# Dynamics of the electrons responsible for the microwave radiation

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Two ensembles of fast electrons can be formed in a magnetic closed loop during a flare. The part of electrons falls in a loss cone and precipitate directly to the lower dense layers of solar atmosphere and generate hard X-ray emission. These electrons do not give the considerable contribution to microwave radiation. Other part of electrons are caught in a magnetic trap. Because of these electrons can trap in a loop for long time their density is much higher than density of electrons in a loss cone. Therefore to study of solar nonthermal microwave radiation we can analyze trapping electrons only.

Temporal dynamics of these electrons will be further considered. Many papers are devoted to the problem of dynamics of fast electrons in closed flare loops. In the majority of them the appropriate kinetic equation is solved numerically (e.g. Zharkov et al, 1995), that does not give a possibility to use the obtained results in the analysis of radio spectra of flares. The analytical solutions were represented by Kennel and Petschek (1966), Lu and Petrosian (1998), Melrose and Brown (1976), MacKinnon (1988, 1991). But in one case for the solution of the kinetic equation the source function describing injection of electrons is used with a delta-function in time (Lu and Petrosian, 1998), in others with a delta-function in a pitch-angle (Kennel and Petschek, 1966; Melrose and Brown, 1976; MacKinnon, 1988). In the last cases it was supposed that the electrons are generated strictly perpendicularly to direction of a magnetic field. It is clear however, that such approach do not reflect all real situations. MacKinnon (1991) took this point into account, but he studied only the electrons in a loss cone, which are responsible for the main part of hard X-ray emission. In present contribution we shall put down a kinetic equation as MacKinnon (1988) did, but we shall solve this equation for an arbitrary source function $S(E,\sigma,t)$, describing injection of electrons:

$$\frac{\partial N}{\partial t}+\frac{\partial}{\partial E}\left(N\alpha\right)+\frac{\partial}{\partial\sigma}\left(N\beta\right)+\frac{N}{t_e}H\left(\sigma_o-\sigma\right)=S\left(E,\sigma,t\right), \qquad (1)$$

where $N$ is the distribution function of the fast electrons, $\alpha = knE^{-1/2}$ is the factor which is taking Coulomb collisions of mildly relativistic electrons into account, $k = 4.9\times10^{-9}$, $n$ is the ambient plasma density, $E$ is electron energy , $s = sin\theta$, where $\theta$ is the pitch-angle of the electron, $\beta = \sigma\alpha/E$, $H$ is the Heaviside step function, $\sigma_o = sin\theta_o$ determines the loss-cone, $t_e = L(me/2E)^{1/2}$ is escaping time for an electron in the loss-cone, $L$ is a loop length. More detailed definition of the model of a magnetic closed loop and assumptions, at which the Equation (1) is true, was indicated by MacKinnon (1988). The most essential assumption is that the electrons should have quite large the pitch-angles. For our consideration it is true, as we analyze radio emission only (we take trapped electrons only with $\theta > \theta_o$ into account) and value $\theta_o$ is large enough ($\approx 50^\circ$) as it follows from Aschwanden et al. (1998).

Let's solve the Equation (1) by the method of characteristics. As the initial condition we assume $S(E,\sigma,0)=0$. For the sake of convenience we denote $2E^{3/2}+3kn(t-s)\equiv E_s$ . Then we have

$$N\left(E,\sigma,t\right)=\frac{1}{kn\sqrt{E}}\int\limits_{E}^{(E_o/2)^{2/3}} e^{-\frac{p^2-E^2}{\sqrt{2m_e}knL}H(\sigma_o-\sigma)}pS\left(p,\frac{\sigma}{E}p,\frac{E_o-2p^{3/2}}{3kn}\right)dp$$

This expression allows to calculate evolution of the fast electrons at an arbitrary source function $S$ rather simply. However, keeping in mind that these results will use for the interpretation of the observation data, we shall carry out further simplification.

As for the source function of fast electrons responsible for flare radio emission, the isotropy and power-law energy dependence of seems natural. We suppose therefore, that

$$S\left(E,\sigma,t\right)=Ch\left(t\right)E^{-\delta}H\left(E-E_{cr}\right)$$

where $C$ is the normalization factor, $E_{cr}$ is the minimum energy of injected electrons. Then

$$N\left(E,\sigma,t\right)=\frac{C}{kn\sqrt{E}}\int\limits_{E}^{(E_o/2)^{2/3}} p^{1-\delta}h\left(\frac{E_o-2p^{3/2}}{3kn}\right)H\left(p-E_{cr}\right)dp$$

In new designation $s=\frac{E_o-2p^{3/2}}{3kn}$, we obtain

$$N\left(E,\sigma,t\right)=\frac{C}{\sqrt{E}}\int\limits_{0}^{t} E_s^{\frac{1-2\sigma}{3}}h\left(s\right)H\left(E_s-E_{cr}\right)dp, \tag{2}$$

Now we shall consider, that $h(t)=t$ for $t\in[0,1]$, $h(t)=2-t$ for $t\in[1,2]$. Outside of the indicated intervals $h(t)=0$. Thus the temporary dependence of a source function has the arrow-headed form. We integrate Equation (2) keeping in mind that hereinafter we shall investigate $N(E,s,t)$ for $t>2$. For a radio-frequency range it is possible also to put $E_s>E_{cr}$. Then we have

$$N\left(E,t\right)=\frac{C\left(E_o^2\left[\frac{E_o}{2}\right]^{\frac{1-2\delta}{3}}+E_2^2\left[\frac{E_2}{2}\right]^{\frac{1-2\delta}{3}}-2E_1^2\left[\frac{E_1}{2}\right]^{\frac{1-2\delta}{3}}\right)}{2\sqrt{E}k^2n^2\left(\delta-2\right)\left(2\delta-7\right)}, \tag{3}$$

Here $E$ in keV, $t$ in seconds, $n$ - in cm$^{-3}$.

More refined study of connection of dynamics of electrons with spectral characteristics of microwave radiation will be indicated in the report. Here we shall analyze the following problems: how the parameter of a power spectrum of electrons varies in due course and how far and fast the distribution function with positive declination in energy loses this declination.

Dependence of a variation of a parameter of a power-law spectrum of the electrons $(\Delta\delta)$ in the energy range $E=50\div250$ keV (just that interval of energies have electrons which are responsible for centimeter and decimeter radiation) for three values of ambient density

$n = 10^9$ , $10^{10}$ , $10^{11}$ $cm^{-3}$ are shown in Fig. 1, 2. In all cases the decrease of $\delta$ occurs. Notice the similarity of curves for different $\delta_n$.

Let's calculate now at which values of $t$ (in sec) the distribution function has a positive declination ($\partial N/\partial E > 0$) for any $E$.

The obtained values, in particular, characterize the duration of generation of radio emission by coherent mechanism.

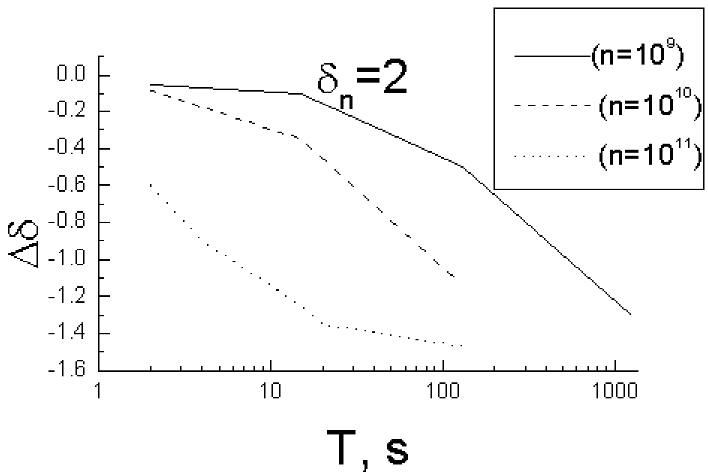


Figure 1: Dependence of a variation of a parameter of a power spectrum of electrons ($\Delta\delta$) in the range $E = 50 \div 250$ keV for three values of ambient density $n = 10^9$, $10^{10}$, $10^{11}$ cm$^{-3}$ and for $\delta_n = 2$

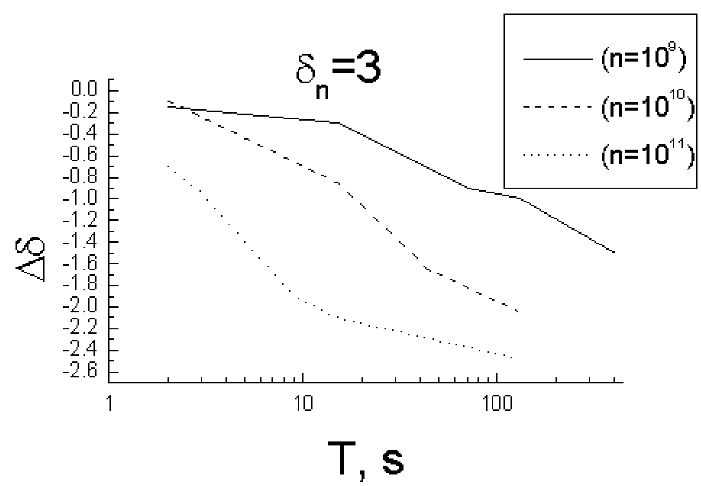


Figure 2: The same for $\delta_n = 3$

Table 1.

| $n$ | $E_{cr} = 5.1$ keV | $E_{cr} = 51$ keV |
|---|---|---|
| $10^{10}$ | 0.23 | 20 |
| $10^{11}$ | 0.023 | 2 |
| $10^{12}$ | 0.0023 | 0.2 |

**References**

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