

# Chandler Wobble in Variations of the Pulkovo Latitude for 170 Years

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**Abstract**—The work studies the Chandler component of polar motion, obtained from variations in the Pulkovo latitude over 170 years (1840–2009). To extend the time series of variations in the Pulkovo latitude back into the past until 1840, we used the first Pulkov observations on the basis of the Reynolds transit instrument in the prime vertical and on the basis of large vertical Ertel circle. We employed different methods of analysis of nonstationary time series, such as wavelet analysis, methods of bandpass filtering, singular spectral analysis, and Fourier and Hilbert transforms. Changes in the Pulkovo latitude from 1904–2006, as inferred from ZTF-135 observations and as calculated from international data, were compared. It was shown that time changes in the amplitude and phase of Chandler polar motion can be studied based on long-term observation time series of latitude at a single observatory, even if these observation records have gaps. We were the first to study the changes in the Chandler wobble for that long time series of variations in the Pulkovo latitude with the help of different methods. The long observation record and the methods of analysis of nonstationary time series had allowed us to identify two similar structures, both well apparent during the periods of 1845–1925 and 1925–2005 in the time variations of phase and amplitude. The presence of this structure indicates that low-frequency regularities may be present in the Chandler polar motion, and one of the manifestations of this may be the well known feature in the region of 1925. The superimposed epoch method was used to estimate the period of variations in the amplitude with a simultaneous change of phase of this oscillation, which was found to be 80 years. In addition, advantages of singular spectral analysis for studying the long-period time series with involved structure are demonstrated.

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## INTRODUCTION

The polar motion of Earth consists of many components, the most significant of which are annual wobble and the Chandler wobble of pole (CW) with the period of 428–435 days. These two periodic motions are easily distinguishable in observations of latitude and have been studied by various authors many times (Fedorov, Yatskiv, 1964; Sidorenkov, 2002; Vondrak, Ron, 2005). The presence of an annual harmonic is explained by seasonal transport of air and water masses over the Earth's surface. The Chandler wobble constitutes quasi-harmonic oscillation with a complex structure and is most often associated with the period of free nutation, the presence of which was theoretically predicted by L. Euler in the late eighteenth century. Assuming that Earth is a solid uniform body, and that external forces act on it, Euler showed that the pole must circumscribe a cone around the principal axis of inertia of the terrestrial spheroid with a period of about 305 days. First, researchers pursued confirmation that the latitude changes, and estimation of the period of its change, relying on the a priori assumption that the Euler period is present. At the Pulkovo Observatory, Kh.I. Peters was the first to calculate the latitude and its changes from his observations on

the basis of a large vertical Ertel circle. The studies of latitude variations were continued by M.O. Nuren. In their study, Peters and Nuren used the Euler-predicted period, and so they failed to detect latitude variations with a period longer than one year. Nuren obtained a period close to 430 days only after all observations were analyzed with the help of a vertical circle, fabricated to compose the 1885 catalog (Ivanov, 1895). These studies preceded the works of S. Chandler, who in 1891 started publishing his papers devoted to variations in latitude, and who was first to claim the existence of about a 428-day period (Chandler, 1891). During three years (1891–1893), he published eight papers in *Astronomical Journal*, which were jointly entitled “On the variation of latitude.” He analyzed 45 short observation series, containing over thirty thousand individual observations, obtained at different observatories around the globe between 1841 and 1890. As a result, he concluded that the variations of latitude contain two oscillations, with periods of about 428 and 365 days. Moreover, later Chandler separated out the annual wobble and found out that the remaining oscillation consists of two harmonics, first with the period of 428.5 days (1.17 year) and amplitude of 0''.14, and the second with the period of 436 days

(1.19 year) and amplitude  $0''.09$  (Chandler, 1901a). Assuming the presence of two harmonics, and taking into account their periods, he suggested a model (Chandler, 1901b) that predicted a new decrease in the amplitude of this oscillation in approximately 1910. Indeed, the CW amplitude had decreased, and the phase had concurrently changed, but almost 20 years later, in approximately 1930. A.Ya. Orlov was the first to detect this feature (Orlov, 1944). The CW has been studied more extensively in monographs (Mank, Mac-Donald, 1964; Lambeck, 1980), as well as in later works (Guinot, 1982; Kurbasova et al., 2002; Guo et al., 2005; Vondrak, Ron, 2005; Brzezinski, 2005).

Works (Miller, 2008; Malkin, Miller, 2009) analyzed the Chandler component, inferred from the longest time series of the coordinates of the pole available from International Earth rotation and Reference systems Service (IERS C01, <http://hpiers.obspm.fr/iers/eop>) for 1846–2008.5, and detected the presence of three regions (in approximately 1846, 1925, and 2005) of sharp change in CW amplitude, accompanied by a change in phase. The presence of the time interval where the amplitude decreases at the beginning of the time series was indicated by S. Chandler, A.A. Ivanova, B. Wanach, H. Kimura, A.S. Vasilyeva, A.Ya. Orlova, and N. Sekiguchi, who studied the variations in latitude of different observatories worldwide in the period from 1836–1860. Work (Miller, Prudnikova, 2010) surveyed the papers of those authors and draw a general conclusion that the amplitude of the Chandler wobble in the period of 1840–1850 was  $\sim 0''.08$ , much lower than for the later period until 1920. Works (Orlov, 1961; Sekiguchi, 1975) showed that the phase of the Chandler wobble changes in this time interval. In his work, Sekiguchi concluded that the CW behaved in this time interval similar to its behavior in 1920s.

Long-term homogeneous observation time series are required to study low-frequency variations in the pole latitude. The Pulkovo observatory has been conducting multiyear astronomic observations with different instruments since 1840. This work studies the Chandler component of the polar motion, inferred from variations in the Pulkovo latitude for 170 years (1840–2009). The analysis is performed using observations with transit instrument in the prime vertical (TIPV), with large vertical circle (LVC), and with a Freiberg–Kondratiev zenith telescope (ZTF-135). By analyzing time series of the coordinates of the pole and variations in the Pulkovo latitude for a maximally accessible time interval with the help of the methods designed for studying the complex nonstationary signals, we detected new long-period regularities in the CWP behavior. We identified three regions where the amplitude decreases simultaneously with change in the phase of this oscillation, suggesting that the well-

known regularity in the middle of the time interval may occur again with a period of about 80 years.

## METHODS USED

Study of the structure of the Chandler wobble had been of top interest in late twentieth century, when the theory of analysis of complex nonstationary signals was not as yet well updated. Researchers used different methods to decompose the polar motion into components. Many methods used for this failed to discriminate a useful signal against noise on the entire observation interval, especially when the noise level varied over the time series, just as it does in the time series of the coordinates of the pole. At present, the dynamic spectral analysis and wavelet analysis are most often used to study the nonstationary time series.

In this work, the Chandler component of the polar motion was isolated using the following methods.

—Singular spectral analysis (SSA) (Golyandina, 2001). This method refers to the class of techniques in natural orthogonal functions, for which the basis functions are determined from the data themselves. In the case of wavelet analysis, the series is decomposed into a chosen basis. The SSA method has good resolution both in time and frequency, can be used to study the time series with complex structure and to trace the behavior of amplitude, phase, and frequency in time. Moreover, the useful signal is well discriminated against noise even if noise varies in different parts of the series. A multidimensional method version can be used to analyze datasets as a whole, by identifying common regularities or distinct features. Works (Vityazev, 2000; Vorotkov et al., 2002) demonstrated the advantages of this method for analysis of time series of Earth's rotation parameters.

—Wavelet analysis (Vityazev, 2001). In the case of wavelet analysis, a basis or a wavelet type should be chosen for each particular task. The Morle wavelet was chosen for analysis of coordinates of the pole; this method is judiciously substantiated in (Kudryashova, 2000) and is compared with the SSA in (Vityazev, 2000).

—Methods of bandpass filtering. In this work, filtering was performed using fifth-order Zolotarev–Cauer elliptic frequency bandpass filter (Ellip, Matlab Signal Processing Toolbox). The reader is referred to (Malkin, Miller, 2009) for examples of application of other filters and for analysis of the results obtained.

Amplitude and phase variations were calculated using complex Hilbert transform (Hilbert, Matlab Signal Processing Toolbox).

Table

Observations	Constituent parts of time series				
	$\Delta\phi_{TIPV}$	$\Delta\phi_{LVC1}$	$\Delta\phi_{LVC2}$	$\Delta\phi_{ZTF-135}$	$\Delta\phi_{C01}$
Time series 1	1840–1843	1842–1848	—	—	1846–2010
Time series 2	1840–1843	1842–1848	—	1904–1941, 1948–2007	1846–1904, 1941–1948, 2006–2010
Time series 3	1840–1843	1842–1848	1863–1875	—	1846–2010
Time series 4	1840–1843	1842–1848	1863–1875	1904–1941, 1948–2007	1846–1904, 1941–1948, 2006–2010

### TIME SERIES STUDIED

In this work, we use the longest records of coordinates of the pole (IERS C01) for 1846–2010 (<http://hpiers.obspm.fr/iers/eop>) at an interval of 0.1 year. These are hybrid time series consisting of separate parts constructed on the basis of observations with different accuracies and densities. The C01 series was compiled using observations at different observatories worldwide, based both on different observations and processing techniques. The time series includes observations performed with the help of both present-day and classical means for determining the Earth Orientation Parameters (EOP). Data for the time interval of 1846–1891.5 were obtained by Rykhlova (1970) from observations in Pulkovo, Greenwich, and Washington.

Measurements of the Pulkovo latitude were calculated from the IERS C01 time series of coordinates  $X_p$ ,  $Y_p$  of the pole according to the formula

$$\begin{aligned}\Delta\phi = \varphi - \varphi_0 &= X_p \cos \lambda - Y_p \sin \lambda \\ &= 0.8631 X_p - 0.5049 Y_p,\end{aligned}\quad (1)$$

where  $X_p$  and  $Y_p$  are coordinates of the pole, and  $\lambda$  is the longitude of the observation ZTF-135.

In addition, we used the homogeneous time series of ZTF-135 observations of Pulkovo latitude ( $\Delta\phi_{ZTF-135}$ ) for 1904.8–2007.0, and four hybrid time series of measurements of Pulkovo latitude (1840.4–2010.0) with 0.1-year step. Due to military activities, ZTF-135 had been demounted in 1941 and relaunched into systematic observations only in 1948, hence the gap in the ZTF-135 observations between 1941 and 1848. This time series exhibits a high stability and homogeneity, especially in the period of 1948–1994.

Selection of all available records of observations of Pulkovo latitude for 1840–1855 (Miller, Prudnikova, 2010) to compile a hybrid time series of variations in the Pulkovo latitude yielded two time series: the observation series of V.Ya. Struve made with the use of Rey-

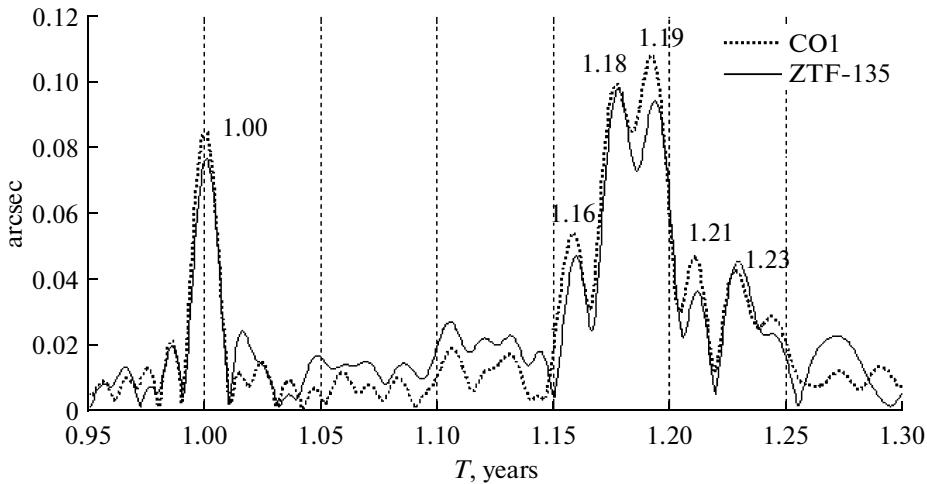
nolds transit instrument in the prime vertical ( $\Delta\phi_{TIPV}$ ) for 1840–1843 and the observation series of Kh.I. Peters made with the help of large vertical Ertel circle ( $\Delta\phi_{LVC1}$ ) for 1842–1848. Also, we used observations by Gylden (1863–1871) and Nuren (1871–1875) made with the help of large vertical circle (Ivanov, 1895) ( $\Delta\phi_{LVC2}$ ). These observations were used to determine absolute stellar coordinates. During determination of the time series of absolute stellar declinations, we also obtained the time series of latitude, which can be used to study polar motion. Constituent subseries of four united records of variations in the Pulkovo latitude are presented in the table.

Entries in time series 1 are as follows:

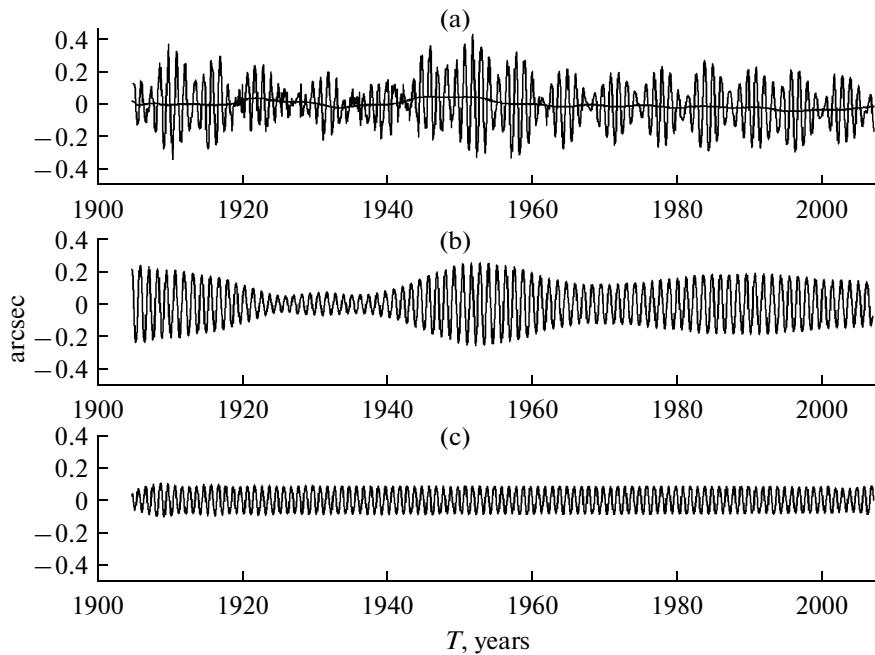
- 1) The series of variations in the Pulkovo latitude, inferred from observations on the basis of transit instrument in the prime vertical for 1840–1843 ( $\Delta\phi_{TIPV}$ ).
- 2) The record of variations in the Pulkovo latitude, inferred from measurements with large vertical circle for 1842–1846 ( $\Delta\phi_{LVC1}$ ).
- 3) The measurement series of Pulkovo latitude, calculated from IERS C01 coordinates of the pole (1846–2010.0) according to formula (1) ( $\Delta\phi_{C01}$ ).

In time series 2, the variations in the Pulkovo latitude, calculated from coordinates of the pole, are replaced with ZTF-135 observations for 1904.8–1941.5 and 1948.7–2007.0 ( $\Delta\phi_{ZTF-135}$ ).

Time series 3 is analogous to record 1, and time series 4 is analogous to record 2, except in the interval 1863.5–1875.5, where variations in the Pulkovo latitude, calculated from the coordinates of the pole, are replaced with the observations of Gylden (1863–1871) and Nuren (1871–1875) with the help of a large vertical circle (Ivanov, 1895) ( $\Delta\phi_{LVC2}$ ).



**Fig. 1.** Fourier analysis of  $\Delta\varphi_{ZTF}$  (solid line) and  $\Delta\varphi_{C01}$  (dashed line) time series in the entire observation period from 1904–2006 for the range of oscillation periods from 0.8 to 1.3 years.



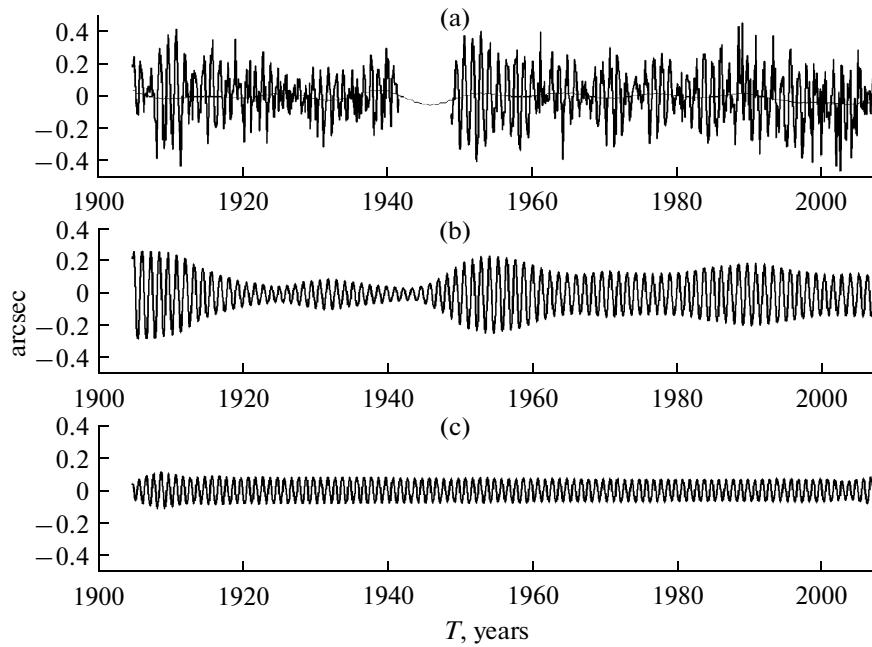
**Fig. 2.** (a) Time series of variations in latitude ( $\Delta\varphi_{C01}$ ), obtained from international data for 1904–2006, as well as its trend; and (b) Chandler and (c) annual wobbles.

#### COMPARISON OF TIME SERIES OF THE PULKOVY LATITUDE, INFERRRED FROM OBSERVATIONS AND INTERNATIONAL TIME SERIES

In this section, we compare the unique time series of ZTF-135 observations of latitude at the Pulkovo observatory, acquired during 102 years, with variations in the Pulkovo latitude, calculated according to formula (1) from IERS C01 time series of coordinates  $X_p$  and  $Y_p$  of the pole for 1904–2006. Work (Miller, Prud-

nikova, 2009) had analyzed this time series more judiciously.

Figure 1 presents the Fourier analysis of time series of the Pulkovo latitude, inferred from ZTF-135 observations ( $\Delta\varphi_{ZTF}$ ) (solid line), and variations in the Pulkovo latitude, calculated from coordinates of the pole according to formula (1) ( $\Delta\varphi_{C01}$ ) (dashed line) for the period of 0.95–1.3 years. The spectral analysis of both time series identifies two regions with maximum frequencies: two main peaks in the region of Chandler



**Fig. 3.** (a) Time series of variations in the Pulkovo latitude ( $\Delta\phi_{ZTF}$ ), inferred from ZTF-135 observations, as well as its trend; and (b) Chandler and (c) annual wobbles.

frequency and annual peak. The two time series show an identical period but differ in amplitude, with annual and Chandler wobble amplitudes (largest peak) both being 50 angular ms larger for  $\Delta\phi_{C01}$ .

Figures 2 and 3 present time series of variations in latitude in terms of  $\Delta\phi_{C01}$  and  $\Delta\phi_{ZTF}$ , respectively, with a 0.1-year step and their SSA decomposition into principal components, with length of the window ( $M = 515$ ) taken to be equal to half of the length of time series for 1904.8–2007.0.

The SSA method is based on transformation of the time series into matrix and its singular decomposition, leading to decomposition of the initial series into additive components. This method involves calculation of the sampling correlation function, whose eigenvalues ( $\lambda_i$ ) are sampling variances of the corresponding principal components. These components are determined such that the first component make the maximum possible contribution to the total dispersion. This transform does not change the sum of the dispersions and just redistributes it so that the first components have the largest dispersion. Thus, the components with small variances can be excluded from analysis. The percentage contribution is calculated from

$$V_i = \frac{\lambda_i}{M} \times 100\%, \quad (2)$$

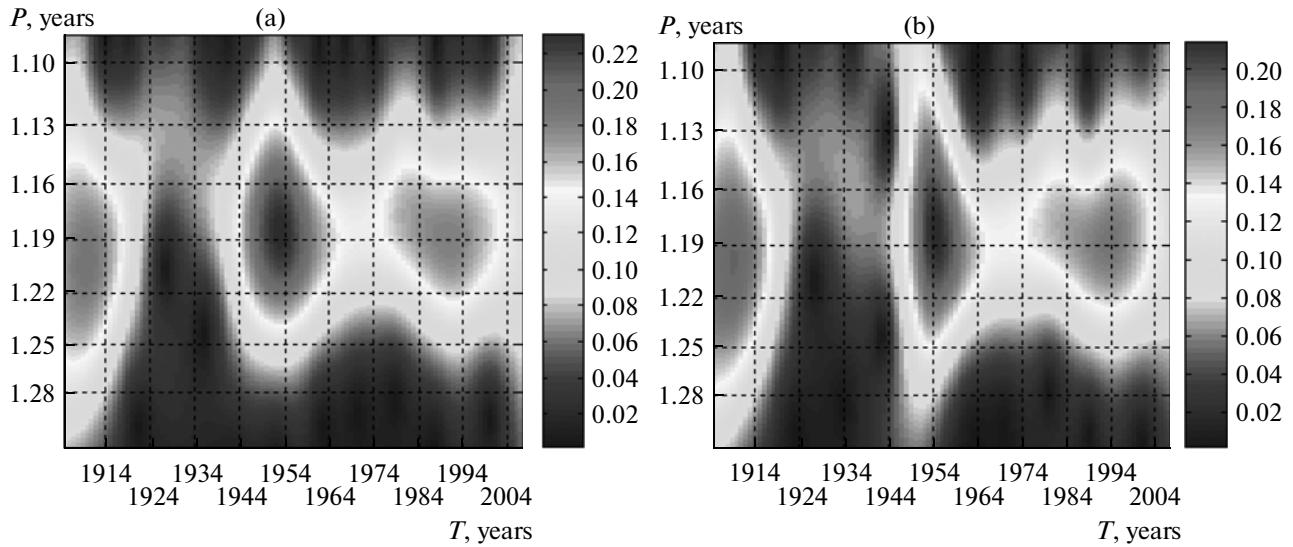
where  $M$  is the length of the window, and  $\lambda_i$  is the  $i$ th eigenvalue.

Using SSA, we obtained the following main components of the polar motion: trend component

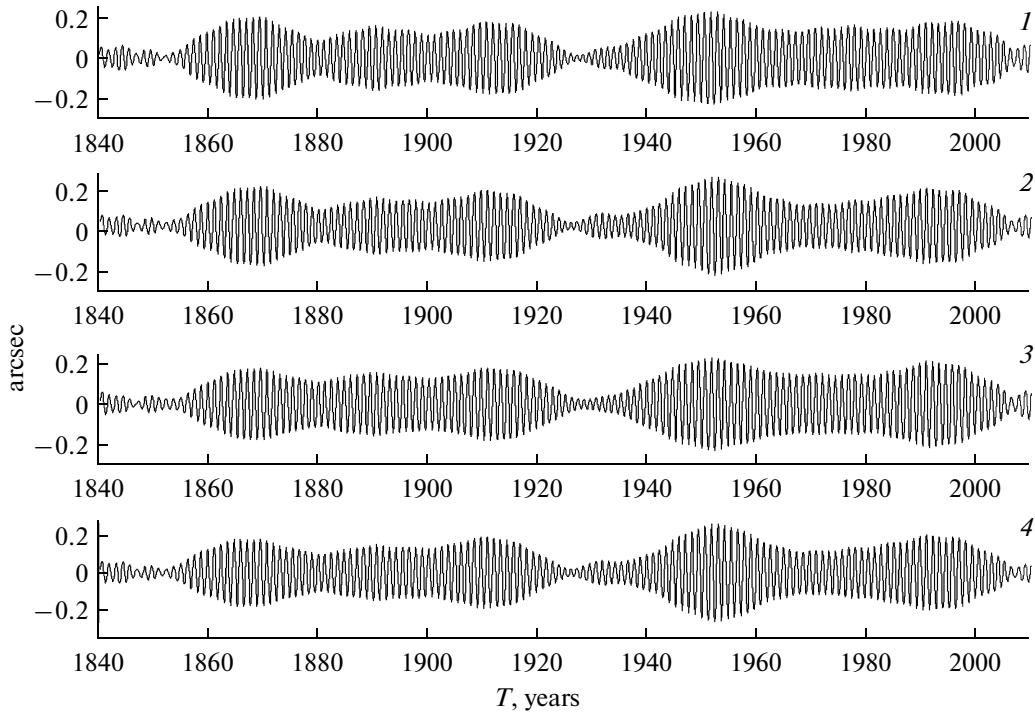
(Figs. 2a and 3a) and Chandler (Figs. 2b and 3b) and annual (Figs. 2c and 3c) wobbles. The sum of the main components contributes ~93% for  $\Delta\phi_{C01}$  time series and ~71% for  $\Delta\phi_{ZTF}$  time series to the initial process. This discrepancy may be, in particular, because ZTF-135 observations are noisier in the entire time interval. From all the figures it is seen that the behaviors of the CW amplitudes, inferred from ZTF-135 observations and calculated from international data, agree well. It should be noted that, using the SSA method, we identified the Chandler and annual wobbles with almost no distortion to the information, even despite an extended absence of observations between 1941.5 and 1948.2 (Fig. 3). When the wavelet analysis is used (Fig. 4), the behavior of the Chandler component is distinctly distorted on the interval of 1941–1948, which corresponds to the gap in the ZTF-135 observations. Thus, SSA can be used to study long-period regularities in the behavior of CW amplitude and phase on the basis of long observation records, even in the presence of extended gaps.

## RESULTS OF CW STUDY

In this work, the CW was studied on the basis of three analysis techniques: SSA, the method of band-pass filtering with the use of fifth-order Zolotarev–Cauer elliptic filter with a passband of  $1.19 \pm 0.8$  yr (Ellip, Matlab Signal Processing Toolbox), and wavelet analysis (Morle basis). For all four hybrid time series of variations in the Pulkovo latitude described in



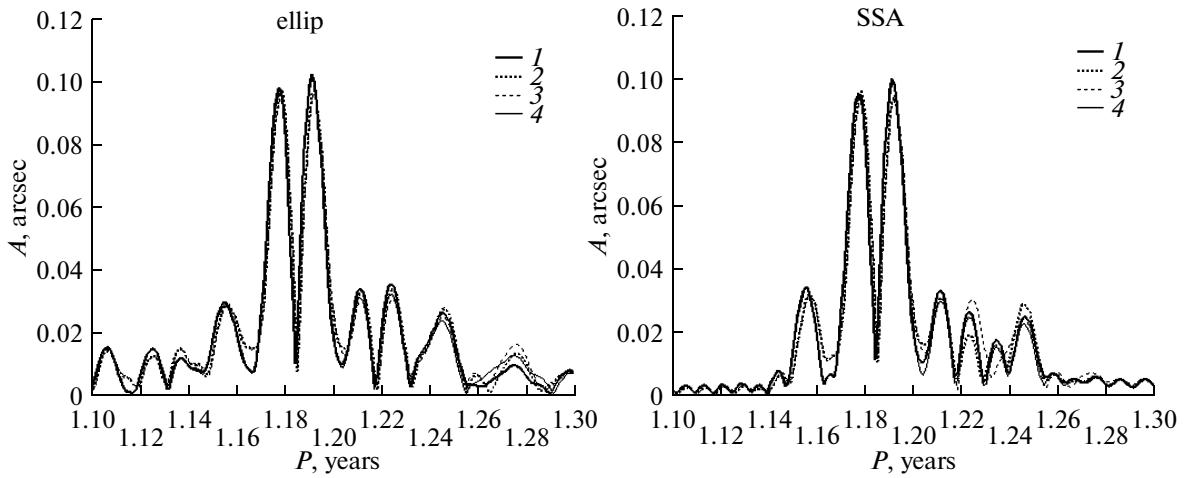
**Fig. 4.** Wavelet analysis of time series of variations in the Pulkovo latitude, inferred from international data ( $\Delta\phi_{C01}$ ) (left panel) and from ZTF-135 observations ( $\Delta\phi_{ZTF}$ ) (right panel).



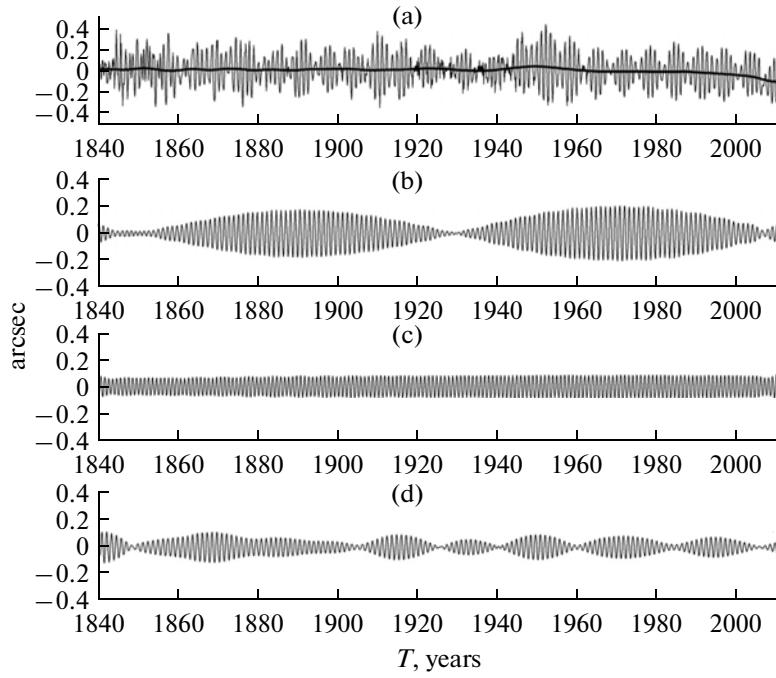
**Fig. 5.** Chandler components, inferred from united time series of variations in the Pulkovo latitude. Numerals on the right of the panels indicate the united time series numbers.

section 3, in SSA treatment the window was as long as half of the time series ( $M=848$ ). Formula (2) was used to estimate the percentage contribution of each constituent time series to the initial process. The sum of the main (Chandler, annual, and trend) components for the hybrid time series 1 and 3 of variations in the Pulkovo latitude was  $\sim 86\%$  ( $\sim 65$ ,  $\sim 19$ , and  $\sim 2\%$

respectively), and this contribution for the time series 2 and 4 was  $\sim 73\%$  ( $\sim 63$ ,  $\sim 9$ , and  $\sim 1\%$ ). Figure 5 presents the Chandler wobble for all four hybrid time series of Pulkovo (the numerals on the right of panels indicate the united time series numbers). The frequency peaks of the Fourier spectra of all these time series practically coincide, though there is some differ-



**Fig. 6.** Spectra of Chandler wobble, obtained from united time series of variations in the Pulkovo latitude with the help of two methods: bandpass filtering (left panel) and SSA (right panel). Numerals indicate time series numbers.

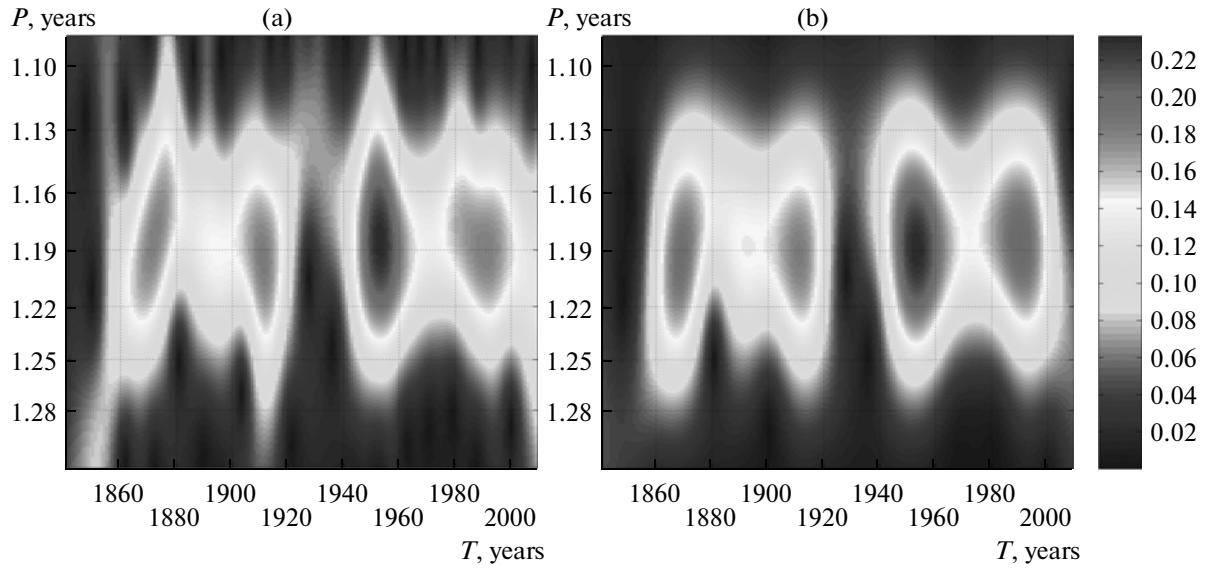


**Fig. 7.** (a) United time series of variations in the Pulkovo latitude (time series 1), including the nonlinear trend; and (b) first Chandler, (c) annual, and (d) second Chandler components, obtained with the use of the SSA method.

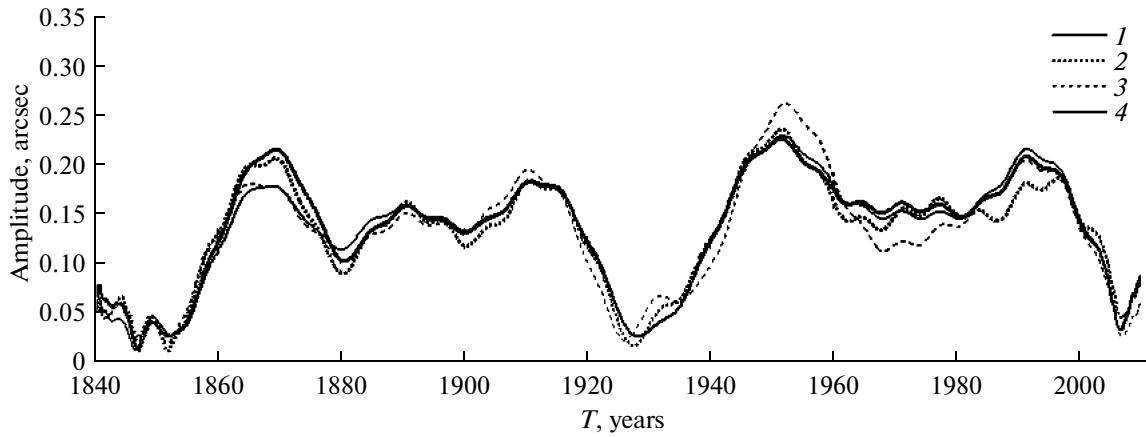
ence in amplitudes (Fig. 6). Two largest central peaks are identical for the two filtering methods used, and a certain difference exists only for low-amplitude ( $A < 40$  angular ms) frequencies.

Using SSA, we cannot only discriminate components, but also analyze the structure of the studied time series, especially in the availability of long observation record. In our case, we for the first time gained comprehensive insight into the structure of the studied time series. Figure 7 presents the hybrid time series 1 of variations in the Pulkov latitude and the structure of

the time series. Figure 7a presents the time series 1 and nonlinear SSA-determined trend (1–2% contribution). Maximum component (contributing ~55%) is the one (Fig. 7b) corresponding to two main peaks of the CW spectrum. This component behaves very similar in two time intervals, 1845–1925 and 1925–2005, with the periods in the first and second interval equal to at 1.183 years (432 days) and 1.185 years (433 days); the maximum amplitude is 184 angular ms in the first interval and 211 angular ms in the second interval. The Chandler component sharply decreases in amplitude



**Fig. 8.** Wavelet analysis of the united time series 1 of variations in the Pulkovo latitude (left panel) and Chandler component (right panel), obtained from this time series.

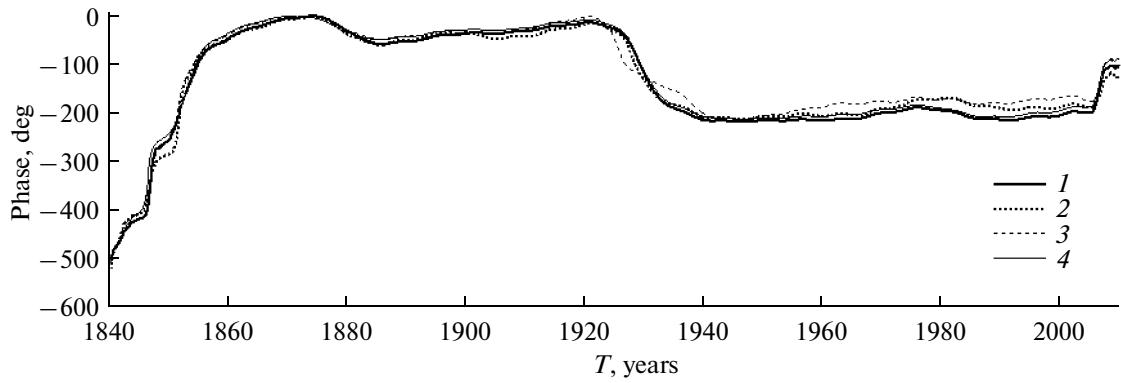


**Fig. 9.** Variations in the amplitude of the Chandler wobble, inferred from united time series of variations in the Pulkovo latitude. Numerals indicate the time series numbers.

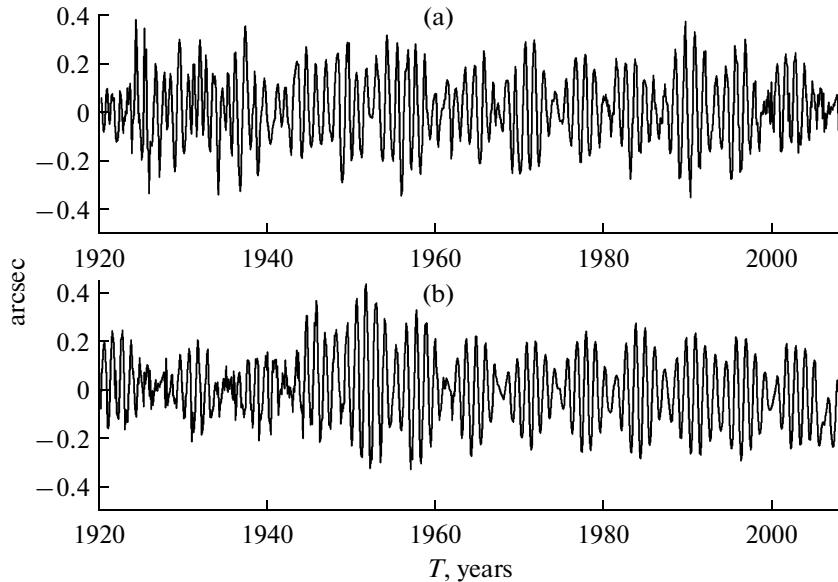
in three temporal regions around 1845, 1925, and 2005. The next component in the contribution ( $\sim 20\%$ ) is the annual wobble (Fig. 7c). The last component of those considered here (Fig. 7d), which corresponds to minor peaks in the CW spectrum, is irregular in character and accounts for  $\sim 10\%$  of the initial time series. Such a partition of the components may point to the presence of two structural components in CW, which may be useful for construction of the model of the polar motion. Work (Vorotkov et al., 2002) also showed that the Chandler component of the polar motion confidently splits into two components, with the averaged amplitude of the minor component being several times lower than that of the major com-

ponent. The minor component has a nonstationary period in the range of 1.1–1.3 years, whereas the period of the major component is stable. Orlov (1961) noted that the apparent variations in the Chandler period could be because the polar motion includes, besides annual and Chandler wobbles, some other neighbor-frequency oscillations whose amplitude should be very low.

Next, we will consider the Chandler oscillation (Fig. 5), representing the sum of the first largest (Fig. 7b) and second weak (Fig. 7d) CW components. We performed the wavelet analysis (using Morle basis) for both the initial time series (time series 1) of variations in the Pulkovo latitude (left panel in Fig. 8), and for its



**Fig. 10.** Variations in the phase of the Chandler wobble, inferred from united time series of variations in the Pulkovo latitude. Numerals indicate the time series numbers.



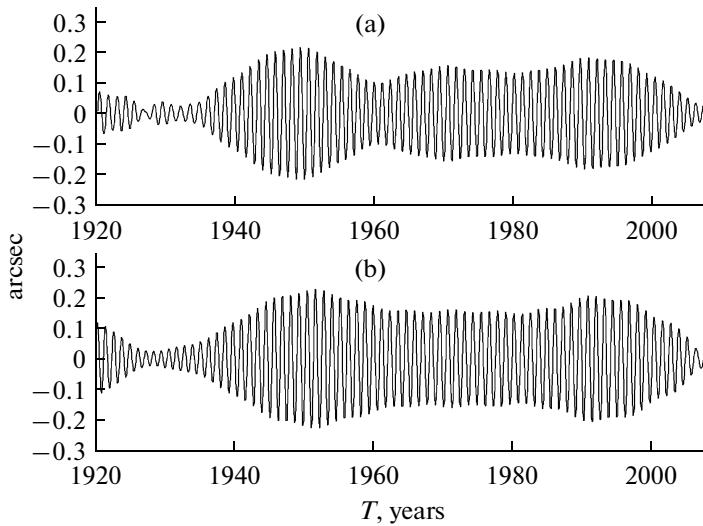
**Fig. 11.** Comparison of time intervals where the CW amplitude decreases at the ends and in the middle of the united time series of variations in the Pulkovo latitude (time series 1).

Chandler component (right panel in Fig. 8). In both cases, CWs behave similar on two time intervals as long as 80 years, with SSA-determined Chandler oscillation exhibiting these intervals more distinctly. Moreover, the start and end of the time intervals of the CW amplitude decrease are readily determined.

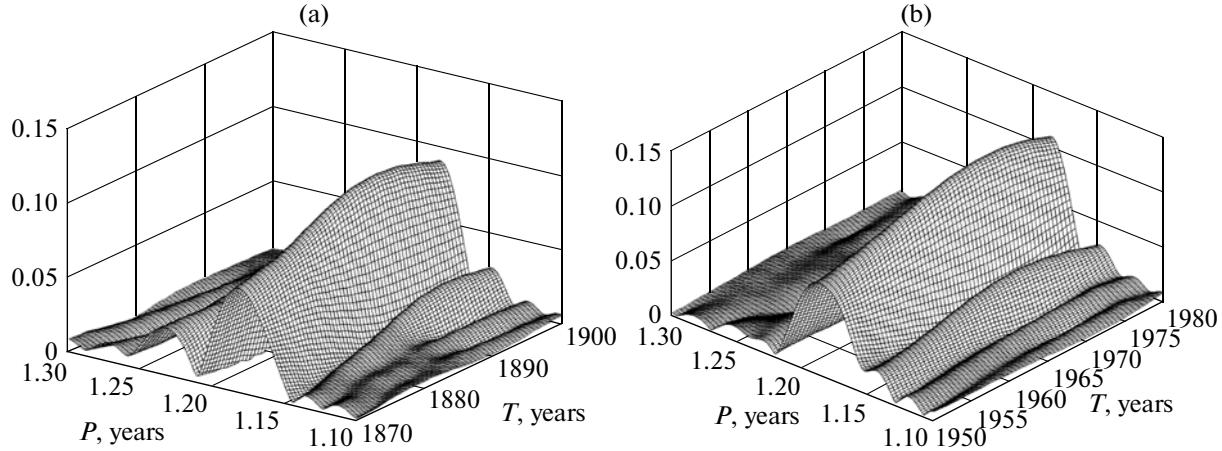
The time variations in the CWP amplitude (Fig. 9) and phase (Fig. 10) were identified in all four studied time series of Pulkovo latitude (Fig. 5) with the help of the Gilbert transform. In Figs. 5, 7b, 8–10, we distinctly see time intervals where CW behaves similarly, as well as three regions where CW strongly decreases in amplitude and simultaneously changes in phase. The timing of the first region at the very beginning of the

time series between 1846 and 1856 agrees well with the findings of (Miller, Prudnikova, 2010). The second, well-known minimum is dated at about 1929. At the very end of the time series, in approximately 2005–2006, the CW amplitude also sharply decreases, and the dependence of the phase may indicate the beginning of next time interval of phase change. The specific features of the behavior of the pole around 2005 were noted in (Lambert, 2006).

Based on the superimposed epoch method, the initial time series, Chandler component (SSA), and changes in the spectrum were used to estimate the correlation  $r$ . The maximum  $r$  value (0.56 for the initial time series, 0.86 for CW, and 0.89 for the spectrum)



**Fig. 12.** Comparison of time intervals where the CW amplitude decreases at the ends and in the middle of the hybrid time series of variations in the Pulkovo latitude (time series 1).



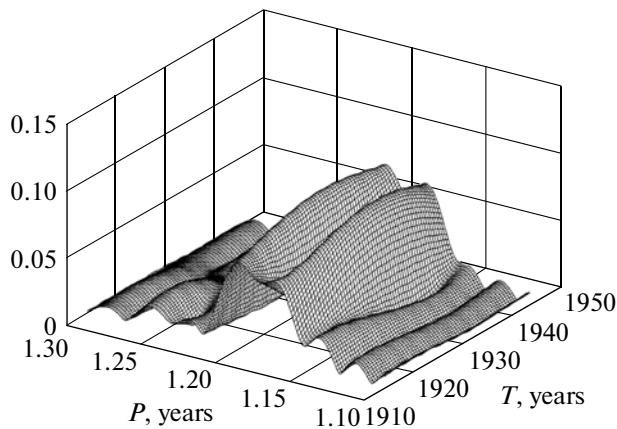
**Fig. 13.** Dynamical Fourier spectrum of the Chandler component of the hybrid time series (time series 1) of variations in the Pulkovo latitude for the first and second 80-year intervals.

corresponds to an 80-year period. These cited correlations are average for all four time series considered here.

Based on the determined 80-year period, we divide the initial time series (time series 1) (Fig. 11) and Chandler component (Fig. 12) obtained from this time series with the help of the SSA, such that the beginning and end of the time series can be compared with analogous intervals in the middle of the time series. From the figures it is seen that Chandler component behave similarly at the beginning and end of the chosen time intervals of 1840–1928 and 1920–2008. Work (Fedorov, Yatskiv, 1964) showed that doubling of the CW peak is associated with change in the

phase in the time interval of 1924–1930. Interestingly, studying the time series of variations in the latitude at different observatories worldwide in the time interval of 1820–1898, Chandler (1901a; 1901b) also found a frequency split in the first time interval due to a change in the phase of this oscillation.

We constructed the dynamic spectrum to study the dynamics of variations in the spectral lines for different chosen time intervals (Figs. 13 and 14). The spectrum was calculated for a 60-year sample with 5-year time lag. From Fig. 13 we see that the spectral peaks in the first and second 80-year intervals behave synchronously, and from Fig. 14 it is clearly seen that the Chandler component doubles when the time interval



**Fig. 14.** Dynamical Fourier spectrum of the Chandler component of the hybrid time series (time series 1) of variations in the Pulkovo latitude for the time interval encompassing the region of phase change.

chosen for analysis encompasses the region of phase change.

## CONCLUSIONS

This work studied the Chandler component of the polar motion according to data of variations in the Pulkovo latitude for 170 years; this record included observations made with the help of transit instrument in the prime vertical, observations with the help of a vertical Ertel circle, and ZTF-135 observations. Using Fourier analysis, wavelet analysis, and singular spectral analysis, we compared variations in the Pulkovo latitude, calculated from ZTF-135 observations and from international data. It was shown that the low-frequency CW variations can be studied using long-term observation series at a single observatory. The SSA method segregates the main components of the polar motion and analyzes the structure of the time series, even if this last one has long-term gaps.

After processing all the data, we obtained united time series of variations in the Pulkovo latitude for 1840–2009, which was found to contain a nonlinear trend and annual and Chandler components. The Chandler component splits into two components, the first of which corresponds to two largest peaks of the Fourier spectrum, and the second corresponding to minor spectral peaks. The first component has a recurrent structure (Fig. 7). This structure was identified thanks to the use of the SSA method and analysis of the unique longest observation record of variations in the Pulkovo latitude for 170 years. The CW period and amplitude were estimated by applying the Fourier analysis to each of these two structural parts on the

intervals of 1845–1925 and 1925–2005. This period turned out to be almost unchanged and to be equal, respectively, to 1.183 years (432 days) and 1.185 years (433 days), and the maximum amplitude was found to be 184 angular ms in the first interval and 211 angular ms in the second interval. In addition, time changes in the CW amplitude and phase were calculated using a Hilbert transform. These calculations showed that sharp change in the CW phase in the middle of the interval is not unique, but rather periodic in character. Despite the fact that they were hypothesized in different papers, the time variations in the phase and amplitude on a 170-year observation record are estimated here for the first time.

Analysis of this structure and its comparison with different geophysical processes may provide new estimates of CW-exciting factors. In view of the very wide range of possible geophysical impacts (Lambeck, 1980), the question of whether or not there is a physical interpretation for the structural features of CW should be the subject of a separate paper.

Thus, this work identified for the first time the recurrent CW structure, which suggests that CW amplitude decreases with a simultaneous change in the CW phase during three periods (in approximately 1846, 1925, and 2005). Based on the superimposed epoch method and dynamical comparison of spectra, the period of variations in CW amplitude with simultaneous change in CW phase was estimated; as a result, these variations were found to have a period of 80 years. For this period, we obtained best correlations ( $\sim 0.9$ ), both for spectra, and for the components themselves. In addition, the advantages of the singular spectral analysis for studying the long-period time series with complex structure are demonstrated.

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