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Features of the Spatial Distribution of Spots in the Solar Cycle and a Model of Dynamo in a Thin Layer

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Abstract—In our earlier works (Ivanov and Miletsky, 2012, 2014), we demonstrated that (1) the evolution of average sunspot latitudes can be described by a universal latitude curve, the shape of which does not depend on the cycle amplitude and (2) at the cycle decline phase, these latitudes correlate well with the current level of solar activity. In this work, we demonstrate that these features of the latitude evolution of the cycle, as well as the empirical Waldmeier rule, can be described by a simple model of a convective α-ω dynamo in a thin spherical layer with the addition of some nonlinearities.

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1. INTRODUCTION

It is well known that the latitude evolution of solar spots in the 11-year solar cycle is described by the Spörer law: the first spots appear at high latitudes, and then the activity center drifts to the equator. In this process, it was found (see, e.g., Eigenson et al., 1948; Vitinskii et al., 1986; Hathaway, 2011) that the behavior of the average sunspot latitude (ASL) varies from cycle to cycle much more weakly than the spot amplitude characteristics.

In our earlier works (Ivanov and Miletsky, 2012; Ivanov and Miletsky, 2014) based on data of the extended Greenwich catalogue for 1874–2013, we demonstrated that ASL behavior can be described sufficiently accurately by a universal latitude curve, the shape of which very slightly depends on the cycle power: $\psi(t) = a\exp(b(t - T_0))$, where $a = 26.6°$ and $b = -0.126$ year^{-1} are parameters common for all cycles and T_0 is the latitude phase reference point for the given cycle (for brevity, we below call this regularity R1). The variable ψ, which indicates the characteristic ASL, can be used here as a characteristic of the cycle phase; it is an alternative to the commonly used time τ passing from the moment of the cycle minimum. In the same works, it was noted that ASLs have one more feature (we call it R2): in the second half of the cycle decline phase, the ASL still does not depend on the cycle amplitude but correlates well with the current level of solar activity.

The second feature is illustrated in Fig. 1, in which panels (a) and (b) depict the index of the sunspot group number G smoothed over 13 solar rotations with a sinusoidal filter from the 12th to 23th solar cycles as a function of the cycle phases τ and ψ, respectively; panel (c) depicts root-mean-square deviations of G from the mean for all cycles (σ_ψ and σ_τ) as functions of these variables; and panel (d) depicts the ratio of these deviations σ_ψ/σ_τ. It is seen from a comparison of panels (a) and (b) that the scatter of trajectories in the second half of the cycle decline phase is less in the second case. In other words, a definite cycle phase τ in different cycles can be associated with values of the index G from a rather wide range, and the characteristic latitude ψ at the cycle decline phase is in correspondence with a more narrow interval of this index. The same is testified by panels (c) and (d) showing that, at this phase, $\sigma_\psi < \sigma_\tau$.

The obtained regularities certainly must impose constraints on the possible mechanisms of solar periodicity. In this work, we study the question of whether the observed features of ASL evolution can be reproduced by the model of the solar convective dynamo. At the initial stage of the study, we are interested mainly in the qualitative aspect of the problem; for this reason, we chose for the study the simplest model of the α-ω dynamo in a thin spherical layer. When required, the model is complemented with nonlinearities.

2. LINEAR MODEL OF THE α-ω DYNAMO IN A THIN SPHERICAL LAYER

The dependences of the toroidal field strength B and vector potential of the poloidal field A on the radius r and colatitude θ for the case of a thin layer can be written in the form $F(r, \theta, t) = f(\theta, t)\exp(kr)/kr$. The dimensionless equations of the dynamo for $A(\theta, t)$ and $B(\theta, t)$ in this model have the form (see, e.g., (Schmitt and Schüssler, 1989; Solanki et al., 2008))

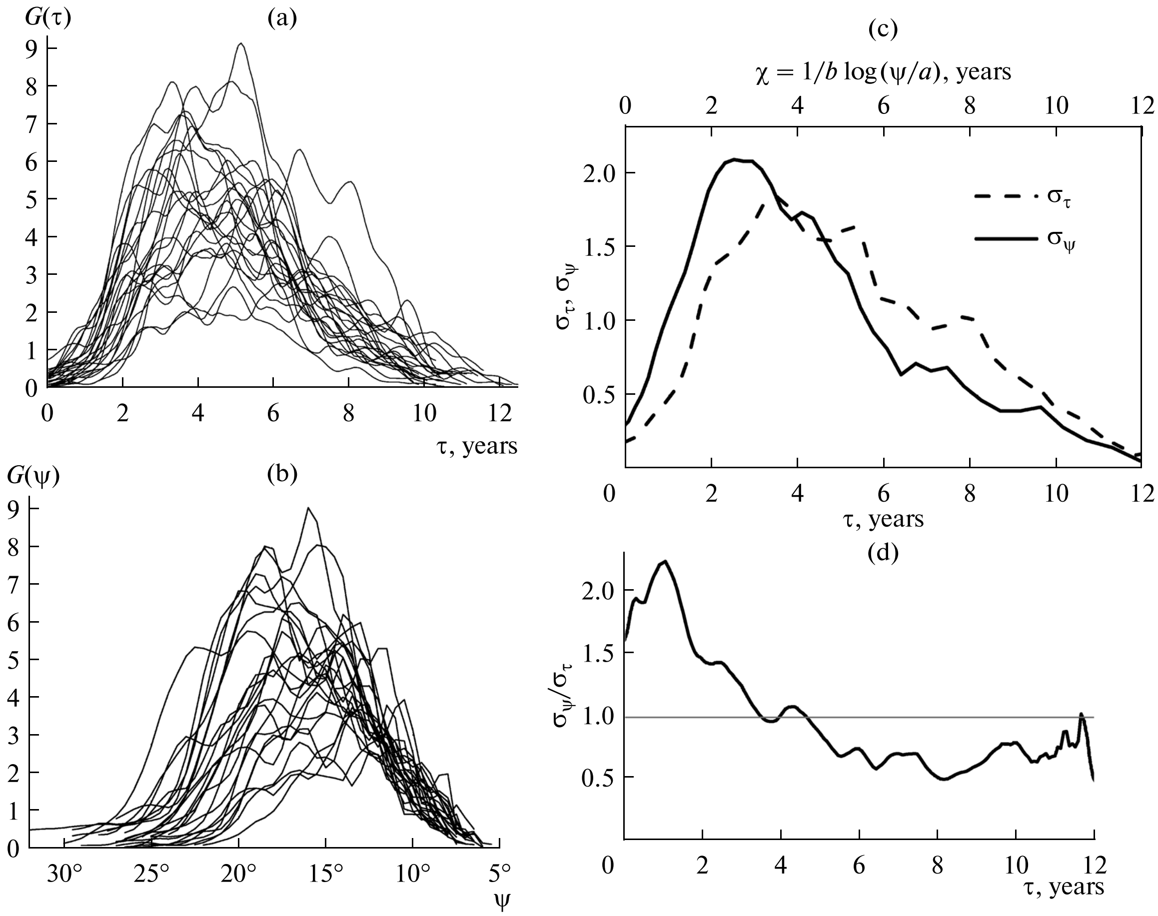

Fig. 1. Relation between the average sunspot latitude and current level of the solar activity in the second half of the cycle phase (see explanations in the text).

$$\frac{\partial B}{\partial t} = D(B) + R_\omega \frac{\partial (A \sin\theta)}{\partial \theta} - R_v \frac{\partial (v_\theta B)}{\partial \theta},$$

$$\frac{\partial A}{\partial t} = D(A) + \alpha B R_\alpha - R_v \frac{v_\theta}{\sin\theta} \frac{\partial (A \sin\theta)}{\partial \theta} + \delta S(\theta, t),$$

where the diffusion operator

$$D = \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial}{\partial\theta} - \frac{1}{\sin^2\theta} - (kR)^2,$$

Reynolds numbers

$$R_\alpha = \frac{\alpha_m R}{\eta}, \quad R_\omega = \frac{\omega' R}{\eta}, \quad R_m = \frac{v_m R}{\eta},$$

R is the radius of the spherical shell, η is the convective viscosity, $\omega' = \partial\omega/\partial r$ is the radial gradient of the angular velocity, $\alpha = \cos\theta$ and $v_\theta \sin 2\theta$ are normalized profiles of the alpha-effect and meridional velocity, respectively, α_m and v_m are the maximum values of these quantities, and $\delta S(\theta, t)$ is the source of the poloidal field potential.

In this model, it is natural to associate the spot formation level observed at a given latitude with the toroidal field strength b of a given sign; to associate the activity level with the same strength averaged over the hemisphere $\langle b \rangle$; and to associate the ASL with the latitude $\varphi = 90° - \theta$ weighted with a weight $|b|$. To obtain cycles with different amplitudes, we complement the model with "secular" periodic variations in a certain parameter with a period $T_{\rm mod} = 3.0$ (in conventional units of time), which, as is seen below, exceeds the main period of the dynamo model T_0 by more than an order of magnitude. For this parameter, we use the source S (below, this type of the model is denoted as VS), the intensity of the α-effect α_m (Vα), or the meridional circulation v_m (Vm). It should be noted that such an artificial introduction of the "secular periodicity," is certainly meant to extend the phase space of the studied models, not to explain actually observed long-periodic solar activity variations.

For all of the models, we set $Rk = 3$. We specify quasi-dipole initial conditions $B(\theta, 0) = 0$ and $A(\theta, 0) = \sin^2\theta$ and study the evolution of the fields on the time interval of $0 \le t \le 6$. In doing this, beginning from cri-

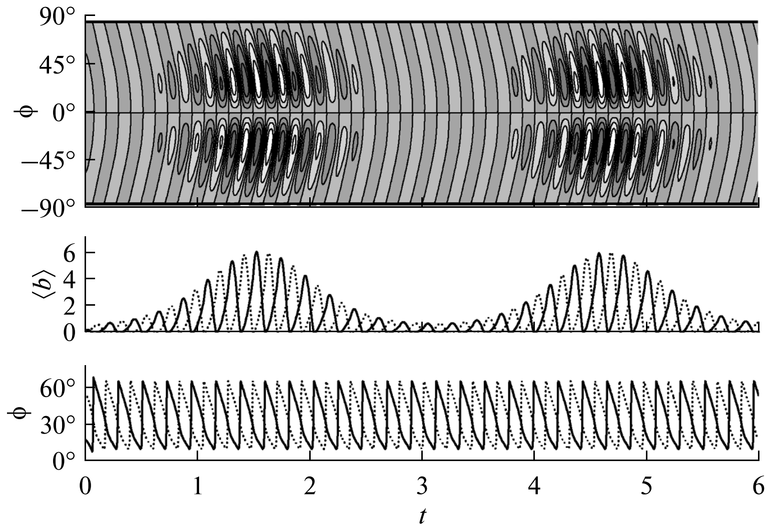

Fig. 2. Latitude–time diagram (upper panel), as well as average strength $\langle b \rangle$ (middle panel) and average latitude φ (lower panel) as functions of time for the linear model.

ticial Reynolds numbers $R_\alpha = -R_\omega \approx 37$, we then gradually increase R_α until the dynamo reaches a steady regime.

We begin the investigation with the linear model of the VS type, in which the critical values $R_\alpha = -R_\omega = 37.43$ are associated with a cycle period of $T_0 = 0.1086$. For this model, Fig. 2 depicts the latitude–time diagram, as well as the dependences of $\langle b \rangle$ and φ on time; Fig. 3, the dependence of $\langle b \rangle$ on cycle phases (a) τ and (b) ψ, as well as (c) of spreads of $\langle b \rangle$ for given values of τ and ψ and (d) ratios of these spreads (these panels are similar in their meaning to the corresponding panels of Fig. 1), as well as (e) dependences of φ on the cycle phase τ. Comparing Figs. 1 and 3, we see that the ASL curve (R1) for the linear model is universal, as should be expected, and R2 regularity does not take place.

3. INTRODUCTION OF NONLINEARITIES INTO THE DYNAMO MODEL

It is evident that the model must be complemented by nonlinearities to reproduce R2 regularity. One well-known way to do this is to assume that the α-effect is suppressed by the toroidal magnetic field (see, e.g., (Charbonneau, 2010)). We take this effect into account by writing $\alpha_m = \alpha_{m0}\exp(-B/B_c)$, where B_c is the critical value of the strength (the αB model). In addition, we study a model with a nonlinearity of another type, in which the α-effect is suppressed in a similar way by the poloidal field: $\alpha_m = \alpha_{m0}\exp(-A/A_c)$ (the αA model). For these types of models, we everywhere set $B_c = 0.4$ and $A_c = 0.4$.

The first two rows of the table show the results of studying these two nonlinearity variants for the three aforementioned types of "secular variations" (VS, Vα, and Vm). The valid latitudinal regularities are shown in corresponding cells of the table. It is seen that both of the regularities above (R1 and R2) take place only for the αA + VS model.

Let us complicate the model requirements by recalling one more effect that must manifest itself in the 11-year cycle model corresponding to observations. This is the Waldmeier rule (Waldmeier, 1935), according to which the coefficient r_W of the correlation between the length of the cycle growth phase and cycle amplitude for observed solar cycles is negative and exceeds 0.6 in absolute value (its exact value depends on the epoch under study and the method of averaging the solar activity index). It is seen from the table, which presents this coefficient, that it has the correct sign for Vα models but none of the three effects (R1, R2, and $r_W < 0$) match in any of the six models.

Let us introduce in the model one more modification by assuming (see, e.g., Solanki et al., 2008) that field tubes of the toroidal field begin to rise to the surface efficiently only if the strength of the field B exceeds the critical value B_b. This situation can be described by assuming that the toroidal fields observed

Latitudinal regularities and correlation for the Waldmeier rule in different types of models

	VS		Vα		Vm	
		r_W		r_W		r_W
αB	R2	+0.94		−0.78	R2	+0.77
αA	Rl,R2	+0.66	Rl	−0.83	R2	+0.69
αA + buoyancy	Rl, R2	−0.61		−0.84	R2	+0.64

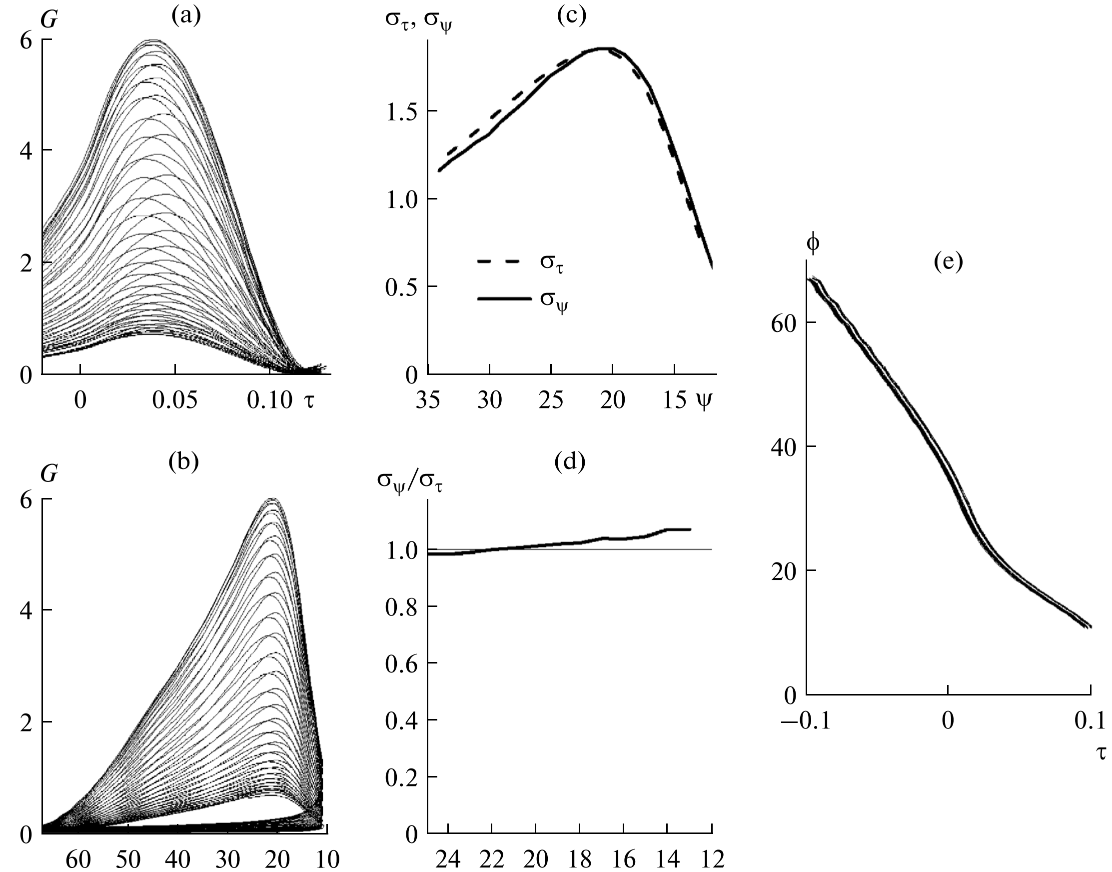

Fig. 3. Relation between the average sunspot latitude and current level of the solar activity (see explanations in the text).

in the photosphere are related to fields generated by the dynamo mechanism in terms of a quasi-step function $B_{\rm obs} = B\exp(-B/B_b)^8)$, where we set $B_b = 0.1$. An analysis of αA models with this additional effect shows that the VS + αA type + buoyancy model satisfies all necessary requirements, including the Waldmeier rule (with a correlation coefficient $r_{\rm W} = -0.61$). Figure 4, which is similar to Fig. 2, depicts the model characteristics of interest.

4. DISCUSSION AND CONCLUSIONS

Thus, we have demonstrated that the simplest models of the α-ω dynamo in a thin spherical layer can describe observed regularities of the latitudinal–temporal evolution of the activity cycle. It is easiest to reproduce the R2 feature (the relation between the activity level and average latitude at the cycle decline phase); it is absent only in models with secular variation in the alpha-effect (Vα).

To obtain the R1 feature (universality of the latitudinal drift law), which naturally arises in linear models, one should restrict oneself to models with suppression of the alpha-effect by the poloidal field. It should be noted that, if the suppression of the alpha-effect by the toroidal field naturally arises in the theory, the assumption about the participation of the weaker poloidal field in this process can look rather artificial. However, our goal in this work was only to show that this regularity can be easily reproduced even in simplest "toy models." Part of the poloidal field in this case is reduced to decreasing the quantity R_α and, therefore, the rate at which toroidal fields are transformed into poloidal. Upon a more detailed investigation with the use of realistic models, this part can be played by some other factor in phase with the poloidal field. Note also that a more complex model must describe the real periodicity feature, according to which $\sigma_\psi > \sigma_\tau$ at the cycle growth phase (see Fig. 1d); this is not reproduced by the simple model that we have described.

Thus, the simple model of the dynamo in a thin layer with secular variation in the source and suppression of the alpha-effect by the poloidal field (VS + αA) can qualitatively reproduce both of the observed latitudinal regularities. If it is complemented by critical buoyancy, it is also able to reproduce the Waldmeier rule. One can suppose that the development of the aforementioned dynamo model with the transition to realistic two-dimensional models will make it possible

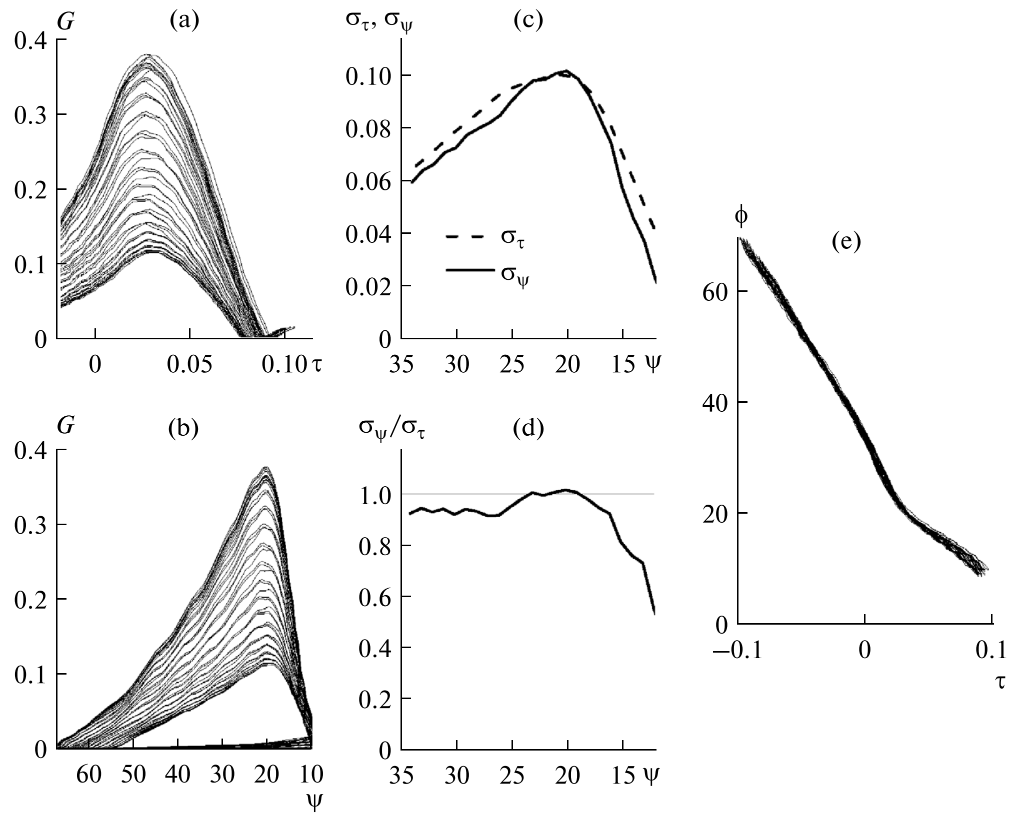

Fig. 4. The same as in Fig. 3 for the VS + αA type + buoyancy nonlinear model.

to find a way also to provide a quantitative description of these effects.

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