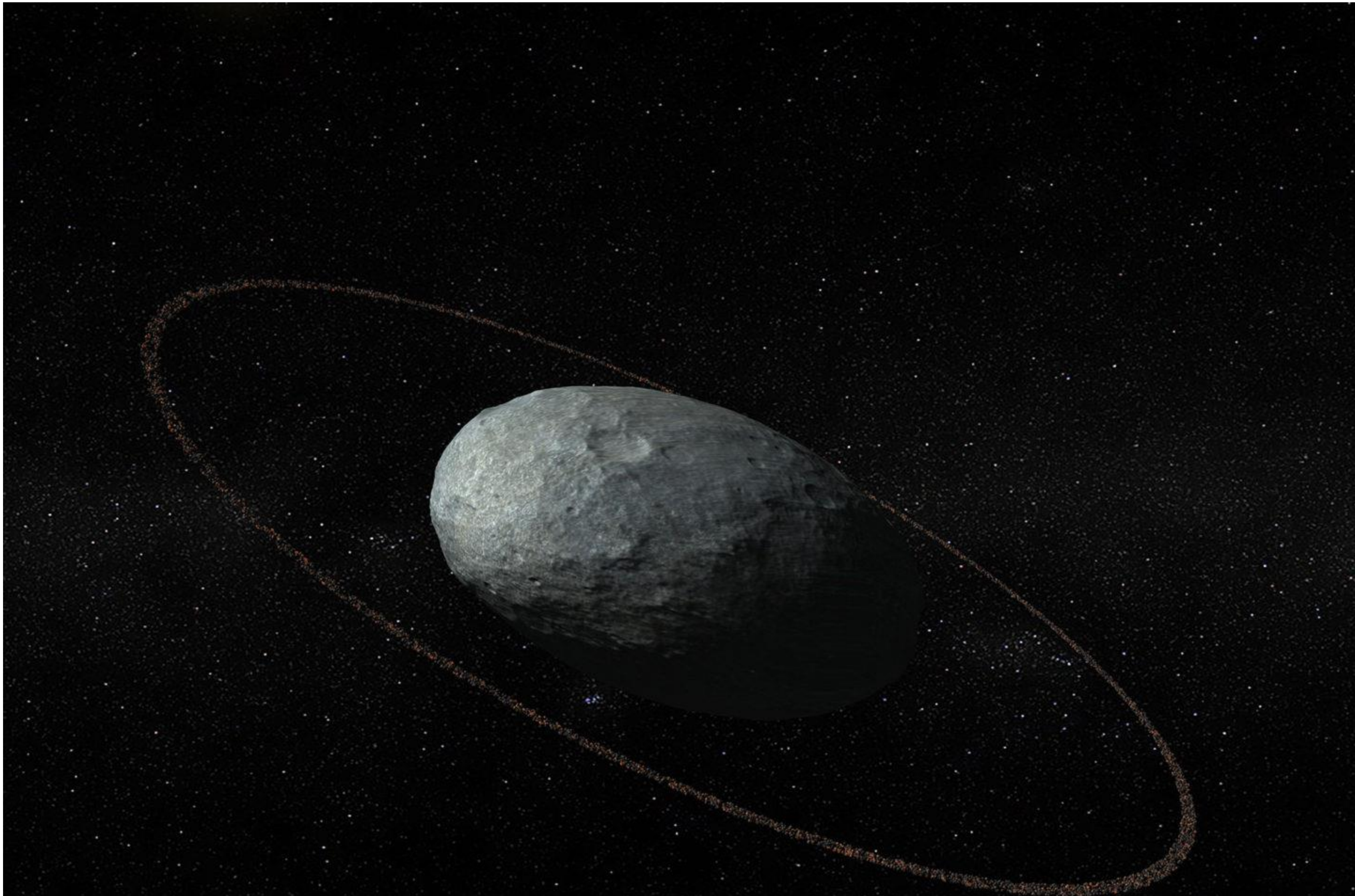
$$a_{2limb} = 569 \pm 13\ km.$$

$$\varphi = -76.3^{0} \pm 1.2^{0}.$$

Figure 2 | Haumea's projected shape. The first time to observe the limb

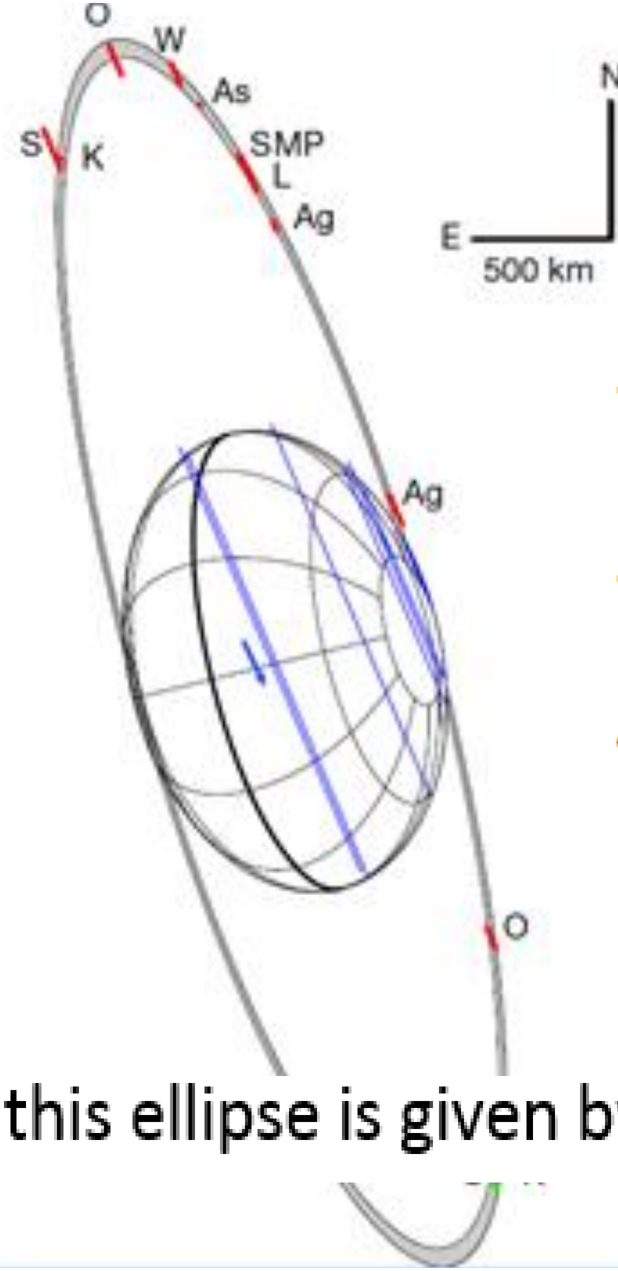
Secondly, observe the ring around the planet

Ortiz et al.Ring around Xaumea



this projection of the ring had semiaxes

Second



$$a'_{1ring} = 2287 \pm^{75}_{45}\ km,$$

$$a'_{2ring} = 541 \pm^{15}_{15}\ km.$$

$$\varphi_{ring} = -74.3^0 \pm 1.3^0.$$

the orientation of this ellipse is given by the position angle of

projection of the ring onto the picture plane

Using all these observational data, as well as photometric data

$$a_{1\text{limb}} = 852 \pm 2\ km,$$

$$a_{2\text{limb}} = 569 \pm 13\ km.$$

$$\varphi = -76.3^0 \pm 1.2^0.$$

$$a'_{1\text{ring}} = 2287 \pm^{75}_{45}\ km,$$

$$\Delta m = 0.32\ mag$$

$$a'_{2\text{ring}} = 541 \pm^{15}_{15}\ km.$$

$$\varphi_{\text{ring}} = -74.3^0 \pm 1.3^0.$$

Based on this information and on the photometry data from (Ragozzine & Brown 2009), in the work (Ortiz et al. 2017) the following values of the semi-axes of the ellipsoid of the Haumea's figure were found

$$a_1 = 1161\,km,$$

$$a_2 = 852\,km, \quad R = (a_1 a_2 a_3)^{\frac{1}{3}} = 797.5\,km.$$

$$a_3 = 513\,km,$$

$$\rho = 1.885\ \mathrm{g/sm}^3.$$

Подчеркнем, что уточненная по (Ortiz et al. 2017) форма планеты заметно отличалась

От формы по (Brown 2009). Например, плотность планеты 1.88 г/см:3 теперь составляла только 72 процента от прежней. **Но самое главное: Хаумеа не попадала теперь на последовательность равновесных эллипсоидов Якоби**

Задача о форме планеты Хаумеа, представляет большой практический и теоретический интерес. Анализ предыдущих исследователей не был полным. **Проблема такова.** Форма и ориентация эллипсоида определяет форму его проекции (лимба) на картинную плоскость. Но как решить обратную задачу:как, зная лимб, восстановить простанственную форму самого эллипсоида. Уже из общих соображений ясно: при эллиптическом лимбе фигура может быть сжатым или вытянутым сфероидом, или же трехосным эллипсоидом. Специалистам по теории фигур равновесия известно, что **эти фигуры обладают разными динамическими свойствами**.

Earlier, two a priori assumptions were made:

1.Ragozzine & Brown 2009: the rotation axis of Haumea is perpendicular to the observer's sightline;

2. It was also believed that Haumea is in a state of relative equilibrium (this is triaxial Jacobi ellipsoid). Based on the assumption (Rabinowitz et al., 2006) found that the figure of this planet has semi-axes

$a_1 = 980\ km,$

$a_2 = 759 km,$ $\rho = 2.58\ \mathrm{g/sm^3}.$

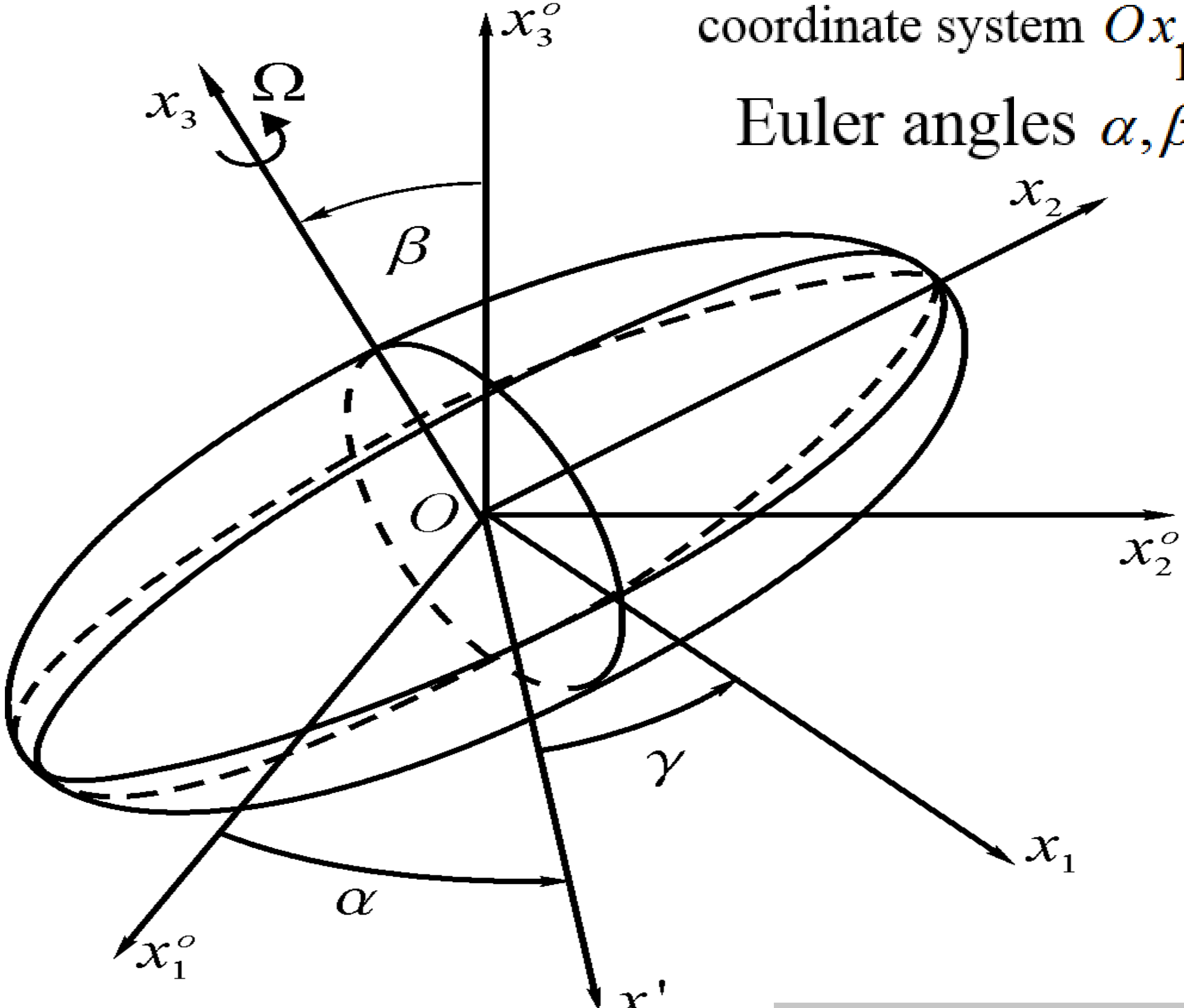
$a_3 = 498\ km, \quad R = (a_1 a_2 a_3)^{\frac{1}{3}} = 718.18\ km$

These dimensions are comparable to Pluto, and the density is much larger than the density of Pluto

Let coordinate system $Ox_1^0x_2^0x_3^0$ – is associated with observer

coordinate system $Ox_1x_2x_3$ of ellipsoid

Euler angles α, β, γ are shown



the picture plane is $Ox_2^0x_3^0$

Let consider an arbitrarily oriented ellipsoid

$$\frac{x_1^2}{a_1^2}+\frac{x_2^2}{a_2^2}+\frac{x_3^2}{a_3^2}=1.$$

The transition matrix from coordinate system of observer to the system of ellipsoid has the form

$$A=\begin{pmatrix} \cos\alpha\ \cos\gamma - \sin\alpha\ \cos\beta\ \sin\gamma & \sin\alpha\ \cos\gamma+\cos\alpha\ \cos\beta\ \sin\gamma & \sin\beta\ \sin\gamma \\ -\cos\alpha\ \sin\gamma-\sin\alpha\ \cos\beta\ \cos\gamma & -\sin\alpha\ \sin\gamma+\cos\alpha\ \cos\beta\ \cos\gamma & \cos\gamma\ \sin\beta \\ \sin\alpha\ \sin\beta & -\cos\alpha\ \sin\beta & \cos\beta \end{pmatrix}.$$

New and old coordinates are related by the formula

$$\boldsymbol{x}=A\mathbf{x}^{\mathbf{o}}.$$

Projection of the ellipsoid and its axis of rotation onto the picture plane

We write equation in the system $Ox_1^0 x_2^0 x_3^0$ and project the visible ellipsoid onto the observer's picture plane $Ox_2^0 x_3^0$.

This projection is (Kondratyev & Ozernoy 1979)

$$\left(R_{12}^2 - R_1 R_2\right) x_2^{0\,2} + 2\left(R_{12} R_{13} - R_1 R_{23}\right) x_2^0 x_3^0 + \left(R_{13}^2 - R_1 R_3\right) x_3^{0\,2} + R_1 = 0,$$

where

$$R_k = \sum_{i=1}^{3} \frac{\alpha_{ik}^2}{a_i^2}, \quad R_{kl} = \sum_{i=1}^{3} \frac{\alpha_{ik}\alpha_{il}}{a_i^2},$$

and α_{ij} are the terms of the rotation matrix A.

Then the position angle between the minor semi-axis of the ellipse and the axis Ox_3^0 is

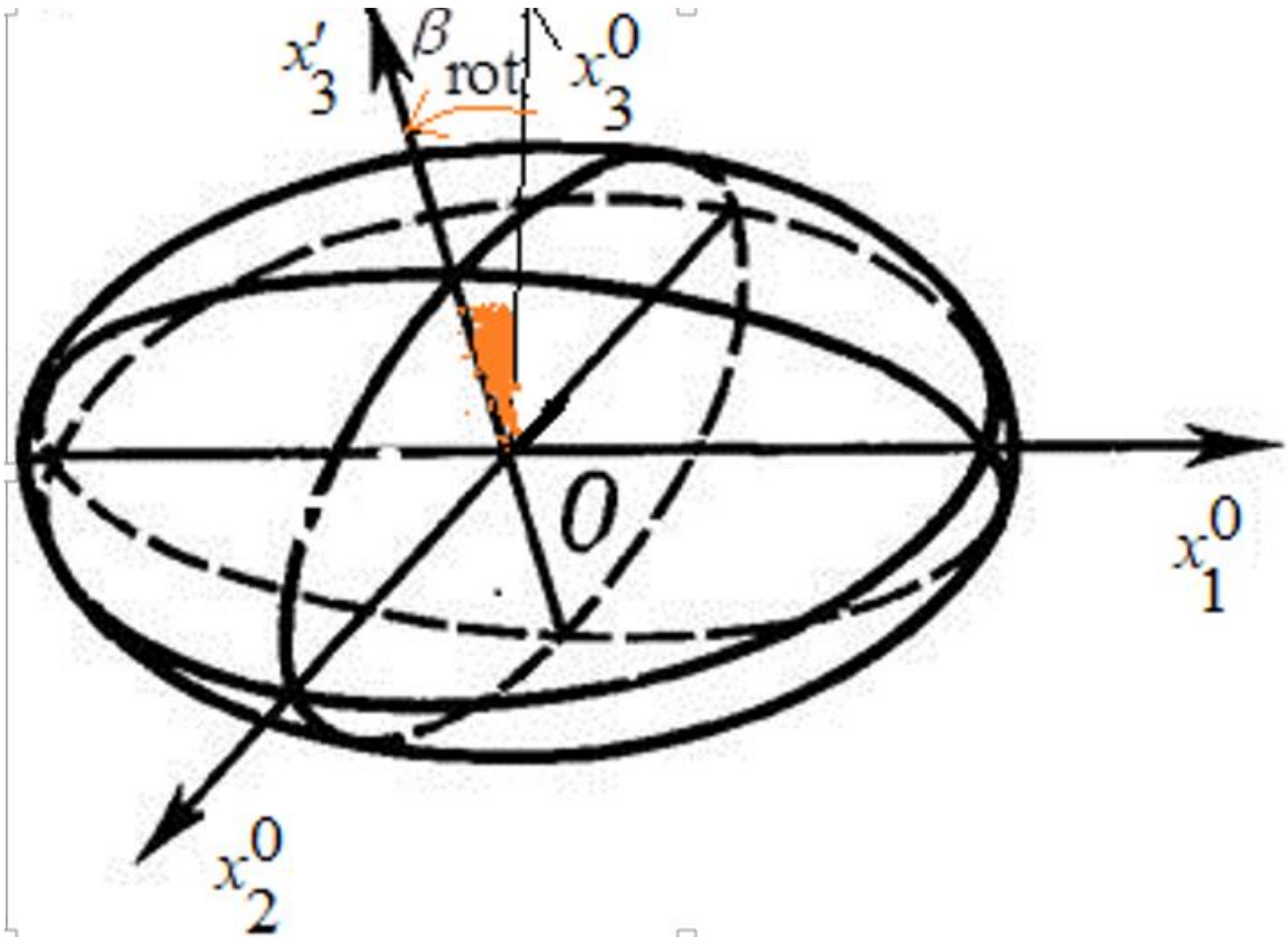
$$\operatorname{tg} 2\varphi = -2\frac{\dfrac{\alpha_{32}\alpha_{33}}{a_1^2 a_2^2}+\dfrac{\alpha_{22}\alpha_{23}}{a_1^2 a_3^2}+\dfrac{\alpha_{12}\alpha_{13}}{a_2^2 a_3^2}}{\dfrac{\alpha_{32}^2-\alpha_{33}^2}{a_1^2 a_2^2}+\dfrac{\alpha_{22}^2-\alpha_{23}^2}{a_1^2 a_3^2}+\dfrac{\alpha_{12}^2-\alpha_{13}^2}{a_2^2 a_3^2}}.$$

The position angle φ_{rot} between the projections on the picture plane of the axis of rotation and the axis Ox_3^0 is

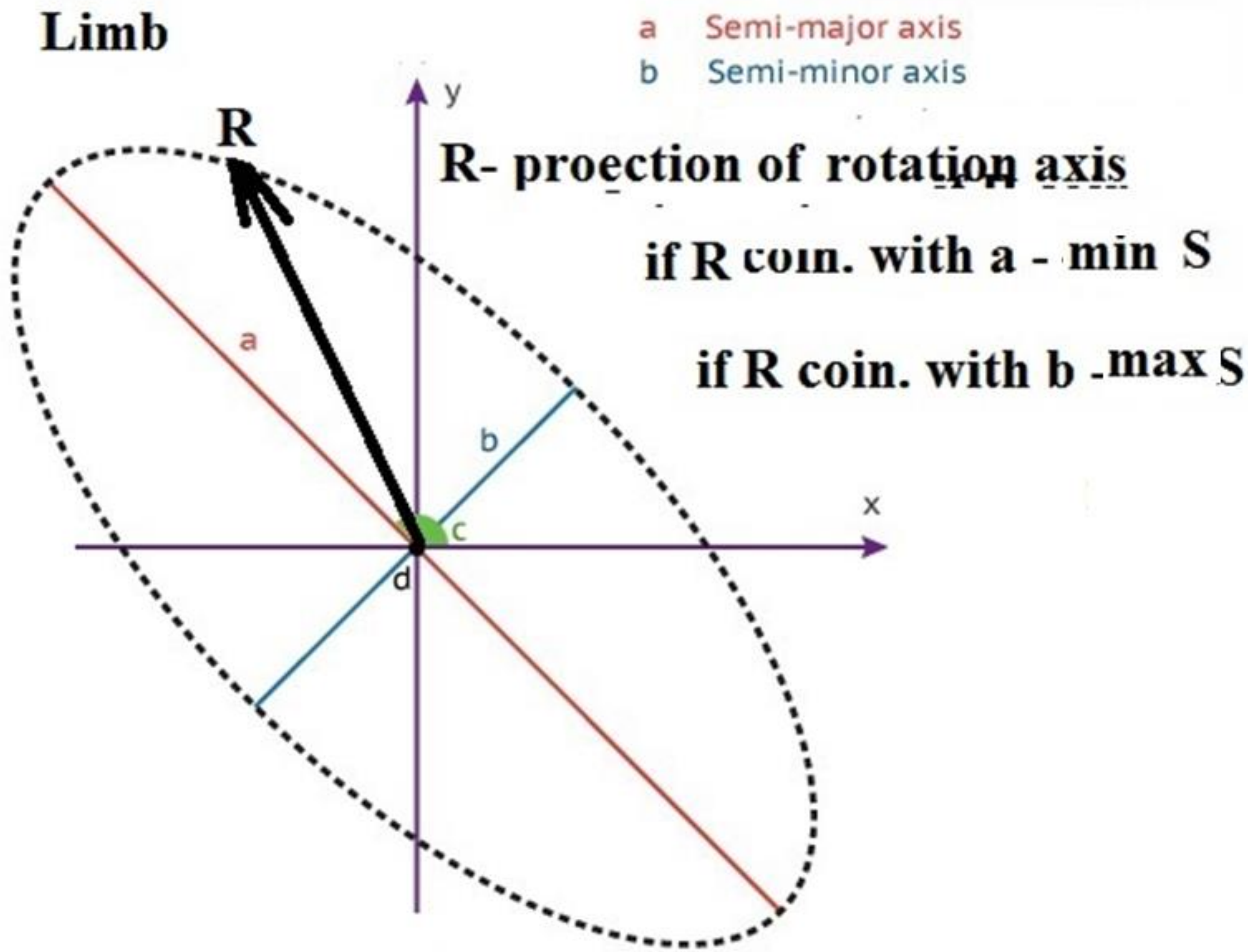
$$tg\, 2\varphi_{\mathrm{rot}} = -\frac{2\cos\alpha\sin\beta\cos\beta}{\cos^2\beta-\cos^2\alpha\sin^2\beta}.$$

In the important case $a_1 = a_2 \geq a_3$, when the body has the form of oblate spheroid, then $\varphi = \varphi_{\mathrm{rot}}$.

The angle between the angular velocity vector of the ellipsoid rotation and the observer's picture plane



$$\beta_{rot} = \arcsin(\sin\alpha \cdot \sin\beta)$$


Limb
a Semi-major axis
b Semi-minor axis
y
R
R- proection of rotation axis
if R coin. with a - min S
if R coin. with b -max S
a
b
x
c
d

Therefore, instead of Euler's angles α, β, γ, we choose three other angles $(\varphi_{\text{rot}}, \beta_{\text{rot}}, \gamma)$. First, we rotate the coordinate system about the axis Ox_1^0 by angle φ_{rot}; then rotate it around the new axis Ox'_2 by angle β_{rot}; finally, we make a rotation around the new axis Ox''_3 by angle γ. The final rotation matrix is

$$A_{x_3^0} = \begin{pmatrix} \cos\gamma & \sin\gamma & 0 \\ -\sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\beta_{rot} & 0 & -\sin\beta_{rot} \\ 0 & 1 & 0 \\ \sin\beta_{rot} & 0 & \cos\beta_{rot} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\varphi_{\text{rot}} & \sin\varphi_{\text{rot}} \\ 0 & -\sin\varphi_{\text{rot}} & \cos\varphi_{\text{rot}} \end{pmatrix},$$

or, after multiplication,

$$A = \begin{pmatrix} \cos\gamma\cos\beta_{\text{rot}} & \sin\gamma\cos\varphi_{\text{rot}} + \cos\gamma\sin\beta_{\text{rot}}\sin\varphi_{\text{rot}} & \sin\varphi_{\text{rot}}\sin\gamma - \cos\gamma\sin\beta_{\text{rot}}\cos\varphi_{\text{rot}} \\ -\sin\gamma\cos\beta_{\text{rot}} & \cos\gamma\cos\varphi_{\text{rot}} - \sin\gamma\sin\beta_{\text{rot}}\sin\varphi_{\text{rot}} & \sin\varphi_{\text{rot}}\cos\gamma + \sin\gamma\sin\beta_{\text{rot}}\cos\varphi_{\text{rot}} \\ \sin\beta_{\text{rot}} & -\cos\beta_{\text{rot}}\sin\varphi_{\text{rot}} & \cos\beta_{\text{rot}}\cos\varphi_{\text{rot}} \end{pmatrix}$$

The system of equations

In principle, part of the work on deriving equations for unknowns in the problem of restoring the shape of an ellipsoid has already been done above. However, the introduction of a new matrix leads to a revision of the problem. So from the positional observations, we immediately find the angle $\sin\beta_{\mathrm{rot}} = \frac{541}{2302} = 0.235013$, from which $\beta_{rot} \approx 13.6^0$. As for the position angle of the small axis of the ellipse of the projection of the ring $\varphi_{\mathrm{ring}} = -74.3^0 \pm 1.3^0$, φ_{rot} it is known directly from observations.

As of January 2017, the parameter of photometry $\Delta m_1 \in [0^m.25, 0^m.35]$, and the direction to Haumea, according to the ephemerides, $\begin{matrix} \alpha = 14^h 12^m 03^s, \\ \delta = +16^0 33' 59'', \end{matrix}$

the angles φ_{rot} and β_{rot} of the matrix are taken on the date.

2). As of February 2009, the photometry parameter for Haumea, according to the ephemerides,

$$\alpha_2 = 13^h 43^m 10^s,$$

$$\delta_2 = +18^0 53' 26'',$$

the angle $\beta_{2\,\text{rot}}$ of the matrix at that date.

3). (α_p, δ_p) is direction to the Haumea pole;

$(a_{1\text{limb}}, a_{2\text{limb}})$ – the semi-axes of Haumea's limb;

φ_{rot} – the positional projection angle on the picture plane of the Haumea rotation axis;

φ – the position angle of the minor axis of Haumea limb;

β_{rot} – the slope of the axis of rotation Haumea to the picture plane;

γ – the rotation angle of Haumea around its axis.

In total, we have eight equations. Among them, two photometric equations:

$$\frac{a_1^2}{a_2^2} \cdot \frac{a_2^2 \sin^2 \beta_{rot} + a_3^2 \cos^2 \beta_{rot}}{a_1^2 \sin^2 \beta_{rot} + a_3^2 \cos^2 \beta_{rot}} = 10^{0.8 \Delta m_1},$$

$$\frac{a_1^2}{a_2^2} \cdot \frac{a_2^2 \sin^2 \beta_{rot} + a_3^2 \cos^2 \beta_{rot}}{a_1^2 \sin^2 \beta_{rot} + a_3^2 \cos^2 \beta_{rot}} = 10^{0.8 \Delta m_2}.$$

and three equations related to the coordinates of the Haumea pole:

$$\left(\cos\alpha_2\cos\delta_2, \sin\alpha_2\cos\delta_2, \sin\delta_2\right)\begin{pmatrix}\cos\alpha_p, \cos\delta_p \\ \sin\alpha_p, \cos\delta_p \\ \sin\delta_p\end{pmatrix} + \sin\beta_{2\mathrm{rot}} = 0,$$

The eight equations include also eight variables:

$$a_1, a_2, a_3, \varphi_{\text{rot}}, \beta_{\text{rot}}, \beta_{2\ \text{rot}}, \alpha_p, \delta_p;$$

In addition, the equations include ten known quantities:

$$a_{1\text{limb}}, a_{2\text{limb}}, \gamma, \varphi, \Delta m_1, \Delta m_2, \alpha, \delta, \alpha_2, \delta_2.$$

When carrying out calculations by this method, one should use the terms of the rotation matrix written above.

It should also be taken into account that observational fact (Ortiz et al. 2017) that at the time of covering the distant star, the brightness of Haumea was minimal (or close to a minimum). As we were kindly informed in a private letter of J. L. Ortiz, the phase of the lag between the minimum brightness of Haumea and the moment of eclipse was 0.037. This gives $\gamma \approx -13.32^0$.

The results of calculations

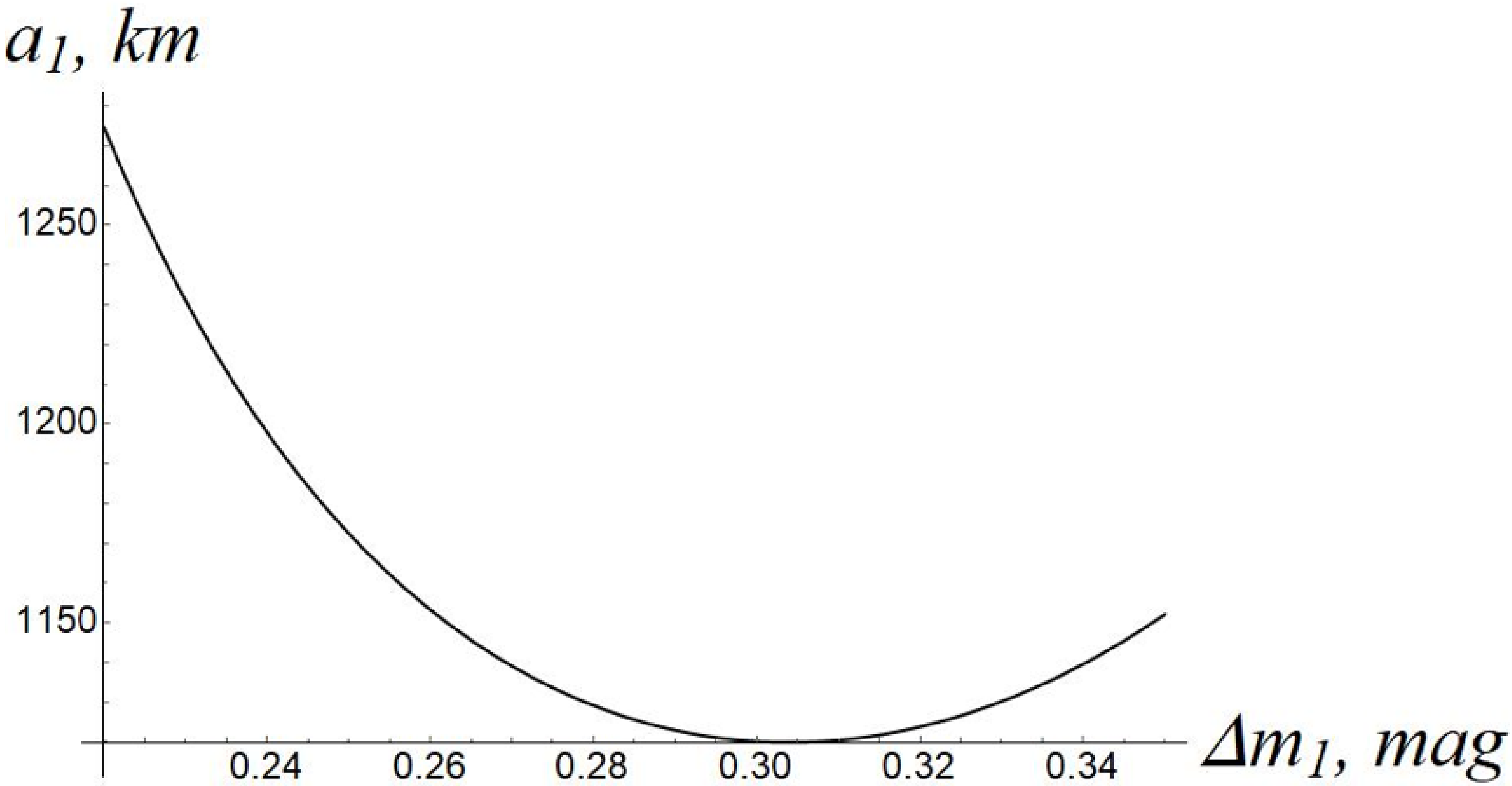


Figure 2. Semi-major axis a_1 (in km) for the ellipsoidal Haumea model as a function of the photometry parameter Δm_1

It is interesting to note that the semi-major axis has a minimum $a_1 \approx 1120\ km$

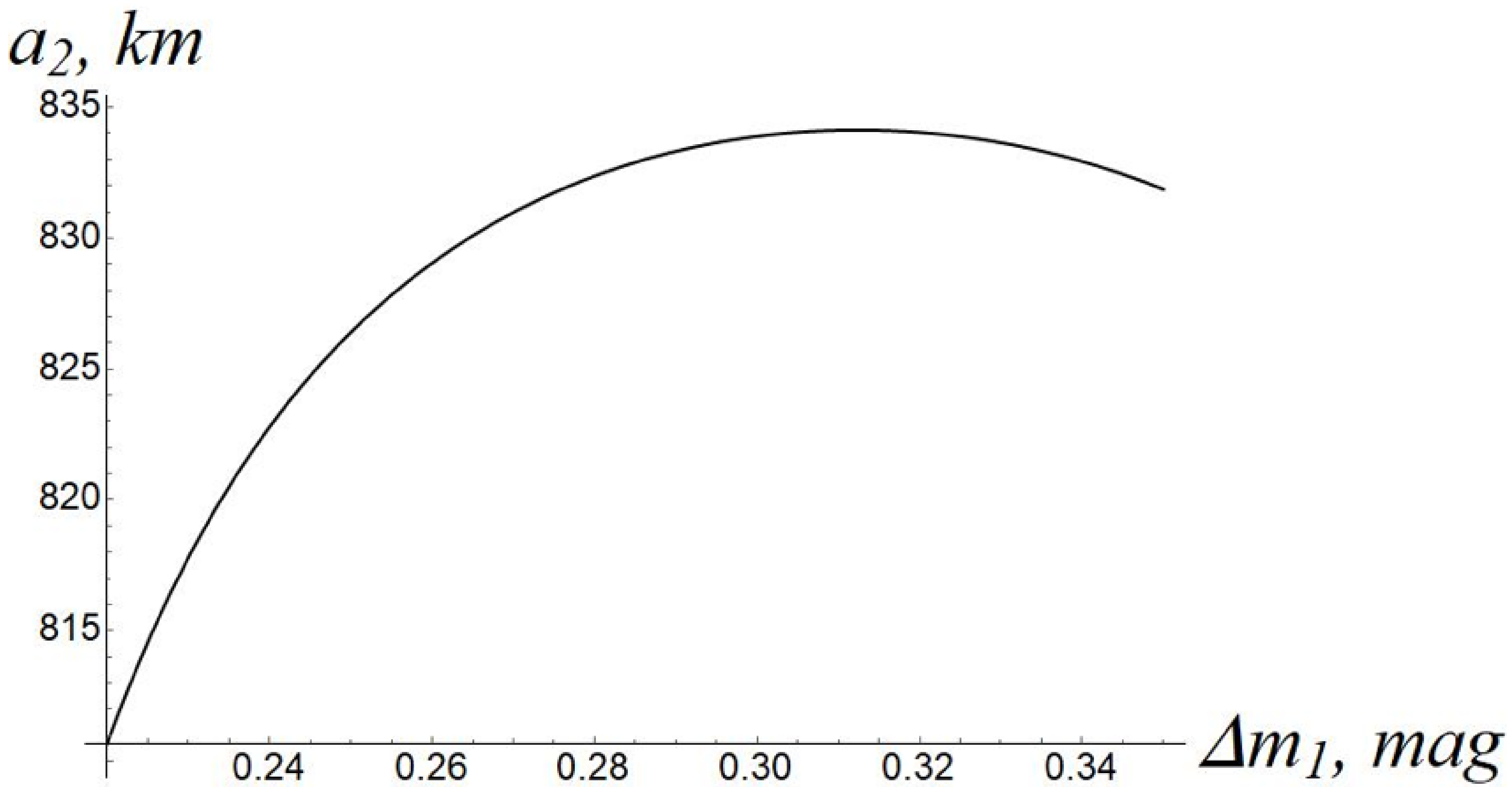


Figure 3. Semi-middle axis a_2 for the ellipsoidal Haumea's model as a function of photometry parameter

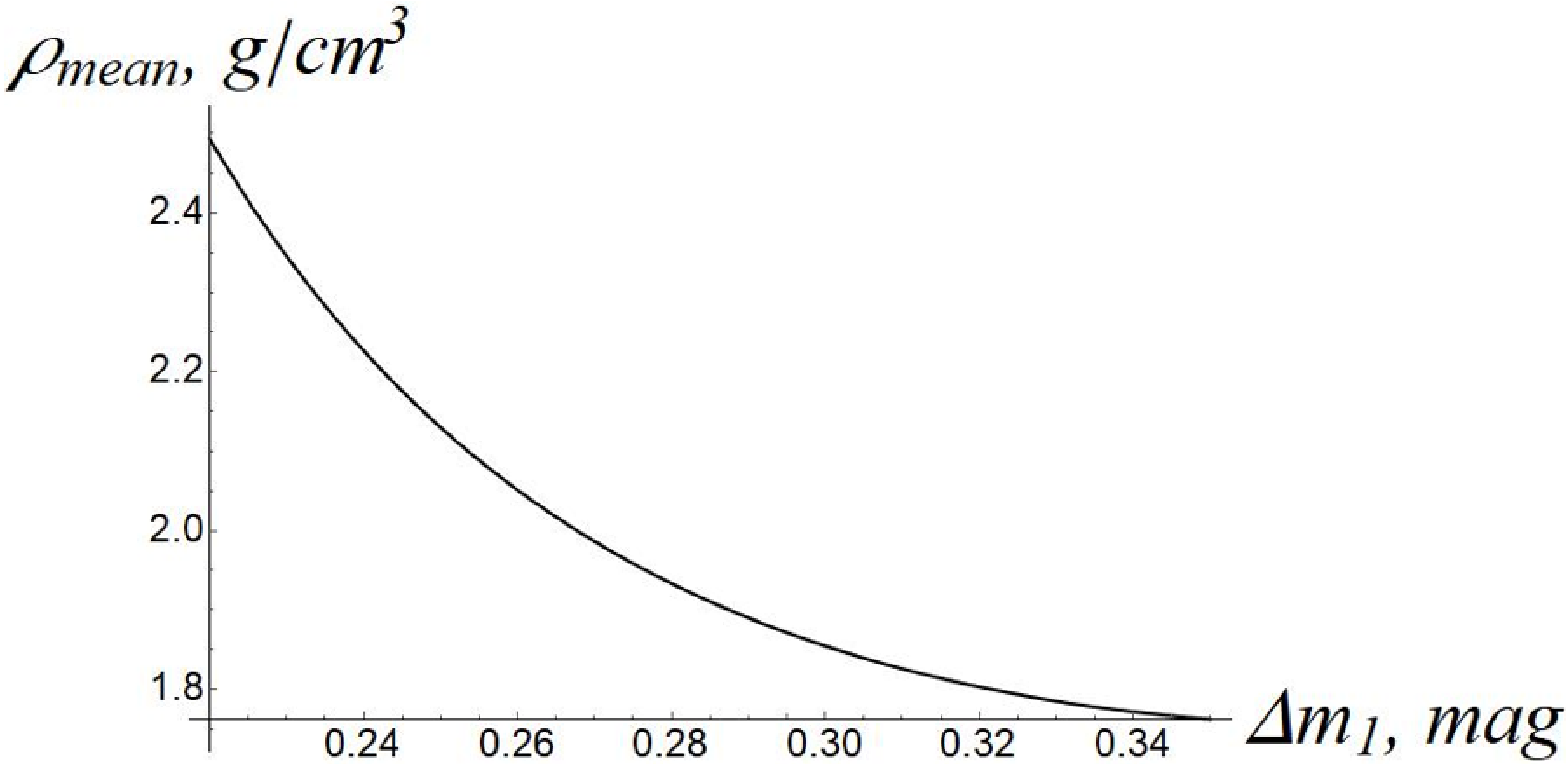
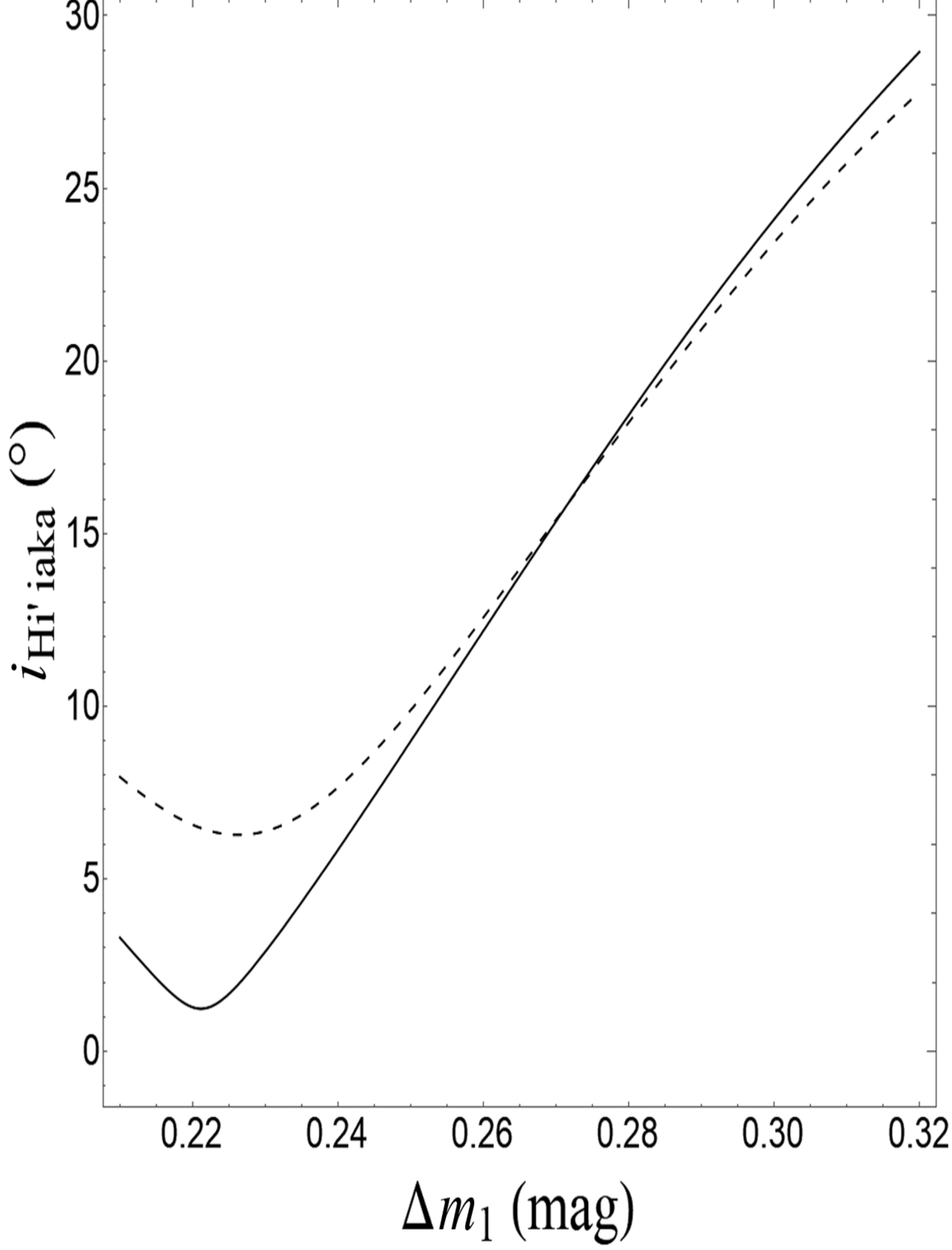


Figure 5. The dependence of the average density ρ_{mean} of the Haumea model on Δm_1



30
25
20
15
10
5
0
0.22
0.24
0.26
0.28
0.30
0.32
$i_{\mathrm{Hi'iaka}}$ (°)
Δm_1 (mag)