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Properties of the Sunspot Latitudinal Distribution Skewness

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Abstract—The properties of the sunspot latitudinal distributions related to skewness have been studied based on the data of the extended Greenwich catalog for 1874–2011. The results of the performed analysis indicate that a significant skewness is present in most annual latitudinal distributions of the sunspot index. In this case, the distribution skewness increases near the 11-year cycle maximum phase. An increase in the sunspot group number is also accompanied by an increase in skewness. In particular, when the sunspot index is large, the number of groups located below midlatitudes is mostly larger than the number of groups above these latitudes and this imbalance increases with increasing total sunspot activity level. In medium and large 11-year cycles, the average distribution skewness for a cycle is always positive and its value is related to the cycle amplitude. This results agree with the theoretical models of the 11-year cycle, where the specific features of the low-latitude meridional circulation are related to the sunspot activity level.

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INTRODUCTION

The Spörer law, signifying the cyclic drift of the main sunspot-generating zone toward the equator, is the most pronounced spatial manifestation of the 11-year cycle. This law is usually characterized by time variations in the mean sunspot group latitude. It was previously (e.g., in (Vitinsky et al., 1986)) noted that the curves, which reflect this dependence for different cycles, “almost coincide.”

The recent works devoted to studying this problem (Hathaway, 2011; Roshchina and Sarychev, 2011; Ivanov and Miletsky, 2012) indicated that the curves describing the mean latitude drift shape in 11-year cycles can be reduced to an integrated mean cyclic curve without significant loss of accuracy by shifting these curves along the time axis and reducing them to a certain initial reference point. In turn, this curve is adequately represented by an exponent equation with two parameters.

We also established (Ivanov et al., 2011) that most unimodal annual latitudinal activity distributions can be approximated by the Gaussian (normal) distribution in a first approximation. If the annual latitudinal sunspot distributions are considered to be Gaussian, the average latitude unambiguously characterizes the position of the sunspot-generating zone center. In this paper, we consider the following approximation: we study the available skewness in the sunspot activity latitudinal distributions since the average latitude ceases to be the only characteristic of this center in the presence of this skewness.

DATA PROCESSING

We used the data on sunspots presented in the Greenwich catalog and its extension NOAA/USAF (http://solarscience.msfc.nasa.gov/greenwch.shtml) for 1874–2011. We selected one year as a time interval used to calculate the latitudinal distribution parameters. The value of the average latitude varies insignificantly during a year; at the same time, this interval gives a representative data sample.

At the first stage, we eliminated most multimodal distributions originating as a result of the repeated latitudinal superposition of adjacent cycles. Each cycle (from cycle 12 to cycle 23) was divided into years, counting off from the minimum point. It turned out that 73% of multimodal (i.e., statistically indeterminate) and only 27% of unimodal distributions fall on the first years of the cycles, counting off from the minimum, and in years 10 and further. On the contrary, only 23% of multimodal distributions and 77% of unimodal distributions (79% in hemisphere N and 75% in hemisphere S) fall on the cycle phases from years 2 to 9. Therefore, to continue the studies, we selected the interval from years 2 to 9 in each of these cycles and obtained 192 annual latitudinal distributions (96 distributions in either solar hemisphere) for these years.

We tried to trace the time variations in the index of skewness, characterizing the deviations of mean latitudes from other distribution center characteristics. As such an index, we selected the difference between the mean (Lmean) and median (Lmed) latitudes (DMM = Lmean – Lmed). This value is entered into the numerator of the expression for the known Pearson’s skewness coefficient ($A = 3(\mathrm{DMM})/\sigma$) and can be considered as an absolute skewness index. We should note that DMM and A correlate well in the obtained distributions (their correlation coefficients are 0.971 and 0.972 in the Northern and Southern hemispheres, respectively). Therefore, further results (highly reli-

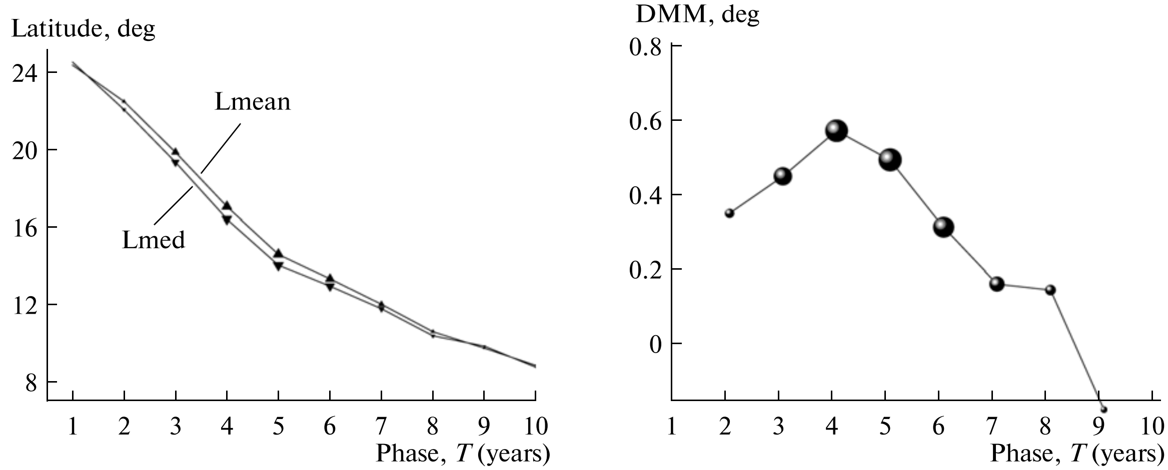

Fig. 1. Values of the mean (Lmean) and median (Lmed) annual latitudes (triangles and upside triangles, respectively; left panel) and difference between these values DMM (right panel) at different phases of the 11-year cycle. The sizes of the symbols are proportional to the activity level (G).

able for DMM) can also be applied to coefficient A. Among 192 (according to the distribution number) values of the DMM difference, 146 distributions (76%) are significant according to the criterion based on the verification of the coefficient A significance. In addition, the DMM differences have the "plus" sign (i.e., Lmed < Lmean) for 141 (73%) of 192 values, which indicates significant different from chance: the confidence level is CL > 99.9%.

To test the distributions for skewness, we used the index of the sunspot group number (G) in addition to DMM (Ivanov and Miletsky, 2011). Let GL and GH be the partial values of the G index for sunspot groups that are located higher (closer to the pole) and lower (closer to the equator) than the mean latitude. In such a case, the GLH = GL – GH difference characterizes the sunspot activity distribution skewness relative to the mean latitude. According to the χ^2 criterion, it turned out that 107 (56%) of 192 GLH values significantly differ from zero (CL > 95%). We should note that the signs of the DMM and GLH indices correlate well and completely coincide (the correlation coefficient is $R = 0.796$, the number of points is $K = 192$; the confidence level CL > 99.99%).

Thus, most annual latitudinal distributions of sunspot activity are asymmetric. In these cases, the median is considered to be a more representative estimate of the distribution center than the mean value (Ageel, 2000).

RESULTS

For either hemisphere in each cycle, we constructed dependences representing variations in the mean and median annual latitudes as a function of the 11-year cycle phase. The curves of the mean (Lmean) and median (Lmed) latitudes, which were obtained as a result of averaging these dependences, are presented in Fig. 1 (left panel). It is interesting that the relative position of the curves is systematic. In the middle of the cycle, the median latitudes (Lmed) are as a rule located lower than the mean ones (Lmean). The curve of the difference between these latitudes (DMM) (the maximum of which falls on the fourth year of the cycle with the Lmean and Lmed values 17.1° and 16.4°, respectively) is presented in Fig. 1 (right panel). It is clear that the skewness index (DMM) value is positive for all phases from the range $2 < T < 9$ and increases toward the cycle center, when the mean latitude is far from the equator and the upper sunspot generation boundary.

A similar phase dependence is also obtained for the skewness index (GLH), which is positive for all phases (except $T = 9$) and has a maximum in the fourth year of the cycle. When GLH is averaged over all years, it accounts for 16.5% of GL.

To study the phase curve shape for different activity levels, we divided the cycles into three groups (according to (Hathaway, 2011)): small (cycles 12–14 and 16; the Wolf number at a maximum is Wmax < 90), medium (cycles 15, 17, 20, and 23; 90 < Wmax < 150), and large (cycles 18, 19, 21, and 22; Wmax > 150). The DMM maximums for small, medium, and large cycles (Fig. 2, left panel) fall on the phases of growth and maximum of these cycles (Fig. 2, right panel). It is evident that the DMM values for large (medium) cycles are as a rule larger than the corresponding phase values for medium (small) cycles.

Then, we studied the existence of an interrelation between DMM and G. The correlation between these indices is significant (CL > 99.9%) but not very strong ($R = 0.39$, $K = 192$). If we distribute G over gradations (0–1, 1–2, etc.) (the regression equation squares and straight line in Fig. 3), the correlation increases considerably ($R = 0.87$, $K = 9$, CL = 99.9%). It is clear (see Fig. 3) that the number of negative DMM values decreases considerably and the distribution skewness value increases with increasing activity level. For G ≥ 6,

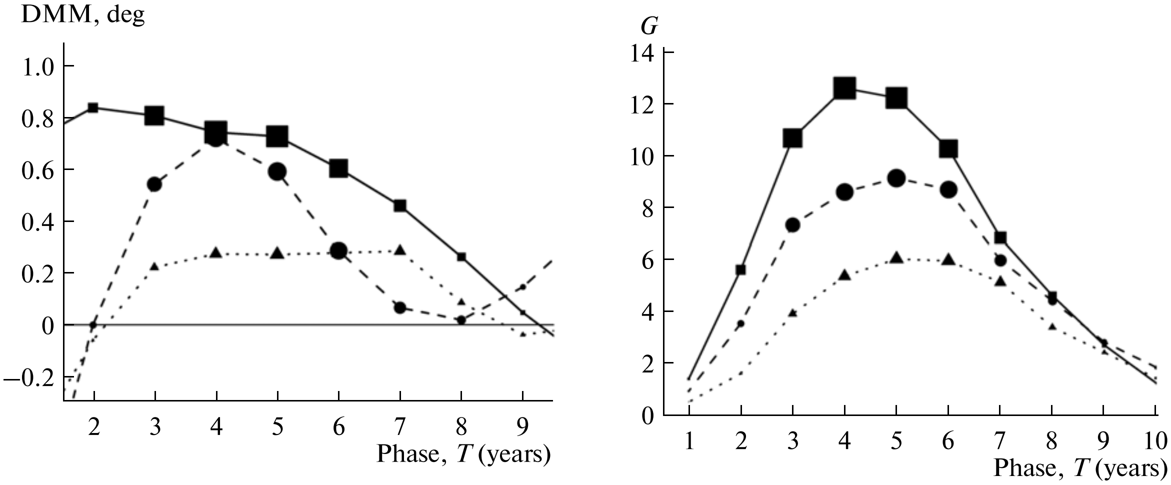

Fig. 2. Values of the differences (DMM) between the mean and median annual latitudes (left panel) and the G index (right panel) at different phases of large (squares and solid lines), medium (circles and dashed lines), and small (triangles and dotted lines) 11-year cycles.

all DMM values are positive; i.e., the median latitude values are always smaller than the mean values in these cases. We should note that the correlation between DMM and G increases pronouncedly ($R = 0.56$, $K = 96$; CL > 99.99%) when hemispheres N and S are combined.

The correlation between GLH and G is higher than between DMM and G ($R = 0.58$, $K = 192$; CL > 99.99%). When the hemispheres are combined, the correlation between GLH and G becomes very significant ($R = 0.76$, $K = 96$; CL > 99.99%). In this case, 39 out of 40 (98%) GLH values are positive for G ≥ Gmean = 6.2. Thus, in this case, the sunspot activity at latitudes lower than the mean latitude is almost always higher than the activity at latitudes higher than the mean latitude.

We also studied the interrelation between DMM and G on the timescales of the 11-year cycles. We calculated the average DMM values over the cycle (for phases of 2–9 years) and constructed the dependence of the DMM values on the maximal annual G values in the corresponding cycles (Gmax) for 12 cycles in either hemisphere. For 24 points (two hemispheres in 12 cycles), the correlation between Gmax and DMM is rather high ($R = 0.71$, $K = 24$, CL > 99.9%). For 24 cycles, DMM < 0 only in the Northern hemisphere during weak cycles (12, 14, 16), whereas skewness is positive and increases with increasing cycle amplitude

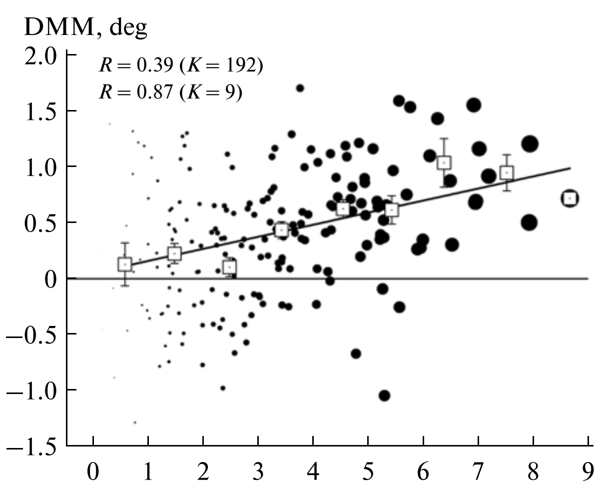

Fig. 3. Values of the differences between the mean and median annual latitudes (DMM), depending on the G index value (circles for the initial G values and squares for G values averaged over gradations 0–1, 1–2, etc.). The sizes of the circles are proportional to the G value.

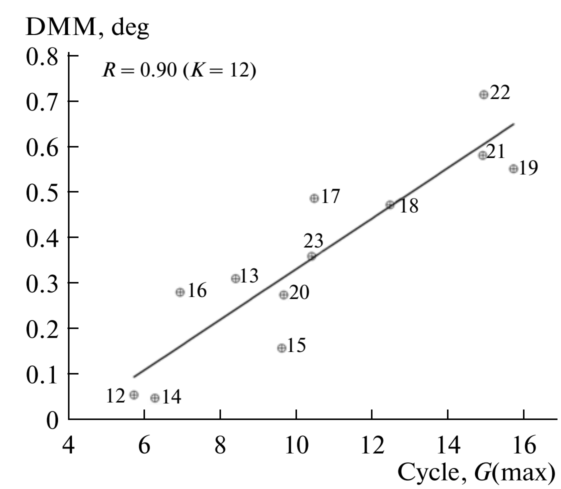

Fig. 4. Values of the differences between the mean and median latitudes (DMM), depending on corresponding cycle maximum G averaged over the cycle (the numbers near circles are their numbers).

in medium and strong cycles. When the hemispheres are combined, the correlation between 12 average DMM values over a cycle and the corresponding cycle amplitudes (Gmax) (Fig. 4) is even closer ($R = 0.90$, $K = 12$, CL > 99.9%). In this case, DMM > 0 for all cycles.

CONCLUSIONS

The performed analysis indicates that considerable skewness is observed in most annual latitudinal distributions of sunspot activity and this skewness increases near the 11-year cycle maximum. An increase in the number of sunspot groups is accompanied by an increase in the distribution skewness. When the sunspot index value is larger than the average level, the number of sunspot groups at latitudes lower than the mean latitude in either hemisphere is as a rule larger than this number at latitudes higher than the mean latitude. In this case, the imbalance increases with increasing total level of sunspot activity. When averaging over a cycle is performed, the correlation between skewness and sunspot activity increases pronouncedly. In medium and large 11-year cycles, the distribution skewness is positive and its value is related to the cycle amplitude.

The achieved results agree with 11-year cycle models, which should be used to relate the specific features of the low-latitude meridional circulation (and, consequently, the latitudinal sunspot drift) to the level of low-latitude (sunspot) solar activity (Nandy, 2011; Nandy et al., 2011). The dependence of the sunspot distribution skewness on the activity level and 11-year cycle phase, detected above, can be among the manifestations of such a correlation.

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