Singular Spectrum Analysis in Astrometry and Geodynamics

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Abstract. The paper presents the possibilities of the Singular Spectrum Analyses on the examples of its application to several astrometric and geodynamic time series. The comparisons of results obtained by other often used methods (Fourier transform, Wavelet Transform, different £lter methods) are given. The Singular Spectrum Analyses method was used for the investigation of the Chandler wobble (CW), which was extracted from the IERS Pole coordinates and latitude variations at Pulkovo. The CW amplitude and phase variations were examined by means of the Hilbert transform. The main conclusion which can be made from this study is: we have found two epochs of deep CW amplitude decreases near 1850 and 2005, which are also accompanied by a large phase jump, similar to well known event in 1920s. The investigation of £rst latitude observations at Pulkovo (1840-1855) was executed with help to gain and analyse the sum of Chandler and annual components from very small quantity of very noisy observations. The SSA is applied for investigation of the zenith troposphere delay time-series derived from observations of several VLBI stations. Combined IVS time-series of the zenith wet and total troposphere delays obtained in IGG were used for analysis. For all stations under consideration the non-linear trends and the seasonal components with annual and semiannual periods were found. Some interesting peculiarities were found to be individual for every stations. Comparison of the trends with meteorological parameters also is presented to show possible similarities.

Keywords: Singular Spectrum Analyses, Chandler Wobble, Polar motion, troposphere zenith delays
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INTRODUCTION

The Singular Spectrum Analyses (SSA) [1] belongs to the class of orthogonal functions methods for which basis functions are calculated from the data themselves. In Fourier and Wavelet Transforms, the most popular tools to study the time series, the decomposition is done on a £xed chosen basis. The SSA method has a good resolution both in time and frequency and permits to study the time series of complex stricture aiming at getting the time-dependent amplitudes, phases and frequencies. Moreover, it permits to separate signal from noise even if level of noise is varied in different part of data, which is typical for astrometric time series. The multidimensional SSA treats the data totality as a whole, yielding general regularities and peculiarities.

With the help of this method we can:

- Recognize certain components in the equally spaced time series. The result of such procedure is a decomposition of the time series into components that usually can be identi£ed as trends, periodical or oscillatory and noise components;
- Analyze structure of the time-series and separate the harmonic and irregular (trend) components;
- Extract components with known period and estimate the value of phase shift and variation of amplitude of pseudo-harmonic signals;
- Find periodicities that are not known in advance;
- The decomposition of the time series is constructed on the base of the unique parameter (the window length).
- To perform the smoothing of the initial data.

Additional information on the SSA method, its abilities and the corresponding software can be found on the site `http://www.gistatgroup.com/cat`. The comparisons of results obtained by other often used methods (Fourier transform, Wavelet Transform, different £lter methods) are given [2], [3].

In the following sections the application of this method to astrometric and geophysical time series is done with emphasis on unique abilities of the SSA as compared to commonly used techniques.

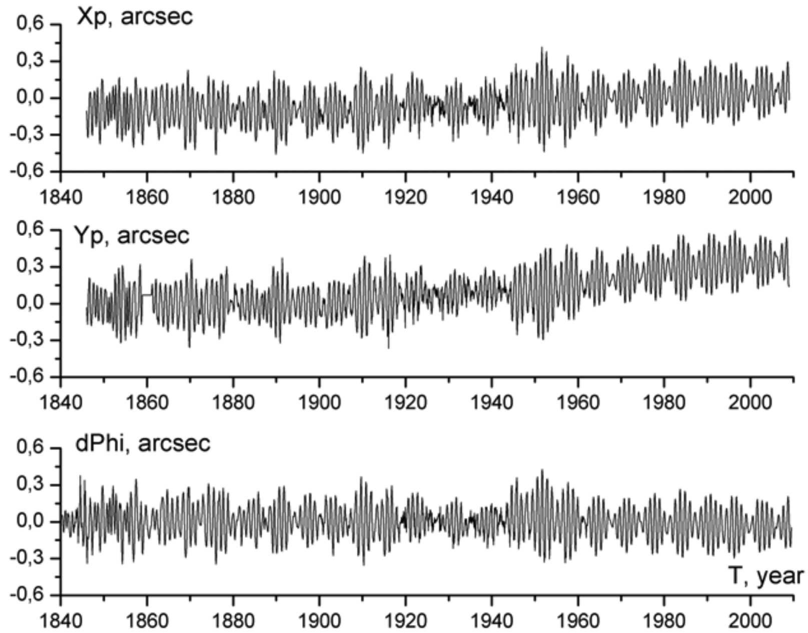

FIGURE 1. Xp, Yp - the EOP (IERS) C01 polar motion series (1846.0-2008.5) and the EOP (IERS) C04 series up to 2009.4; dPhi - the combined series of Pulkovo latitude variations (1840.4-2009.5).

INVESTIGATIONS OF THE CHANDLER WOBBLE

Investigations of the anomalies in the Earth rotation, in particular, the polar motion components, play an important role in our understanding of the processes that drive changes in the Earth's surface, interiors, atmosphere and ocean. This example is primarily aimed at investigation of the major Chandler wobble (CW) amplitude and phase variations [3], [4] from the longest interval of observations. In our analysis, we investigated the whole 167-year (International Earth Rotation and Reference Systems Service) Polar coordinates time series and the 170-year latitude variations at Pulkovo (£g. 1). The rate of sampling is 0.1 yr.

The CW signal was extracted from these time series using two digital £lters: singular spectrum analysis (SSA) and Design elliptical or Cauer digital £lters `ellip` from the MATLAB Signal Processing Toolbox (£g. 2). The £g. 3 presents the power spectra of the time series exposed in £g. 2. The CW signal looks similar in all £ltered series. However, some differences can be seen near the ends of the interval. The wavelet transform (the Morlet basis) was used for analysis of the initial series of the Pulkovo latitude variations (left, £g. 4) and its Chandler Wobble (CW) component derived with the help of SSA (right, £g. 4). It is seen that the CW component extracted from the initial time series have the structure more pronounced than the CW in the initial series. Thus obtained CW series were used to study the CW amplitude and phase variations (£g. 5).

Let us consider a general CW model with the time-dependent amplitude and phase

$$\begin{aligned} X_p(t) &= A(t)\cos\Phi(t)\,, \\ Y_p(t) &= A(t)\sin\Phi(t)\,, \end{aligned} \tag{1}$$

where X_p and Y_p are the Pole coordinates.

In this case, the amplitude may be written as

$$A(t) = \sqrt{X_p(t)^2 + Y_p(t)^2}\,, \tag{2}$$

In contrast to the computation of the CW amplitude, the calculation of the CW phase is not such an unambiguous procedure. Hilbert transform methods were applied to evaluate the CW phase variations. For this work we used the function `hilbert` from the MATLAB Signal Processing Toolbox. All the methods used gave similar results, with some differences at the ends of the interval. These discrepancies can be explained by different edge effects of the methods used.

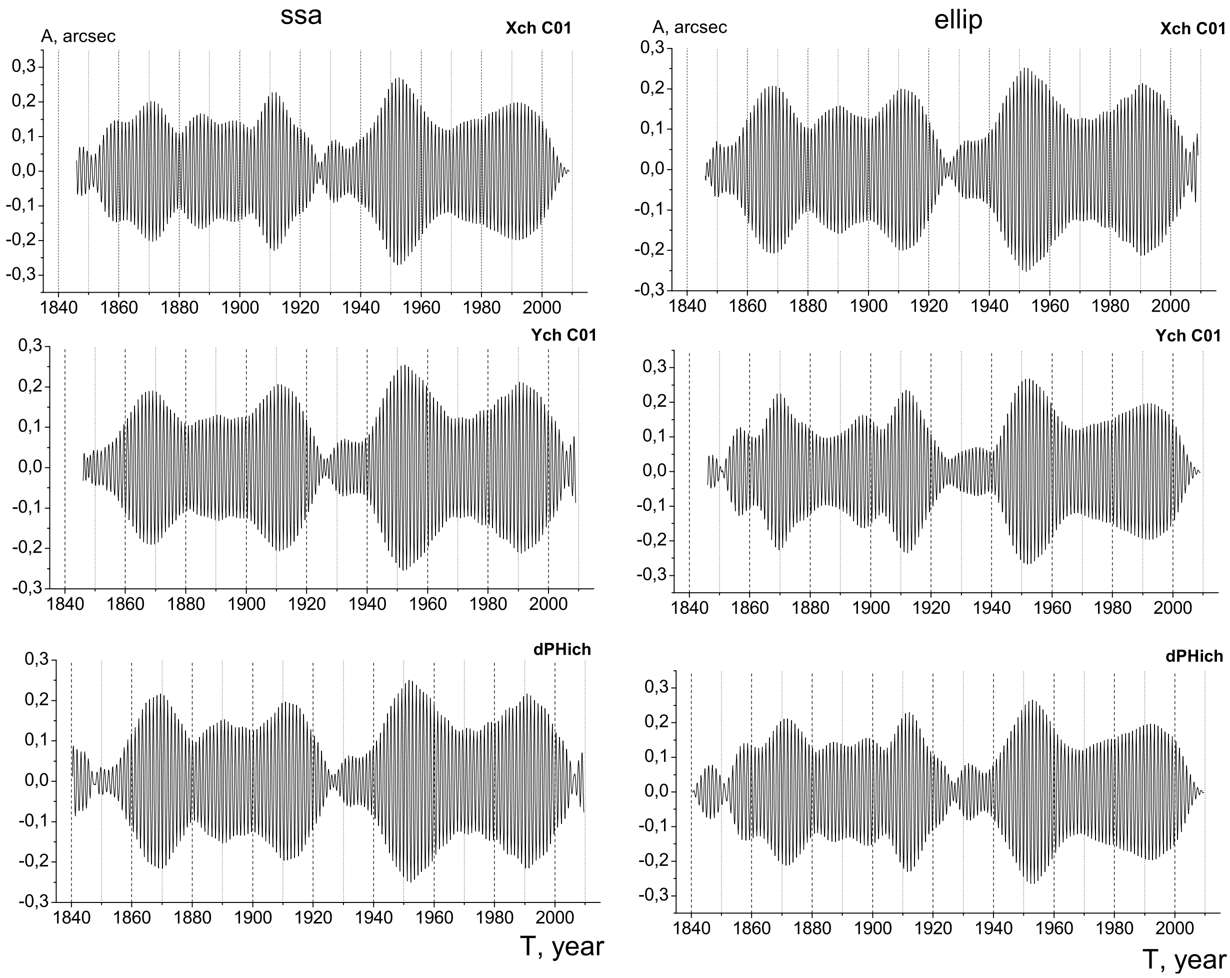

FIGURE 2. The Chandlerian components in the latitude variations and the coordinates of the pole Xp, Yp obtained (left): by the SSA method; (right): by elliptical digital £lter Ellip.

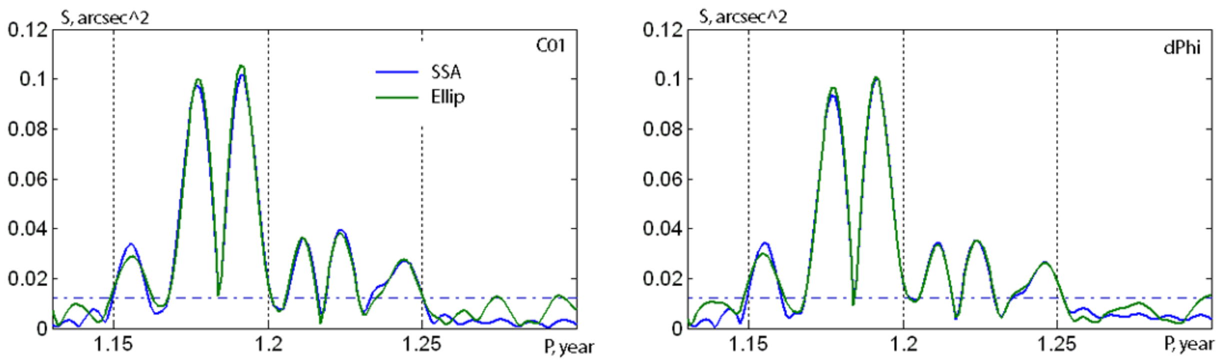

FIGURE 3. The power spectra of the time series from £g. 2. The CW signal looks similar in all £ltered series. However, some differences can be seen near the ends of the interval.

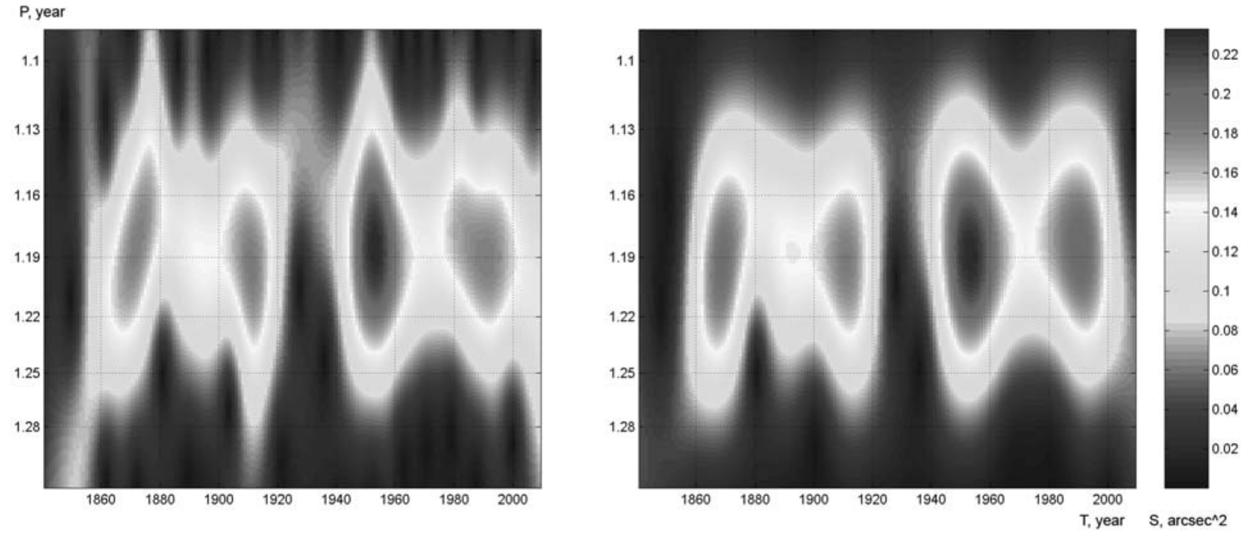

FIGURE 4. The wavelet transforms (the Morlet basis) of the initial series of the Pulkovo latitude variations (left) and its Chandler Wobble (CW) component derived with the help of SSA (right). It is seen that the SSA extracted CW component has the structure more pronounced than the CW in the initial series.

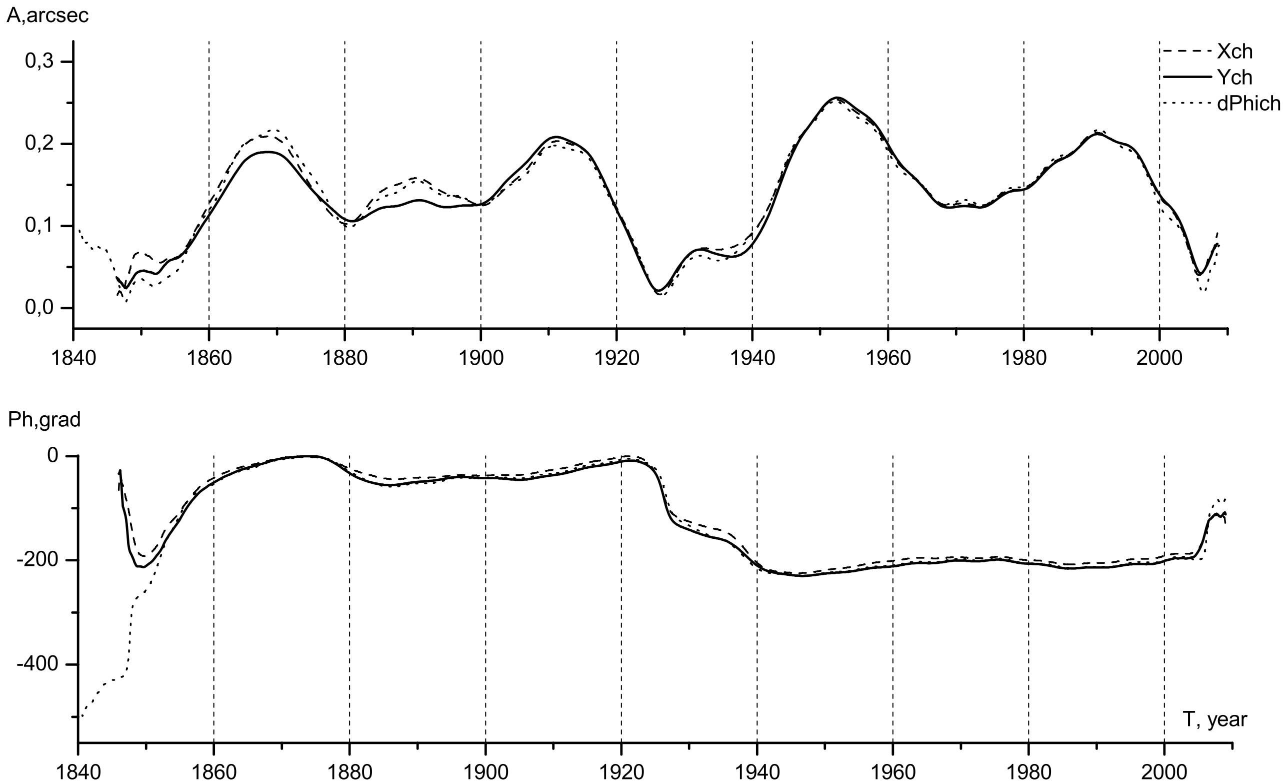

FIGURE 5. The CW amplitude and phase variations computed with the SSA.

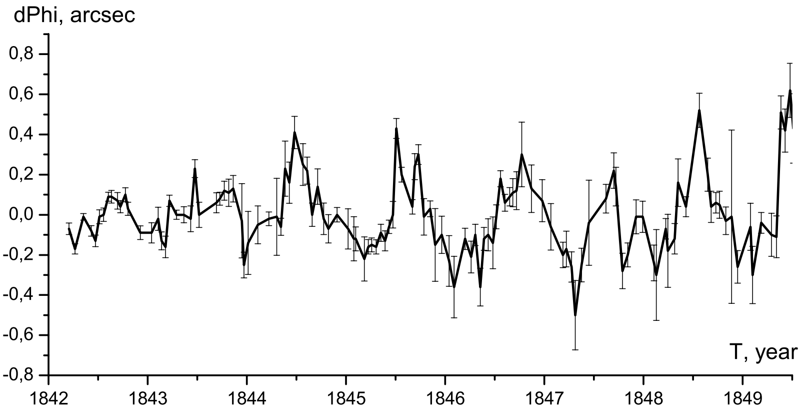

FIGURE 6. The Pulkovo latitude variations obtained from X. I. Peters's observations with Ertel vertical circle by A. A. Ivanov.

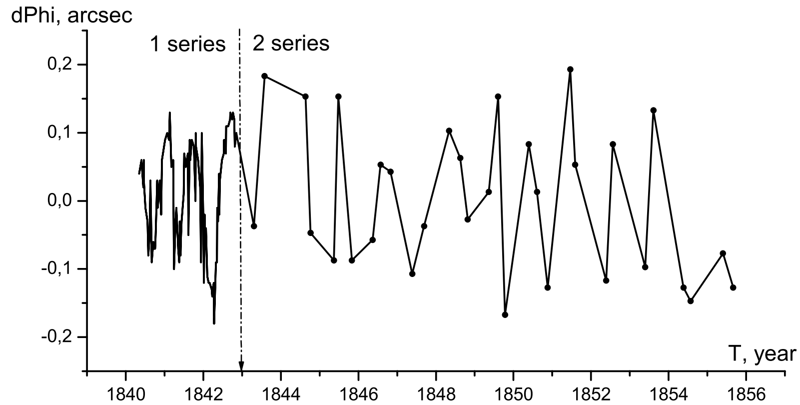

FIGURE 7. The Pulkovo latitude variations obtained from V. J. Struve observations with Repsold transit instrument in the £rst vertical by B. Wanah.

The main conclusion which can be made from this study is: we have found two epochs of deep CW amplitude decreases near 1850 and 2005, which are also accompanied by a large phase jump, similar to well known event in 1920s. Thus, the latter no more can be regarded as an unique event. Unfortunately, both periods of the phase disturbances described in this example are located at the edges of the interval covered by the IERS EOP series. As for the end of the interval, the next decade will allow us to quantify the phase jump in the beginning of the 21th century.

On the other hand, an additional study including an extension of the polar motion series into the past, seems to be extremely important to improve our knowledge about polar motion in the 19th century. From Sekiguchi [8], Orlov [9], Kimura [10] and other authors it follows that there exist several latitude series obtained in the £rst half of the 19th century, which can be used to extend the IERS C01 polar motion series back to 1830s.

THE ANALYZE OF THE EARLY LATITUDE OBSERVATIONS AT PULKOVO

The latitude observations at Pulkovo began in 1840. The Ertel vertical circle and the Repsold transit instrument in the £rst vertical were used for the purpose. W. Struve in 1840-1856 and X. I. Peters in 1842-1849 were the £rst observers on these instruments. The latitude variations obtained from X. I. Peters's observations with Ertel vertical circle by A. A. Ivanov (£g. 6) and from V. J. Struve's observations with Repsold transit instrument in the £rst vertical by B. Wanah (£g. 7) were analysed to appreciate the possibility of their using for investigating the polar motion during maximal accessible length of realization.

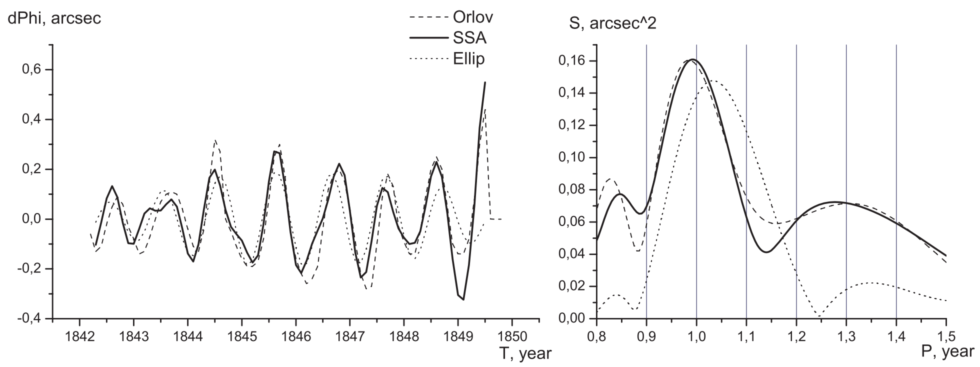

FIGURE 8. Filtered latitude variations of Pulkovo (left) and their spectra (right) obtained from observations with Ertel vertical circle.

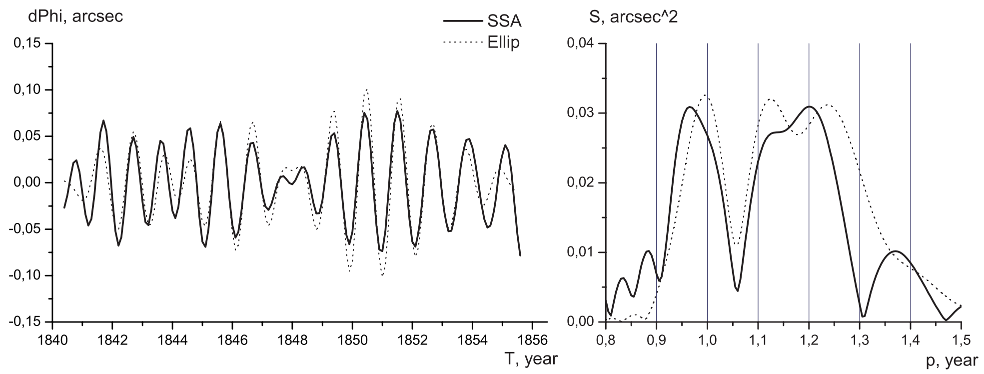

FIGURE 9. Filtered latitude variations of Pulkovo (left) and their spectra (right) obtained from observations with Ertel vertical circle.

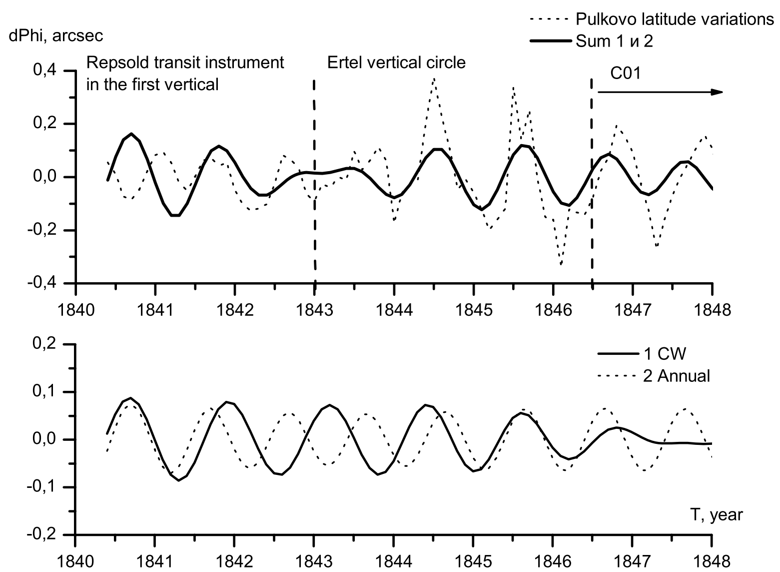

FIGURE 10. The set of the Pulkovo latitude variations for 1840-1848 which can be used to prolong the latitude variations back to 1840.

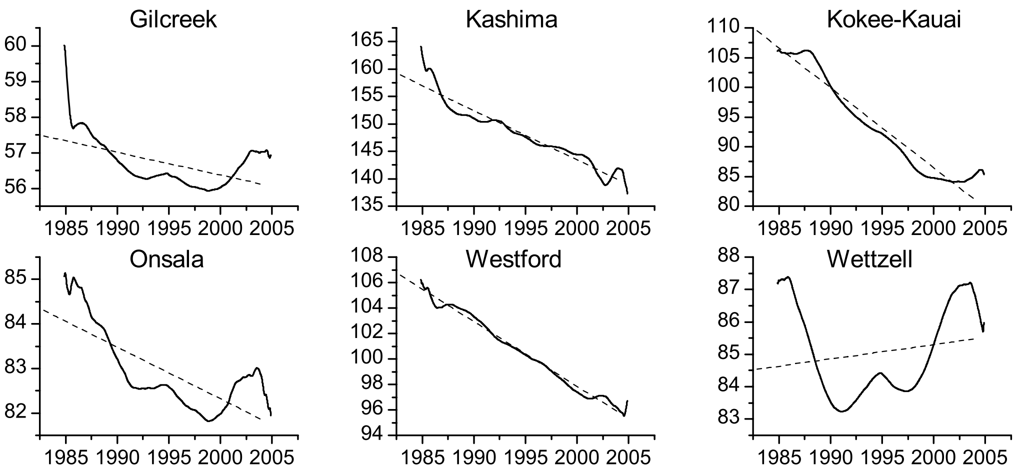

FIGURE 11. SSA trends of ZWD (solid line) and their linear approximation (dashed line). Unit: mm.

The SSA and ELLIP methods were also used to extract and analyze the sum of Chandler and annual latitude variations. The results of the £ltering latitude variations of Pulkovo (left) and their spectra (right) obtained from observations with Ertel vertical circle and with Repsold transit instrument in the £rst vertical are given on £g. 8, £g. 9.

In spite of small number of observations we succeeded to separate the signals. The results obtained with the SSA £lter turned out to be similar to those calculated with the Orlov's £lter. The Orlov's £lter is used for forming the series IERS C01. In our opinion, the SSA £lter is more preferable. The set 1840-1848 was used to form time series of Pulkovo latitude variations for 1840-2009 (£g. 10) which was described in [5].

USING THE SSA FOR INVESTIGATION OF THE TROPOSPHERE PARAMETERS

The VLBI data provides possibility to estimate the troposphere zenith delays (ZD) with the millimeter accuracy. The troposphere delay is one of the most troublesome errors in the analysis of the space-geodetic technique data measured at radio wavelengths. Therefore the application of new mathematical methods for treating the troposphere delay and other meteorological parameters are highly desirable.

One of the main features of the SSA method is the possibility of detecting even weak, but signi£cant trends. We applied the SSA method for investigation of the zenith wet troposphere delay time series. The combined IVS time-series of the zenith wet (ZWD) and total troposphere delays (ZTD), obtained at IGG, were used for analysis[7].

Six VLBI stations (Gilcreek, Kashima, Kokee-Kauai, Onsala, Westford, Wettzell) with the longest time series of troposphere zenith delays were selected for study. The geographic location of the stations was taken into consideration. For all stations we used the same time interval 1984.88 - 2004.87 (the total span is 20 years with N=2000 points), and the maximum window length M=1000. With the help of the SSA method the non-linear trends (£g. 11) and the seasonal components with annual (£g. 14) and semiannual (£g. 15) periods were found in all series.

The fact that the stations located in the same geographic region (Onsala and Wettzell) reveal similar trend features is of a special interest. Moreover, all trends have the same small curving about year 1995. Original series obtained by analysis centers BKG, GSFC, IAA, MAO show the same properties [6].

The SSA decomposition affords interesting comparison of non-linear trends in the ZD with the trends in the meteorological parameters taken from the VMF1 £les provided by the IGG . For example, the SSA trends of meteorological parameters for Wettzell and Gilcreek stations were obtained for this purpose.

Figure 12 shows the trends for the hydrostatic zenith delay, pressure at the site from the VMF1 and ZTD-ZWD computed from the IVS combined series. The coincidence of the curves is obvious for Wettzell. For Gilcreek, one can see the difference in shape of the curves. Figure 13 shows the trends for wet zenith delay, temperature at the site, water vapor pressure at the site from the VMF1 and ZWD. The ZWD trend is very close to trend of water vapor pressure for the Wettzell. But for the Gilcreek the similar curves do not show such an agreement. The Gilcreek is the only station where we failed to extract the trend from the site temperature series.

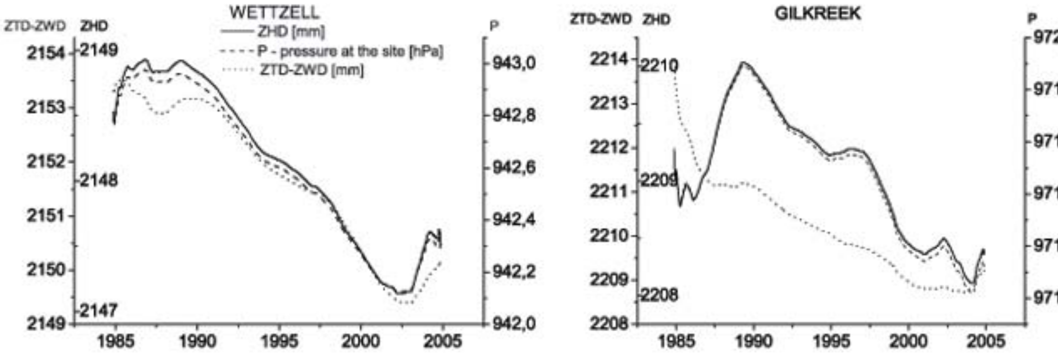

FIGURE 12. SSA trends of hydrostatic zenith delay, pressure at the site and ZTD-ZWD.

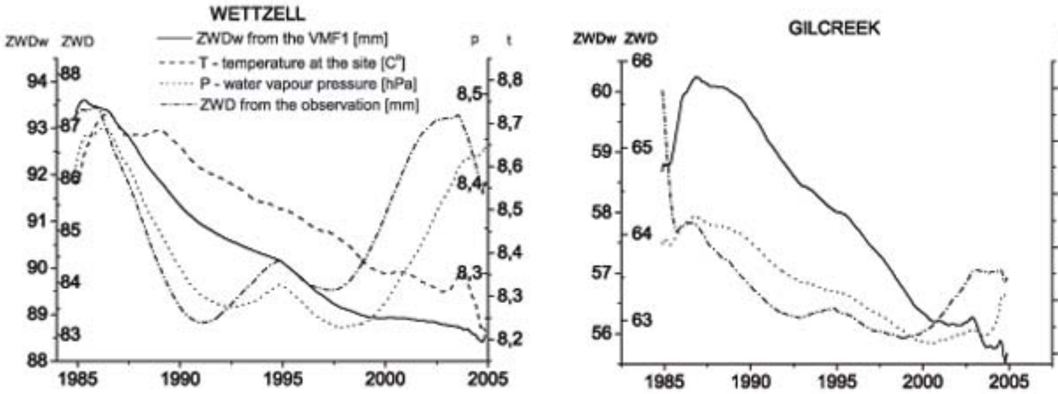

FIGURE 13. SSA trends of wet zenith delay, temperature at the site, water vapor pressure and ZWD.

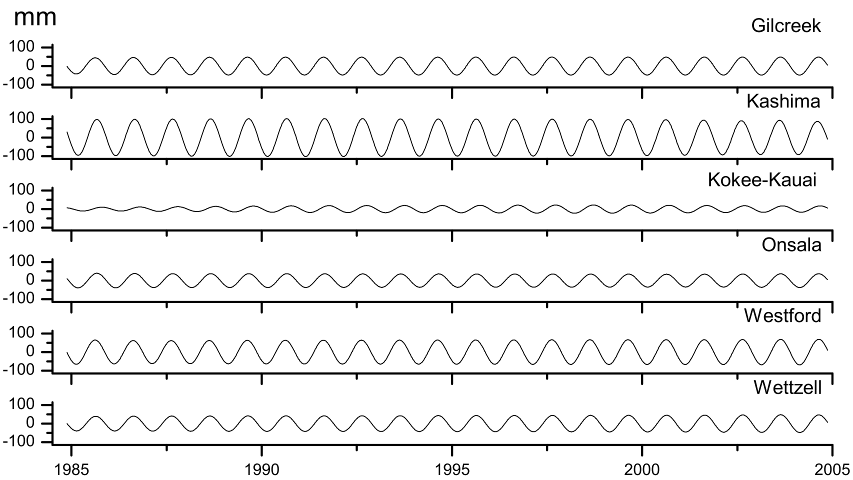

FIGURE 14. The annual components in reconstructed ZWD series.

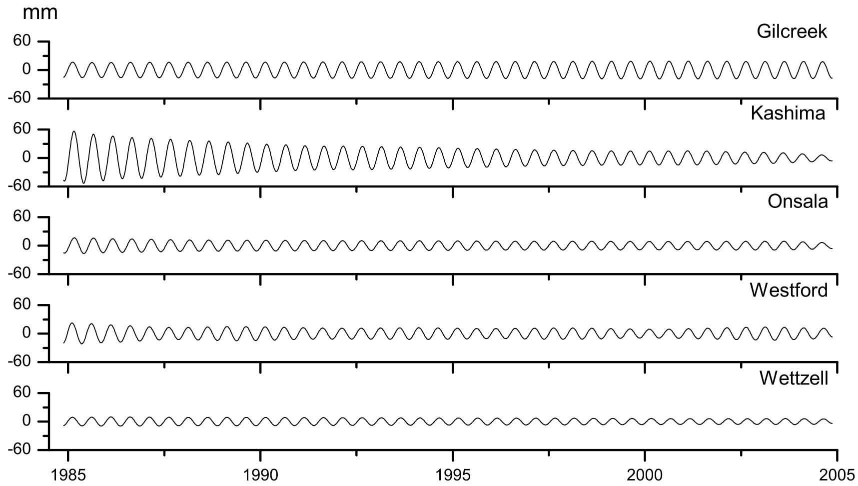

FIGURE 15. The semiannual components in reconstructed ZWD series.

The annual and semi-annual components ZWD have a steady phase but variations of amplitude. It should be mentioned that the contribution of the semiannual component is comparatively small, near noise level, and therefore this component needs more careful study. For Kokee-Kauai semiannual component was not found, and so for ZTD at Onsala. For Kashima, the amplitude of the semiannual component is larger in the beginning of the time interval than at the rest of interval.

CONCLUSIONS

In this paper, we have examined an ability of the SSA method for three examples. Non-linear trends and variations of the amplitude of well know components have been detected. Some interesting peculiarities in their behavior have individual character for every examples of site.

The believe that the advantages of the SSA over the commonly used techniques are:

- the base orthogonal functions of the SSA are generated by initial series oneself,
- it allows us to investigate the time series structure in more detail than other digital £lters;
- it is possible to evaluate a phase shift and variation of amplitude of pseudo-harmonic signals;
- it permits to separate signal from noise even if level of noise is high, in particular, it allows to use very old observations for modern investigation;
- it allows to £nd the peculiarities in behavior of non-linear trends.

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