Sky temperature resolution by microwave radiometry

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Introduction

The threshold sensitivity of a radiometer is determined by well known equation (Karlov & Manenkov, 1966)

$$\langle \Delta E_\nu \rangle = h\nu \left[\frac{\langle n_\nu \rangle + 1}{N_\nu \Delta\nu \, \Delta t} \left(\langle n_\nu \rangle + \frac{G_0 - 1}{G_0} \right) \right]^{1/2} . \qquad (1)$$

Here we denote $\langle \Delta E_\nu \rangle$as the minimum detectable increment of radiation spectral density E_ν at a frequency ν. N_ν is the mode number of the front end device (detector or amplifier) having the gain G_0 within operating frequency bandwidth of $\Delta\nu$; Δt is the integration time; $\langle n_\nu \rangle$ is the mean number of photons in input radiation. The angle brackets imply the ensemble average.

Equation (1) suggests that the radiometer sensitivity is limited by external thermal radiation fluctuations (so called "ideal" radiometer case). By definition (Karlov & Manenkov, 1966; Kislyakov, 1997), the ideal radiometer sensitivity does not essentially depend on the presence of input amplifier; so that hereafter we put $G_0 = 1$. Besides, for the limit temperature resolution to be achieved such broadband input device as detector is preferable.

Taking into account that the radiation power increment $\Delta P_\nu = \langle \Delta E_\nu \rangle N_\nu \Delta\nu$ the following expression can be derived from Eq. (1) (Kislyakov, 1997):

$$\Delta P_\nu = h\nu \sqrt{\frac{\langle n_\nu \rangle}{\Delta t} (\langle n_\nu \rangle + 1) N_\nu \Delta\nu} \underset{\langle n_\nu \rangle \to 0}{\longrightarrow} \frac{h\nu}{\Delta t} \sqrt{N_\nu \langle n_\nu \rangle} . \qquad (2)$$

It was assumed that $\Delta\nu \, \Delta t \simeq 1$. The transition $\langle n_\nu \rangle \to 0$ implies the seldom photons in external radiation. As it follows from Eq. (2), the radiometer sensitivity is unlimited if $\langle n_\nu \rangle \to 0$. However, the photon number in every electrodynamic system has a lower limit because of zero-field fluctuations with energy spectral density of $N_\nu h\nu/2$. On the other hand, the zero-field photons are absent in a radiation flux (they are not involved in energy transport). Including the zero-field fluctuations into consideration we obtain a new equation, valid under $\langle n_\nu \rangle \ll 1$, instead of Eq. (2):

$$\Delta P_\nu \simeq N_\nu \frac{h\nu}{\Delta t} \sqrt{\left(\frac{\langle n_\nu \rangle}{N_\nu} + \frac{1}{4} \right) \Delta\nu \Delta t} \underset{\langle n_\nu \rangle \to 0}{\longrightarrow} N_\nu \frac{h\nu}{2\Delta t} \qquad (3)$$

in accordance with physically evident expression $\Delta P_\nu = h\nu/\tau$, where τ is the time constant, under $N_\nu \equiv 1$. Equation (3) gives a correction to the expression for limit detector sensitivity (Kislyakov, 1997).

Temperature resolution

The differential temperature resolution by thermal emission measurements can be estimated using the expression

$$\Delta T = \frac{2\mathrm{sh}(z/2)}{z}\,\frac{T}{\sqrt{N_\nu\,\Delta\nu\,\Delta t}}. \tag{4}$$

derived from Equation (1) with its left part substituted by $\langle \Delta E_\nu \rangle = h\nu \frac{d}{dT}\langle n_\nu \rangle_T \Delta T$, assuming $G_0 = 1$, and $\langle n_\nu \rangle_T = [\exp(z) - 1]^{-1}$, where $z = h\nu/kT$. Equation (4) is valid under $\Delta\nu \ll \nu_T = kT/h$. Under constant T, the value ν_T gives approximately the width of thermal spectrum. Evidently, the radiometer frequency bandwidth must be close to ν_T in order to minimize the value ΔT. Total thermal spectral sensitivity (TS) of a detector radiometer can be derived from its detection capability (DC) given by

$$D_\mathrm{T} = \frac{T}{\Delta T} = F(z)\sqrt{N_\nu \Delta\nu\,\Delta t}, \tag{5}$$

where $F(z) = z[2\mathrm{sh}(z/2)]^{-1}$. Making both sides of Eq. (5) squared, letting $\Delta\nu = d\nu = \nu_T dz$ and integrating the right part of Eq. (5) over z we obtain the expression for integrated DC:

$$\overline{D}_\mathrm{T} = \sqrt{\nu_\mathrm{T}\Delta t}\left[\int_0^\infty (1 + az^2)F^2(z)\,dz\right]^{1/2} = \sqrt{\varphi(T)\,\nu_\mathrm{T}\Delta t}, \tag{6}$$

where $a = 2\pi\, S_n/\lambda_\mathrm{T}^2$, S_n is the detector operating area and $\lambda_\mathrm{T} = ch/kT$. The mode number N_ν give rise to the factor $(1 + az^2)$ in Eq. (6). If $N_\nu \equiv 1$, then $a = 0$ and $\varphi(T) \equiv 3{,}29$. It is easy to see that TS is determined by ratio $T/\overline{D}_\mathrm{T}$, therefore,

$$\delta T'_T = \frac{T}{\overline{D}_T} = \frac{T}{\sqrt{\varphi(T)\nu_T \Delta t}}. \tag{7}$$

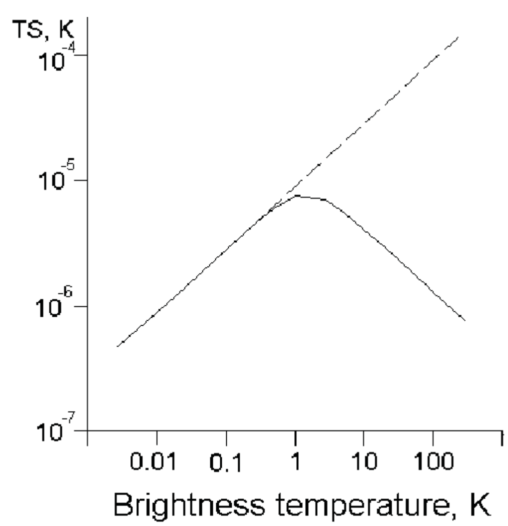

Figure 1

Fig. 1 presents two curves calculated from Eq. (7) under $a = 0$ (the single-mode case, dashed line) and for $S_n = 0{,}1$ cm^2 (multi-mode case). In both cases $\Delta t = 1$ s. For single-mode detector, the TS grows monotonically so that $\delta T'_{_T} \propto \sqrt{T}$ while the multi-mode detector has a maximum in TS under $T \sim 1$ K. Obviously, the T-position of this maximum is strongly dependent on S_n. Only right wing of the curve $\delta T'_{_T}$ corresponds to multi-mode operation of detector.

Nonideal detector

One should take into account that equations Eqs. (2 and 3) present the radiometer sensitivity with an ideal detector as front end. It means that thermal and shot noise of a detector were neglected. However, under $T \to 0$ both noise sources will be of importance as the input thermal noise will be negligibly small. The detector radiometer sensitivity in the case of detector thermal noise taken into account equals to (Kislyakov, 1997)

$$\Delta T = \frac{(e^z - 1)^2}{z^2 e^z} \frac{2}{M\, N_\nu \Delta\nu} \sqrt{\frac{T_0}{k}\,\Delta f}. \tag{8}$$

where $M = \beta/\sqrt{Y_0}$ is the detector figure of merit; β is detector effectiveness; Y_0, T_0 are differential conductivity and physical temperature of a detector; Δf is the output frequency bandwidth of radiometer. Eq. (8) is valid under $hf \ll kT_0$. The detector effectiveness can be formally described by expression (Kislyakov, 1997)

$$\beta = \frac{q_e}{h\nu}\left[1 - \exp\left(-\frac{h\nu}{kT_0}\right)\right]. \tag{9}$$

Eq. (9) incorporates two extreme cases:

$$\begin{array}{cc} 1)\,\text{classic detector}: h\nu \ll kT_0 & 2)\,\text{quantum detector}: h\nu \gg kT_0 \\ \beta = q_e/kT_0 & \beta = q_e/h\nu \end{array} \tag{10}$$

where q_e is the electron charge. Eq. (9) shows that unlimited increasing of $\beta = q_e/kT_0$ in classic case under $T_0 \to 0$ is impossible. Analogously to TS calculations presented above, one can obtain the following expression in classic case

$$\delta T''_{_T} = \frac{T\, I_n}{q_e\, \nu_{_T}\, \psi(T, T_0)}. \tag{11}$$

Here we introduce the detector noise current $I_n = \sqrt{4kT_0 Y_0 \Delta f}$ and some new function

$$\psi(T, T_0) = \int\limits_0^\infty \frac{z\, e^z (1 + a z^2)}{(e^z - 1)^2} (1 - e^{-bz})\, dz,$$

where $b = T/T_0$. Fig. 2 presents the single- (top curves) and multi-mode functions $\delta T''_{_\mathrm{T}}(T)$.

All the curves at Fig. 2 correspond to $Y_0^{-1} = 10^2\ \Omega$, $\Delta f = 1$ Hz and $S_n = 0{,}1$ cm^2. The multi-mode functions change their slope at the point $T = T_1$, where the input noise transformed by detector begin to dominate over its thermal noise. The magnitude of T_1 is strongly depended on detector operating temperature T_0. Therefore, the curves $\delta T'_{_T}(T)$

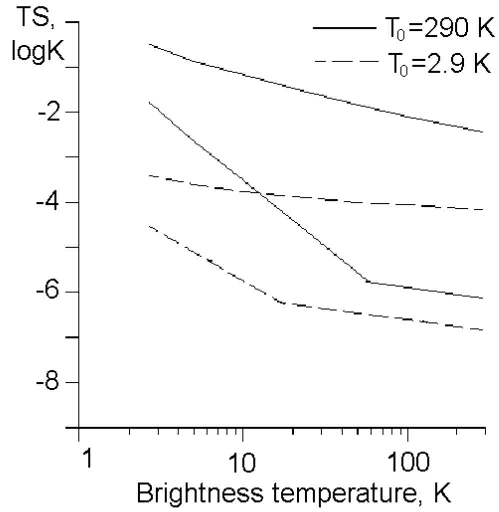

Figure 2

and $\delta T''_{_T}(T)$ in multi-mode case are coincident under $T > T_1$. Analogously, the single-mode curves should change their behavior under $T > T_2 \gg T_1$. Notice that T_2 has the same sense for single-mode case as T_1 in multi-mode one. If $T > T_2$ then $\delta T''_{_T}(T) \propto \sqrt{T}$ in accordance with dashed curve at Fig. 1. It is worth to note that the quantum detector case can be considered using for β the 2) formula from (10) and replacing the expression for I_n with corresponding equation for shot noise current (see Kislyakov, 1997).

Sky temperature resolution

Under sky effective temperature T_s measurements, one should take into account the antenna angle resolution $\psi \simeq \lambda/2D$ (single-mode case) or $\psi \simeq d/D$ (multi-mode case), where D is antenna diameter and d – the detector crossection. Assuming gaussian beam shape and gaussian distribution of T_s over sky with the angle correlation radius of θ, the following equations for minimum detectable temperature variations ΔT_s can be obtained (Kislyakov & Shvetsov, 1974)

$$\begin{aligned} \Delta T_{s1} &= \Delta T_{_T}\sqrt{1+\frac{\lambda^2}{4D^2\theta^2}}, \\ \Delta T_{sN} &= \Delta T_{_T}\frac{\lambda}{d\sqrt{2\pi}}\sqrt{1+\frac{d^2}{D^2\theta^2}}, \end{aligned} \tag{12}$$

where ΔT_{s1} and ΔT_{sN} correspond to single- and multi-mode cases, respectively. Both quontities are frequency dependent, so that we derive, as previously, two expressions for sky temperature inhomogenieties detectability:

$$\begin{aligned} \overline{D}_{s1} &= \sqrt{\nu_{_T}\Delta t}\left[\int_0^\infty (1+\frac{p^2}{z^2})^{-1}F^2(z)\,dz\right]^{1/2}, \\ \overline{D}_{sN} &= \sqrt{\nu_{_T}\Delta t}\frac{d}{\lambda_{_T}}(1+\frac{d^2}{D^2\theta^2})^{-1/2}\sqrt{\frac{\pi}{2}}\left[\int_0^\infty \frac{1+az^2}{\mathrm{sh}^2(z/2)}\,dz\right]^{1/2}, \end{aligned} \tag{13}$$

where $p = \lambda_{_T}/2D\theta$. Fig. 3 presents the functions $\delta T_{s1}(T) = T/\overline{D}_{s1}$ (top curve) and $\delta T_{sN}(T) = T/\overline{D}_{sN}$ calculated using (13) under following parameters: $\Delta t = 1$ sec, $d =$

0.3 cm, $D = 10$ m and $\theta = 3 \cdot 10^{-3}$. It is easy to see that these parameters allow to realize the multimode operation under $T \geq 1$ K. As it could be expected, $\delta T_{s1}(T)$ and $\delta T_{sN}(T)$ behave very similarly to corresponding dependences on Fig. 1. The next plots on Fig. 4 give an impression on angular dependences of δT_{s1} and δT_{sN}.

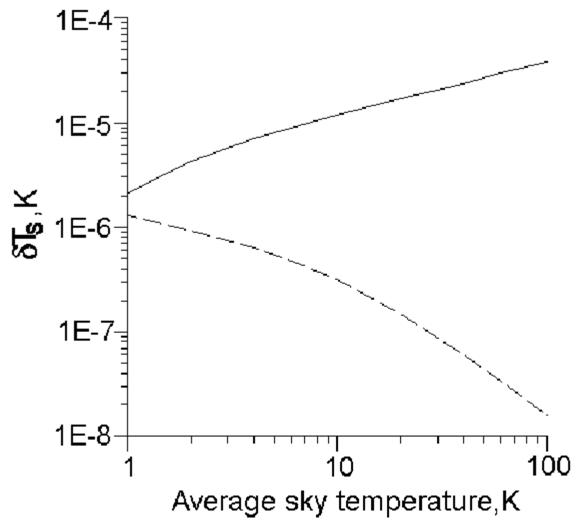

Figure 3

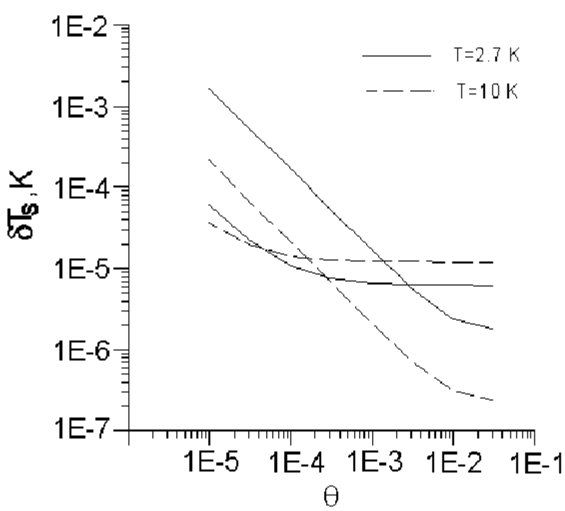

Figure 4

One can see here the sequence of antenna angular resolution effects. The curves of Fig. 4 were calculated using the same values of such parameters as Δt, d and D. As it was mentioned above, the multimode antenna resolution is limited by ratio d/D thus explaining the sharp increase in $\delta T_{sN}(\theta)$ function under $\theta \to 0$, and in this case a single-mode telescope is preferable. However, under $\theta \to \infty$, the $\delta T_{sN}(\theta)$ is proportional to decreasing factor of $d/D\theta$ thus leading to $\delta T_{sN}(\theta) \ll \delta T_{s1}(\theta)$. The lack of site does not allow to analyze the nonideal detector case.

Final remarks

As it follows from consideration presented above, the choice of appropriate radiometer and telescope type should be made in accordance with average temperature and brightness spatial distribution of a source. For instance, the operating waverange of a telescope under $T = 2.7$ K (the "Big Bang" black body radiation temperature) should be close to $\lambda = 1.5$ mm and the receiver bandwidth should be of the order of $\nu_T \simeq 58$ GHz. Depending on the spatial spectrum to be investigated, the single-mode or multi-mode operation for a given D may be preferred. It is worth to note that in the case of metagalactic background radiation the spatial scale of cosmological interest is rather extended: from $\sim 90°$ (quadrupole anisotropy) up to few sec of arc (see, for instance, Partridge, 1986). Another example, is the Galaxy diffused matter thermal emission with the expected brightness temperature of about 10 K. In this case the maximum of Planck curve is close to $\lambda = 0.3$ mm and the corresponding $\nu_T \simeq 200$ GHz. The spatial scales of galactic dark nebulae lie approximately within $10° \div 10''$. The curves on Fig. 4 help to choose the telescope scheme in this case.

This contribution is an attempt to establish some limits in sky temperature resolution under microwave measurements as dependent on source parameters. The only parameter of a telescope has been taken into account the diameter. All other reasons restricting the receiver sensitivity and antenna angular resolution (say, the Earth atmosphere influences or hard ware problems) were neglected. The detailed analysis taking all possible factors into account can lead to other conclusion on the proper telescope scheme. However, the limit values of δT_{s1} and δT_{sN} will remain as a quality indicator of a telescope.

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