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Amplitude–Time Relations at Different Latitudes in the 11-Year Cycle of Solar Activity

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Abstract—On the basis of sunspots data for cycles 12–23, it is shown that the correlation between the amplitude of the 11-year cycle and lengths of intervals that determine the location of the cyclic curve of average latitudes relative to the cycle's maximum and minimums is strongly dependent on the latitude. The correlation extrema are reached at latitudes 15° and 20°, which are the most representative. The equations for calculating the amplitude of the 11-year cycle are composed based on the information on the intervals corresponding to latitude 15°. It is found that the correlation between the amplitude of the 11-year cycle and the latitude values of the sunspot group number index is significant at middle and high latitudes (15°–25°) and is small at low latitudes (5°–10°). This agrees with the statement that the level of sunspot activity is not correlated with the cycle amplitude at the declining phase of the 11-year cycle (for latitudes <12°) and depends only on the current value of the average latitude. A high correlation ($R = 0.88$ for latitudes ±8°) is found between the activity level of the current 11-year cycle at low latitudes and the amplitude of the next one. The amplitude of the 24th cycle in GSN units (GSN(24) = 91) is estimated by the corresponding regression equation.

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1. INTRODUCTION

Recent studies (Hathaway, 2011; Roshchina and Sarychev, 2011; Ivanov and Miletsky, 2014) showed that the trajectory of the average latitudes of sunspot groups, which characterizes the law of sunspot latitude drift during the 11-year cycle (Sporer's law), can be highly accurately approximated by a functional dependence that is common for all cycles, in particular, by the exponential one. Recently we (Ivanov and Miletsky, 2014) showed that it is convenient to take as the reference point of the cycle phase the time when the exponent that describes the average latitude drift of sunspots arrives at certain "reference" latitude values ±26.6°. We called this time the Latitude Phase Reference Point (LPRP). We note that the exponent on average arrives at these latitudes at the time of minimum (for cycles 12 –23). However, in each particular cycle, this time is shifted relative to the minimum either forward or backward. In this approach the LPRP is determined by the position of the exponent approximating the average latitude cyclic drift.

We obtained equations that link the amplitude of the 11-year cycle and the intervals that determine the location of the exponent relative to the cycle maximum and minima (Miletsky and Ivanov, 2014).

Generally speaking, calculating the LPRP of a given cycle, and, consequently, the position of the approximating exponent, requires complete data on the latitudes of each sunspot group in it. However, it would be useful (in particular, for making forecasts) to have other reference times related to the average sunspot latitude that can be calculated without waiting until the end of a cycle. In this work, we introduce times when the average sunspot latitudes reach certain selected values and find correlations between their positions in the 11-year cycles and the amplitudes of the latter.

2. DATA AND THEIR PROCESSING

The source of sunspot data was the Greenwich catalog and its NOAA/USAF continuation (http://solarscience.msfc.nasa.gov/greenwch.shtml) for the period 1874–2014. Using these data, we obtained a series of rotation average values of mean latitudes and the sunspot group number index (G). Using a 13-point window with sinusoidal weights, we formed smoothed series, for which we found the times and values of cyclic minimums (TGmin, Gmin) and maximums (TGmax, Gmax).

Next, we selected a range of latitudes: 25°, 20°, 15°, 10°, and 5°. For each cycle we found the moments T25(n), T20(n), T15(n), T10(n), and T5(n) at which the smoothed average latitudes reach the selected values during the 11-year cyclic evolution. At these times we also calculated the values of the sunspot group number index GT25(n), GT20(n), GT15(n), GT10(n), and GT5(n) that showed the activity level at these times.

We denote the time of minimum of the 11-year cycle with number n preceding its maximum as TGmin(n) and the subsequent time of minimum as

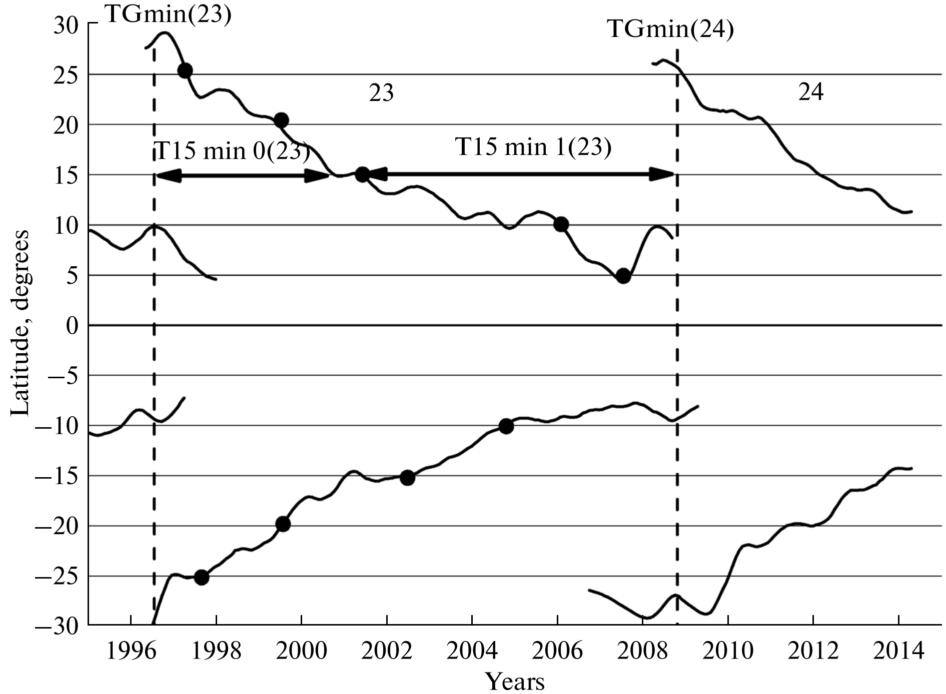

Fig. 1. Smoothed average latitude curves for the northern (N) and southern (S) hemispheres of the 23rd cycle and the points of their intersection (the circles) with the latitudes 25°, 20°, 15°, and 10°.

TGmin(n + 1). The length of the time interval T15min0(n) from preceding minimum TGmin(n) to point T15(n) is equal to the difference T15min0(n) = T15(n) – TGmin(n). Similar definitions are introduced for intervals T25min0(n), T20min0(n), T10min0(n), T05min0(n), and we will use the notation TLAmin0 in the case when the latitude band LA is not specified.

The length of the interval T15min1(n) in the nth cycle from time T15(n) to subsequent minimum TGmin(n + 1) equals the difference T15min1(n) = TGmin(n + 1) – T15(n). Similar designations are introduced for intervals T25min1(n), T20min1(n), T10min1(n), T05min1(n), and TLAmin1.

Figure 1 presents a scheme of intervals T15min0(23) and T15min1(23) for the 23rd cycle.

The length of the time interval T15max(n) from time T15(n) in the nth cycle to its maximum (TGmax(n)) equals the difference T15max(n) = TGmax(n) – T15(n). Similar designations are introduced for intervals T25max(n), T20max(n), T10max(n), T05max(n), and TLAmax.

The correlation coefficients that characterize the relation between the cycle amplitude Gmax and the length of the intervals mentioned are calculated for cycles 12–23 (k = 12) in each latitude band.

3. RESULTS

The correlation (Fig. 2) between the cycle amplitude Gmax(n) and the lengths of intervals T25min0(n), T20min0(n), T15min0(n), T10min0(n), and T05min0(n) strongly depends on the latitude and is significant only for latitudes 15° and 20°. Here, its values for latitude 15° (both in the case of independent consideration of the N and S hemispheres and for the entire Sun) are higher than the values obtained for the approximating exponent (the horizontal lines in the figures). For the data taken for the entire Sun, we have R(Gmax, T15min0) = +0.74.

As in the previous case, the correlation (Fig. 3) between the cycle amplitude Gmax(n) and intervals T25min1(n), T20min1(n), T15min1(n), and T10min1(n) depends on the latitude and is significant only for latitudes 15° and 20°. Its values for latitude 15° nearly reach the correlation coefficients calculated for the approximating exponent (horizontal lines in the figures). For the data taken for the entire Sun, we have R(Gmax, T20min1) = –0.78.

Previously we found a latitudinal analog of the Waldmeier rule in which the LPRP rather than the cycle minimum was taken as the beginning of the cycle reference point (Ivanov, Miletsky, 2014). For this analog of the rule, the correlation between the amplitude of the cycle and the length of its growth branch calculated from the LPRP was noticeably stronger than for the traditional Waldmeier rule.

The correlation (Fig. 4) between the cycle amplitude Gmax(n) and the lengths of intervals T25max(n), T20max(n), T15max(n), and T10max(n) depends on the latitude and is the most significant for latitudes 15°

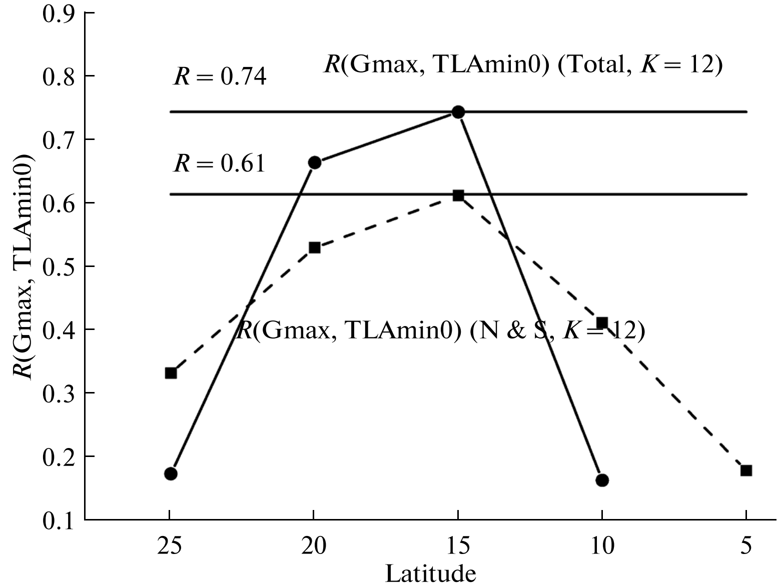

Fig. 2. Latitude dependence of the correlation between the cycle amplitude Gmax(n) and lengths of intervals T25min0(n), T20min0(n), T15min0(n), T10min0(n), and T05min0(n). The dashed curve and squares correspond to the independent consideration of the data for the N and S hemispheres, and the solid curve and the circles represent the data taken for the entire Sun.

and 20°. Here, the extrema at latitude 15° (both in the case of independent consideration of the N and S hemispheres and for the entire Sun) even exceed (in absolute value) the correlation coefficients obtained for the calculation of the cycle phase from the LPRP (R(Gmax, T15max) = –0.87.

This indicates that the Waldmeier rule can be generalized. As an interval related to the cycle amplitude, we can select not only the interval representing the difference between the time of maximum and the preceding minimum (or the LPRP) but also an interval that equals the difference between the times of maximum and any latitudinal moment TLA(n) that satisfies condition T20max(n) < TLA(n) < T15max(n).

Thus, in the studied cases, latitudes 15° and 20° are the most representative in the sense of reaching

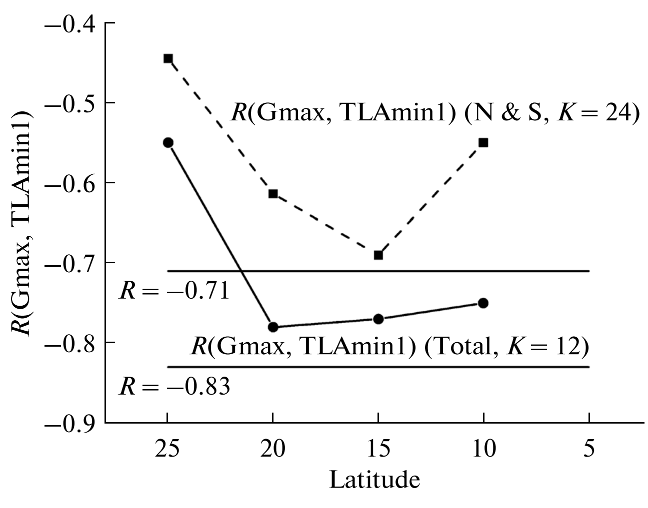

Fig. 3. Latitude dependence of the correlation between the cycle amplitude Gmax(n) and lengths of intervals T25min1(n), T20min1(n), T15min1(n), and T10min1(n). The dashed curve and the squares correspond to the independent consideration of the data for the N and S hemispheres, and the solid curve and the circles designate the data taken for the entire Sun in total.

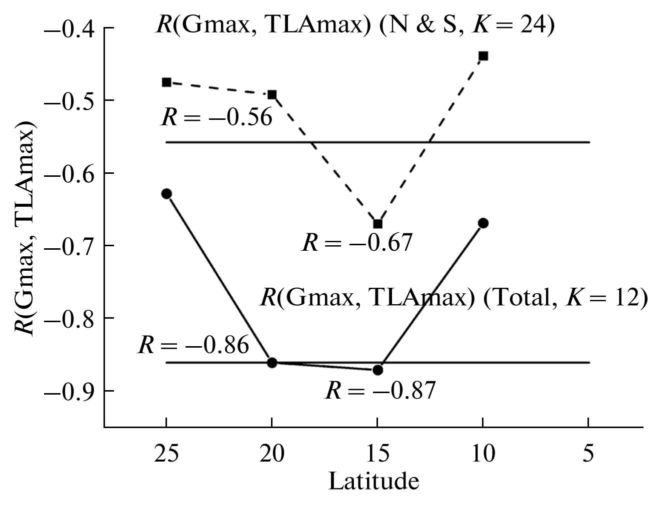

Fig. 4. Latitude dependence of the correlation between the cycle amplitude Gmax(n) and lengths of intervals T25max(n), T20max(n), T15max(n), and T10max(n). The dashed curve and the squares correspond to the independent consideration of the data for the N and S hemispheres, and the solid curve and the circles mark the data taken for the entire Sun.

extrema of correlation. Therefore, to derive the respective equations, it is sufficient to calculate the times at which latitudes 15° and/or 20° are reached by the average latitude curve in each cycle.

Using these results, we obtained (for cycles 12–23) equations of linear regression to determine the cycle amplitude Gmax based on the information on the lengths of time intervals T15min0 and T15min1, as well as T15min0 and T15max. For the data for the entire Sun ($K = 12$), the equation is written as

$$\mathrm{Gmax} = \mathrm{A0} + \mathrm{A1T15min0} - \mathrm{A2T15min1},$$

where A0 = (10.7 ± 9.7), A1 = (2.1 ± 1.4), A2 = (1.4 ± 0.7), ($R = 0.82$, $k = 12$, $SD = 1.9$).

When the N and S hemispheres of the Sun ($K = 24$) are considered independently, the equation has the form

$$\mathrm{Gmax} = \mathrm{B0} + \mathrm{B1T15min0} - \mathrm{B2T15min1},$$

where B0 = (8.2 ± 3.6), B1 = (0.48 ± 0.4), B2 = (0.76 ± 0.31), ($R = 0.71$, $k = 24$, $SD = 1.2$).

In a similar way, for T15min0 and T15max and for the entire Sun ($K = 12$), we obtain

$$\mathrm{Gmax} = \mathrm{C0} + \mathrm{C1T15min0} - \mathrm{C2T15max},$$

where C0 = (2.6 ± 4.8), C1 = (1.8 ± 1.1), C2 = (1.6 ± 0.5), ($R = 0.88$, $k = 12$, $SD = 1.5$).

When the N and S hemispheres ($K = 24$) are considered independently, we have

$$\mathrm{Gmax} = \mathrm{D0} + \mathrm{D1T15min0} - \mathrm{D2T15max},$$

where D0 = (2.0 ± 1.5), D1 = (0.78 ± 0.32), D2 = (0.68 ± 0.23), ($R = 0.74$, $k = 24$, $SD = 1.2$).

For the latitude bands of 25°, 20°, 15°, 10°, and 5°, we calculated the correlation coefficients for the amplitude in the 11-year cycle Gmax(n) with the values of the sunspot group number index GT25(n), GT20(n), GT15(n), GT10(n), and GT5(n) at these latitudes.

The correlation is significant (Fig. 5) at the average and high latitudes (15°–25°) and is maximal at latitude 15°. The correlation is insignificant at low latitudes (5°–10°). It agrees with the statement (Ivanov and Miletsky, 2014) that sunspot activity is not related to the cycle amplitude in the declining phase of the 11-year cycle (for latitudes <12°) and depends only on the current value of the average latitude.

We checked the hypothesis on the possible relation between the activity level at low latitudes of the nth 11-year cycle with the amplitude of the subsequent ($n + 1$)-th one. To do this, we calculated the "hemisphere" values of the sunspot group number index G6N, G6S, G7N, G7S, ... G10N, G10S at the times T6N, T6S, T7N, T7S, ... T10N, and T10S. We computed the correlation coefficients between the total values (taken in the nth cycle) G6N + G6S, ..., G10N+G10S and the amplitude of the next cycle Gmax($n + 1$); the correlation (Fig. 6) was insignificant (and small) at the latitudes 9°–10° but high at the latitudes 6°–8°.

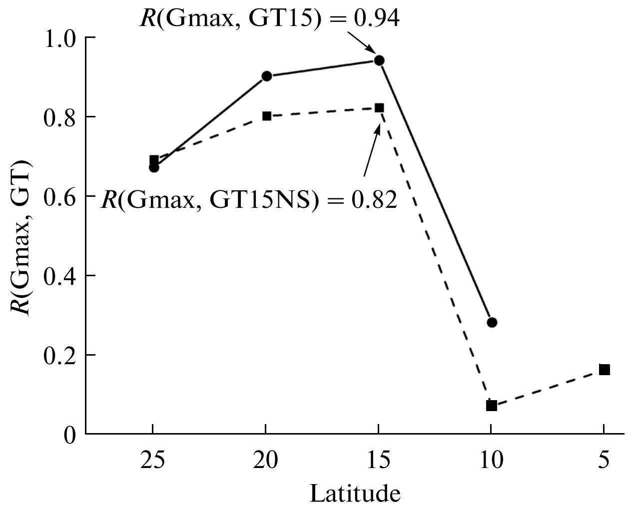

Fig. 5. Latitude dependence of the correlation between the cycle amplitude Gmax(n) and values GT25(n), GT20(n), GT15(n), GT10(n), and GT5(n). The dashed curve and the squares correspond to the independent consideration of the data for the N and S hemispheres, and the solid curve and the circles denote the data taken for the entire Sun.

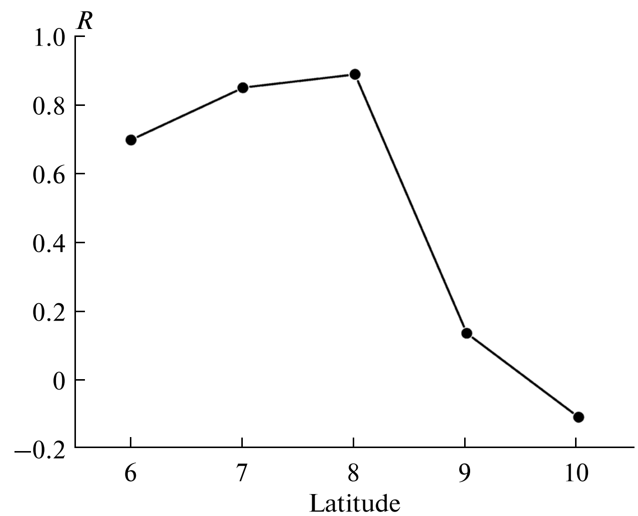

Fig. 6. Latitude dependence of the correlation between the ($n + 1$)-th cycle amplitude Gmax($n + 1$) and values of the nth cycle G6N(n) + G6S(n), ..., G10N(n) + G10S(n).

For the latitude 8°, where the correlation was maximal ($R = 0.88$), we composed a regression equation Gmax($n + 1$) = 1.24+8.07(G8N + G8S) and used it to calculate the 24th cycle amplitude GSNmax(24) = 91.

This result agrees with the conclusions made on the basis of the magnetic flux transport model by Jiang et al. (2014), where on the example of two model latitudes (8° and 18°) was shown that if the parameters are equal, the bipolar magnetic regions (BMRs) appearing at low (8°) latitudes contribute substantially more to the amplitude of the current magnetic axial dipole than to the average (18°) latitude BMRs. Hence, they are likely to correlate more strongly with the amplitude of the next sunspot cycle.

4. CONCLUSIONS

It was shown for the range of latitudes 25°, 20°, 15°, 10°, and 5° that the correlations between the amplitude of the 11-year cycle and the intervals that determine the location of the average latitude curve on the time axis relative to the cycle maximum and minimums depend on latitude. In terms of reaching the correlation extrema, latitudes 15° and 20° prove to be the most representative.

Regression equations were composed for cycles 12–23, which made possible to calculate the amplitude of the 11-year cycle based on information on the lengths of intervals corresponding to the latitude of 15°.

It was established that the correlation between the amplitude of the 11-year cycle and the latitude values of the sunspot group number index is significant at the average and high latitudes (15°–25°) and is small at the low latitudes (5°–10°). This conforms to the statement that the sunspot activity level is not related to the cycle amplitude at the declining phase of the 11-year cycle (for the latitudes <12°) and depends only on the current value of the average latitude.

The hypothesis on the relation between the activity level at low latitudes of the current 11-year cycle and the amplitude of the subsequent one was confirmed.

The biggest correlation was the one for the amplitude of the $(n + 1)$-th cycle $\mathrm{Gmax}(n + 1)$ with values $\mathrm{G8N}(n) + \mathrm{G8S}(n)$ of the nth cycle for latitude ±8°. It was used as a basis to calculate the amplitude of the 24th cycle $\mathrm{GSNmax}(24) = 91$, which is a satisfactory estimate in accordance with the currently available information.

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REFERENCES

Hathaway, D.H., A standard law for the equatorward drift of the sunspot zones, *Solar Physics.*, 2011, vol. 273, pp. 221–230.

Ivanov, V.G. and Miletsky, E.V., Sporer's law and relationship between the latitude and amplitude parameters of solar activity, *Geomagn. and Aeronomy*, 2014, vol. 54, no. 7, pp. 907–914.

Jiang, J., Hathaway, D.H., Cameron, R.H., Solanki, S.K., Gizon, L., and Upton, L., Magnetic flux transport at the solar surface, *Space Science Reviews*, 2014, vol. 186, nos. 1–4, pp. 491–523.

Miletsky, E.V. and Ivanov, V.G., Interaction between the amplitude and length of the 11-year sunspot cycle, *Geomagn. and Aeronomy*, 2014, vol. 54, no. 7, pp. 1000–1005.

Roshchina, E.M. and Sarychev, A.P., Sporer's law and the rhythm of sunspot cycles, *Solar System Research*, 2011, vol. 45, pp. 365–371.

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