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Nucleon and Nuclear Structure-dependent Corrections to precision tests at low energy: Dispersing Uncertainties

STANDARD MODEL

OF ELEMENTARY PARTICLES AND FUNDAMENTAL INTERACTIONS

FERMIONS
matter constituents
spin = 1/2, 3/2, 5/2, ...

Leptons spin =1/2

Flavor	Mass GeV/c^2	Electric charge
ν_L lightest neutrino*	$(0-0.13)\times10^{-9}$	0
e electron	0.000511	–1
ν_M middle neutrino*	$(0.009-0.13)\times10^{-9}$	0
μ muon	0.106	–1
ν_H heaviest neutrino*	$(0.04-0.14)\times10^{-9}$	0
τ tau	1.777	–1

Quarks spin =1/2

Flavor	Approx. Mass GeV/c^2	Electric charge
u up	0.002	2/3
d down	0.005	–1/3
c charm	1.3	2/3
s strange	0.1	–1/3
t top	173	2/3
b bottom	4.2	–1/3

BOSONS
force carriers
spin = 0, 1, 2, ...

Unified Electroweak spin = 1

Name	Mass GeV/c^2	Electric charge
γ photon	0	0
W^-	80.39	–1
W^+ (W bosons)	80.39	+1
Z^0 Z boson	91.188	0

Strong (color) spin =1

Name	Mass GeV/c^2	Electric charge
g gluon	0	0

Higgs Boson spin = 0

Name	Mass GeV/c^2	Electric Charge
H^0, H^+	≈ 126	$0, +1$

Properties of the Interactions

The strengths of the interactions (forces) are shown relative to the strength of the electromagnetic force for two u quarks separated by the specified distances.

Property	Gravitational Interaction	Weak Interaction	Electromagnetic Interaction	Strong Interaction
		(Electroweak)		
Acts on:	Mass – Energy	Flavor	Electric Charge	Color Charge
Particles experiencing:	All	Quarks, Leptons	Electrically Charged	Quarks, Gluons
Particles mediating:	Graviton (not yet observed)	W^+ W^- Z^0	γ	Gluons
Strength at 10^{-18} m	10^{-41}	0.8	1	25
Strength at 3×10^{-17} m	10^{-41}	10^{-4}	1	60

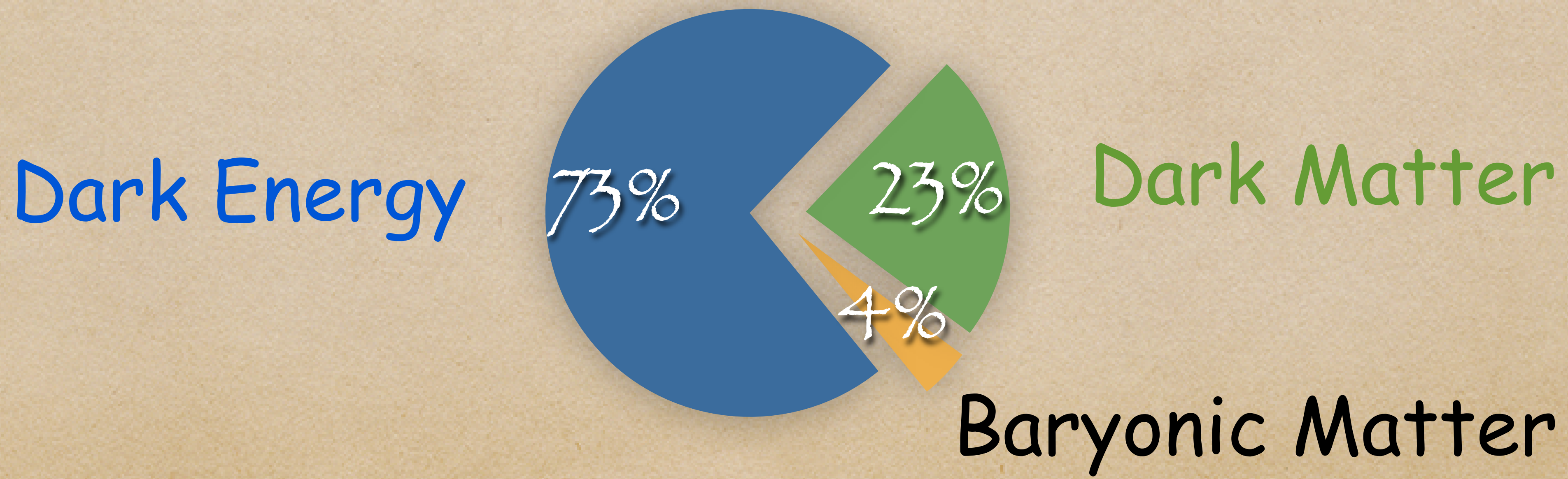
IT'S NOT THE END OF THE STORY...

Standard Model is incomplete:

- SM turns out to be extremely fine-tuned (hierarchy problem)

- CP violation and matter-antimatter asymmetry (SM is symmetric w.r.t. matter-antimatter but the Universe is not)

- Cosmology: known matter is only small part of the Universe!

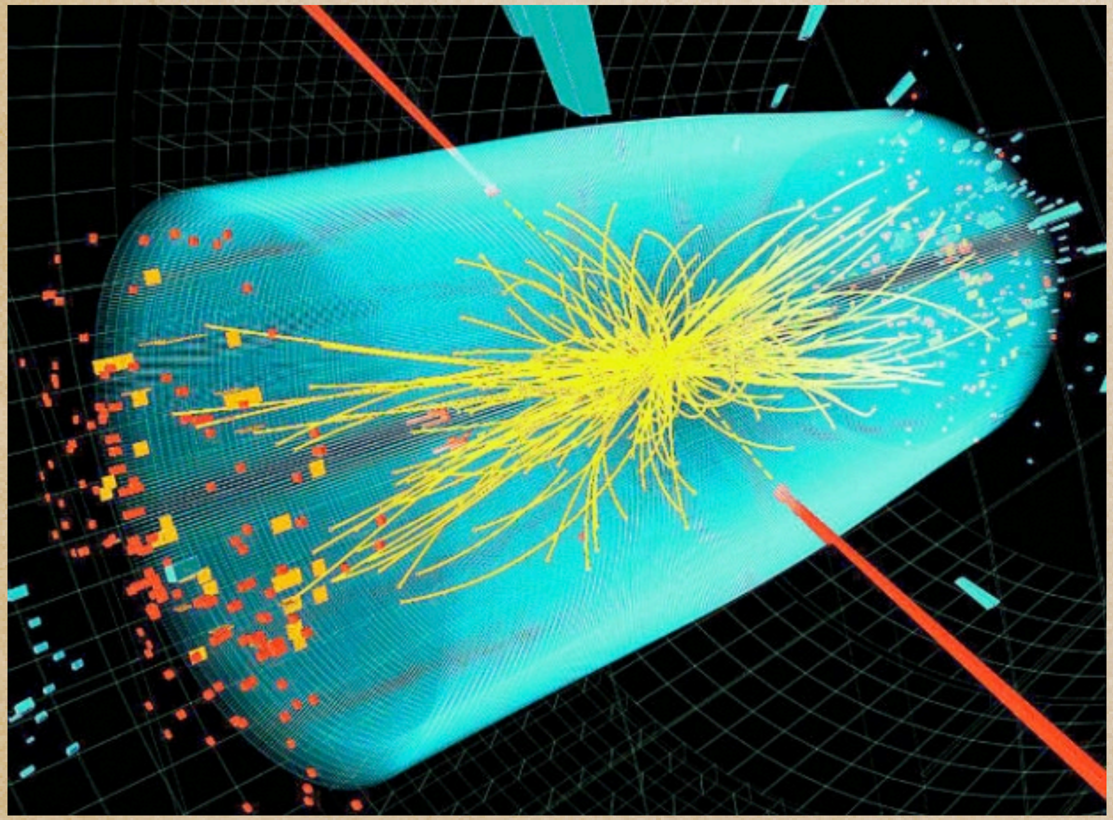


SEARCHES FOR NEW PHYSICS

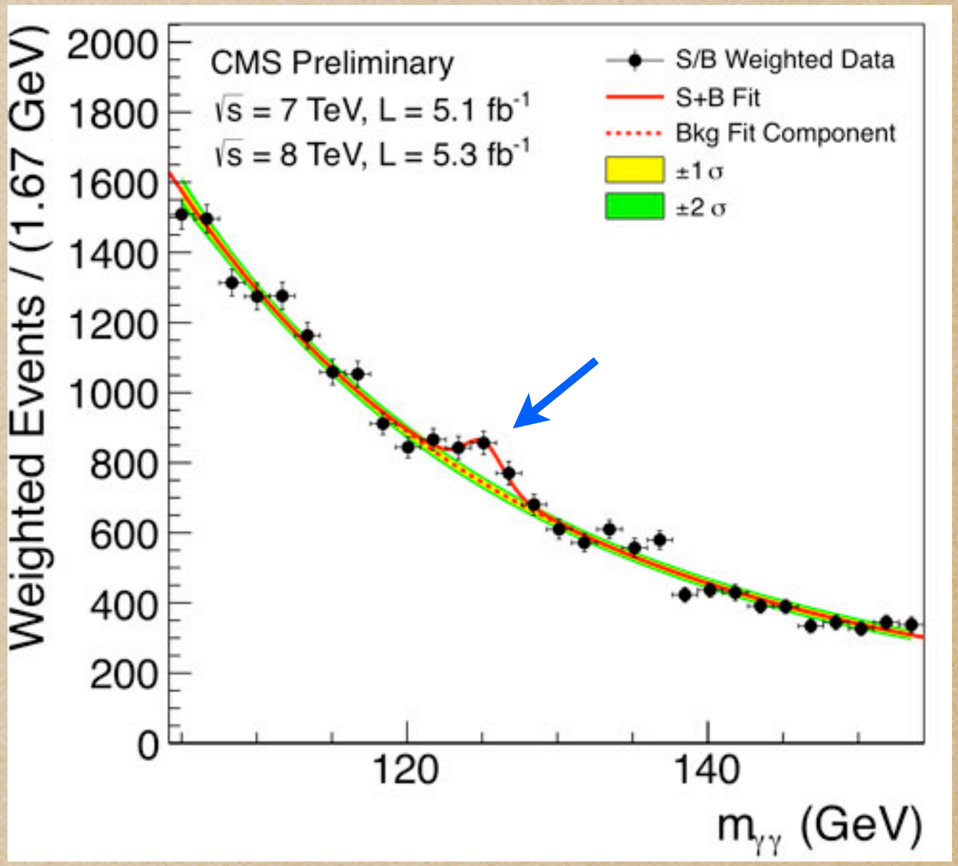
- Collider searches:

  accelerate known particles to produce heavy new particles

A typical p-p event in the CMS detector @ LHC



Higgs boson with mass ~ 126 GeV observed



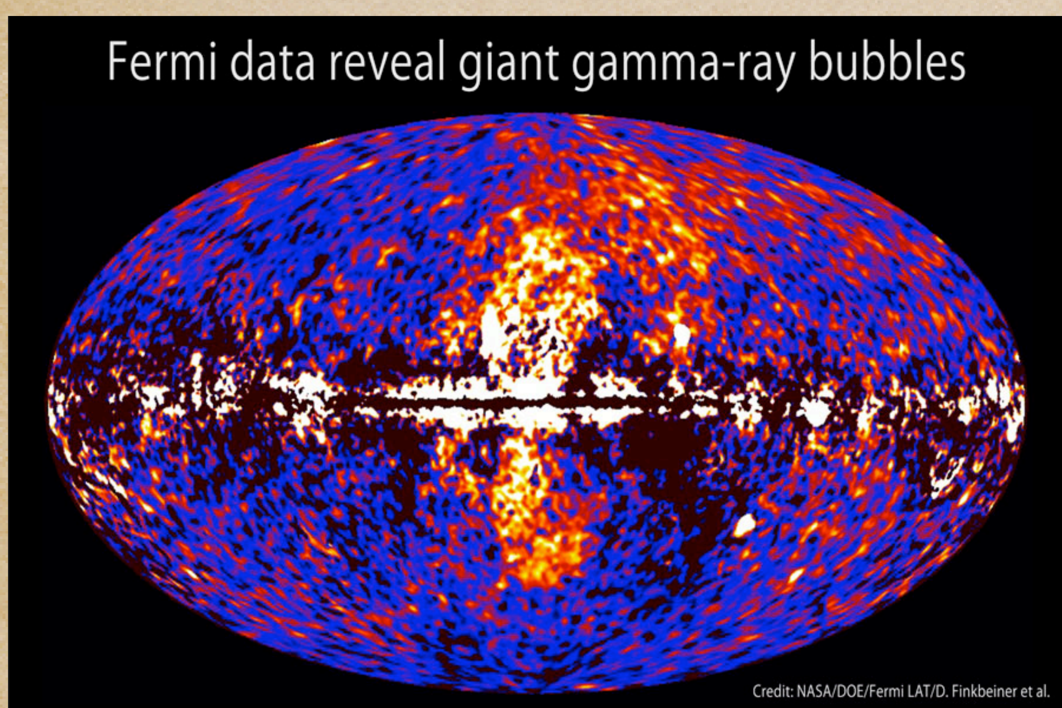
No new particles (few 100's GeV) observed yet - quest continues

SEARCHES FOR NEW PHYSICS

- Astrophysics searches:

  observe signals of new particles coming from the space

Fermi
Gamma-ray
Space Telescope



No new particles observed yet

< 300 GeV

- quest continues

SEARCHES FOR NEW PHYSICS

Low energy tests:

deviations from SM predictions

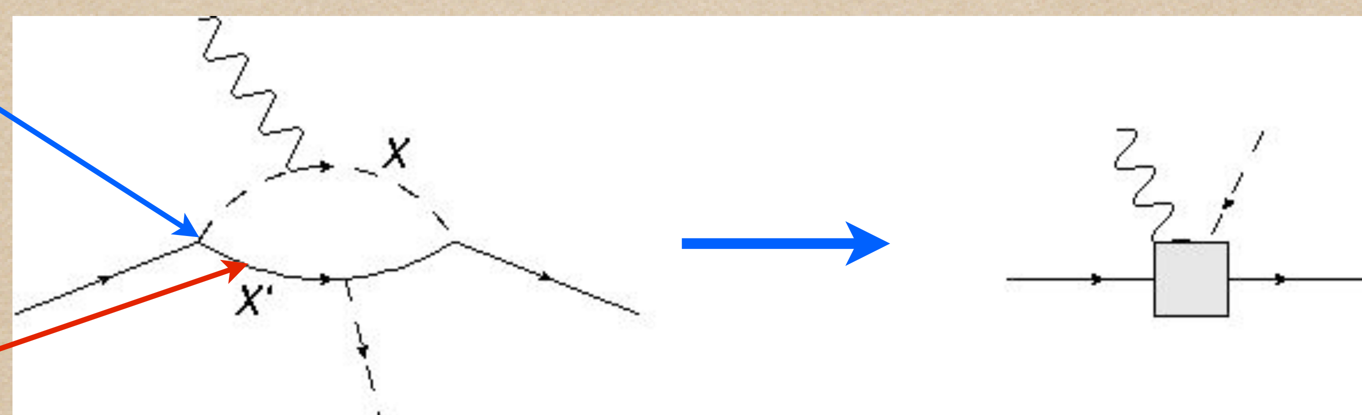
set constraint onto quantum fluctuations due to unknown heavy particles

?

Theory (SM)

Coupling g

Mass Λ



$$\sim \frac{g^2}{\Lambda^2}$$

Precision = scale of New Physics

Precision in BOTH experiment and SM theory becomes crucial

LOW ENERGY PRECISION TESTS

Parity-Violating Electron Scattering

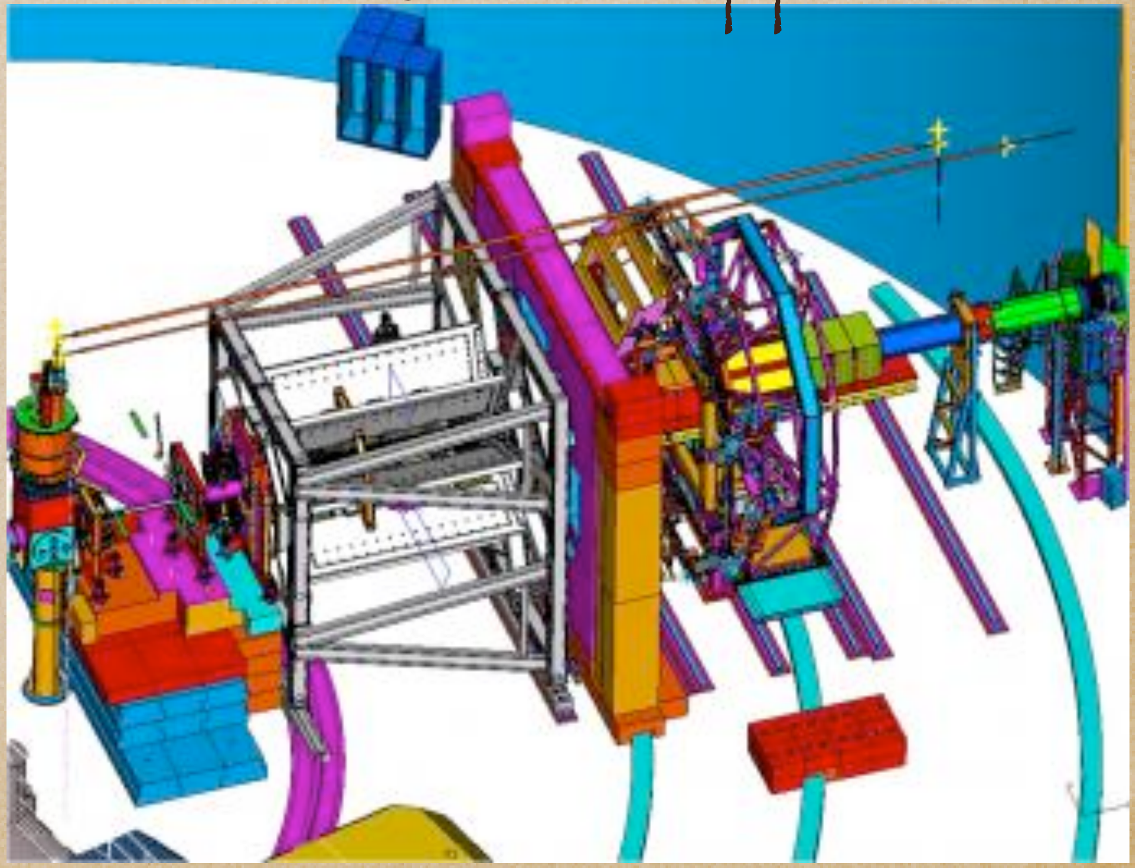
QWEAK, PVDIS, HAPPEX, PREX, G0 @ JLab; A4 @ Mainz; SAMPLE @ MIT-Bates; E158 @ SLAC;

Future: QWEAK @ Mainz, PVDIS (Hall C), Møller @ JLab

Measurements of the PV asymmetry in elastic electron scattering to extract:
- weak mixing angle;
- nucleon's strange FFs;

Probe the scale of New Physics
- few TeV

QWEAK apparatus

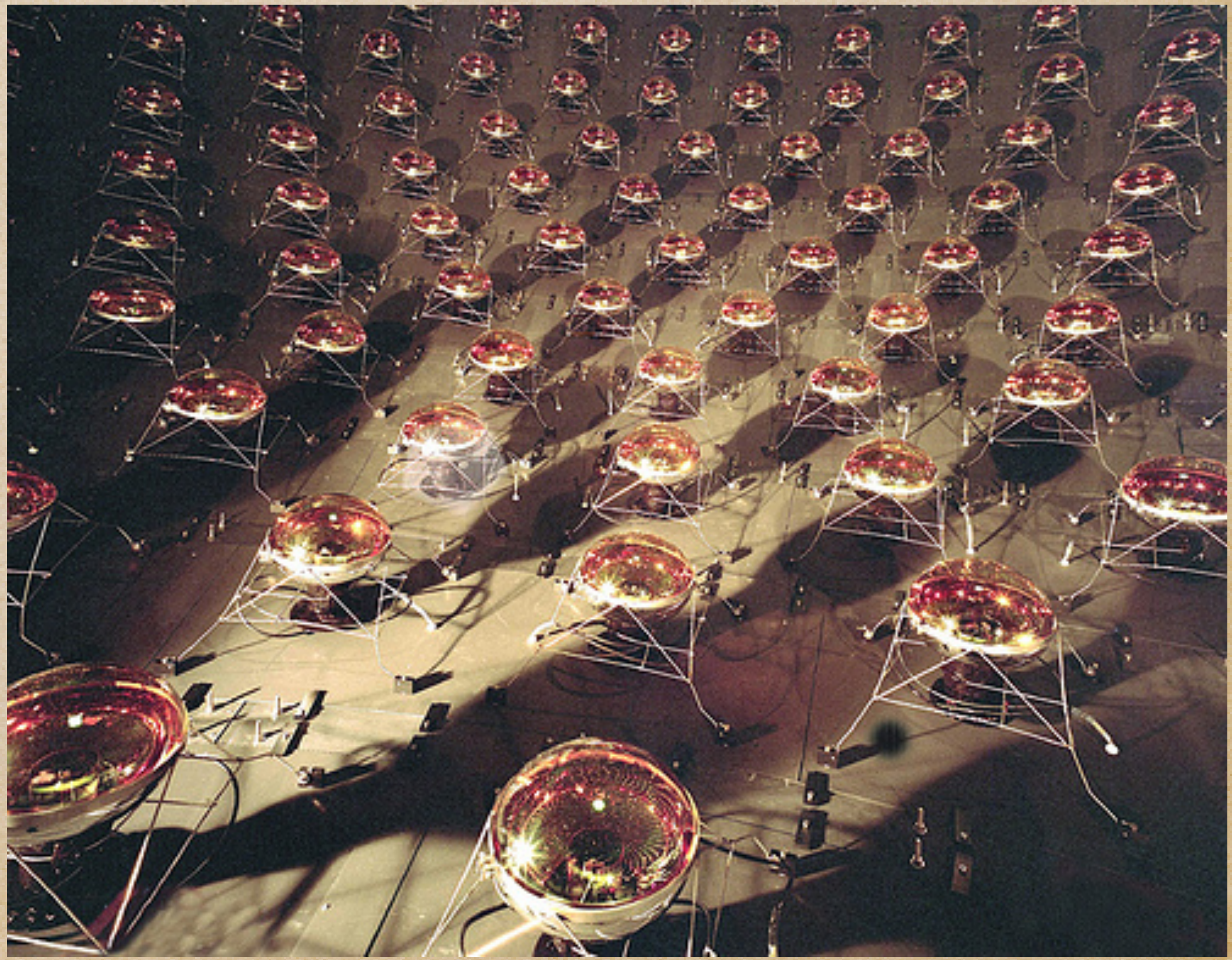


LOW ENERGY PRECISION TESTS

Neutrino Oscillation Experiments

NuTeV, LSND, Daya Bay, MiniBooNE, SciBooNE, NOvA, SNO, Super-K, OPERA, NEMO, MINOS, Ice Cube, Borexino, ANTARES, Double Chooz

MiniBooNE detector



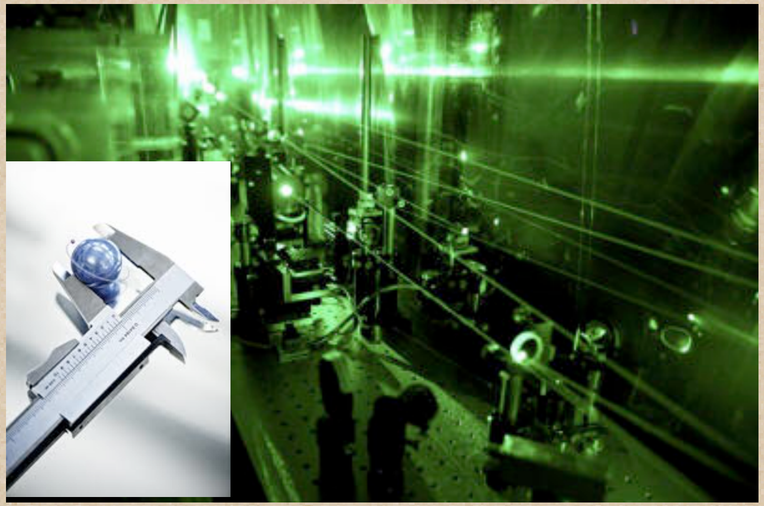
Measurements of neutrino oscillation parameters:

- masses;
- neutrino mixing angles;
- Dirac vs. Majorana neutrino

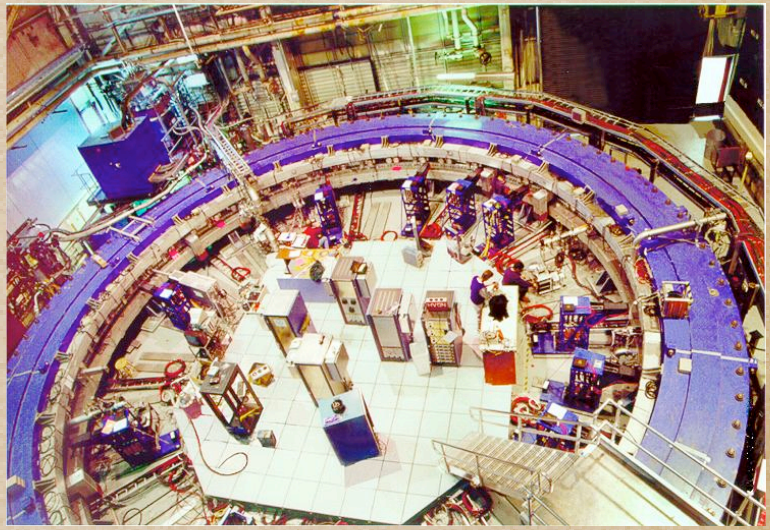
LOW ENERGY PRECISION TESTS

Precision tests of QED

Lamb shift in muonic atoms @ PSI

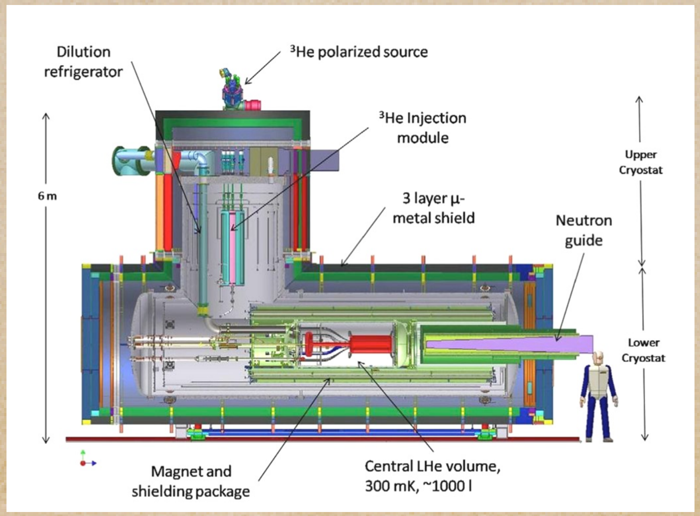


Muonic g-2 @ BNL, Fermilab



New Physics at Low Energies

nEDM, eEDM - @ PSI, ORNL, ILL,...



Dark Photon Search @ JLab, Mainz, ...

THEORY SUPPORT TO LOW ENERGY TESTS

Leave to the experimentalists gauging the scale

Theorists: gauge the weights!

- Precision calculations of observables in the kinematics of atomic, nuclear and hadronic experiments are needed!

- Appropriate model-independent methods for strong interacting systems with reliable error estimation

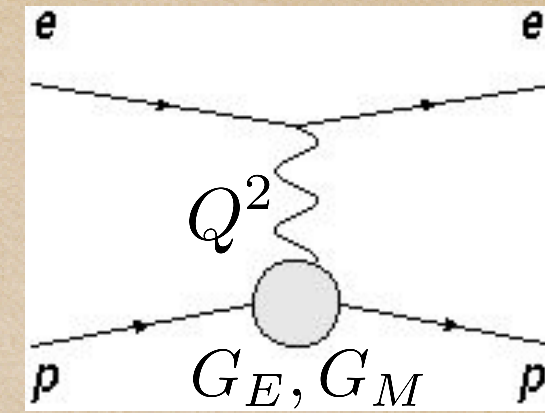
- Requires understanding QCD in the non-perturbative regime

Proton Radius Puzzle

Proton Charge Radius from Electron Scattering

Elastic e-p scattering

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right) = \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_{\mathrm{Mott}} \frac{\varepsilon G_E^2 + \tau G_M^2}{\varepsilon\left(1+\tau\right)}$$



Charge/magnetic radii

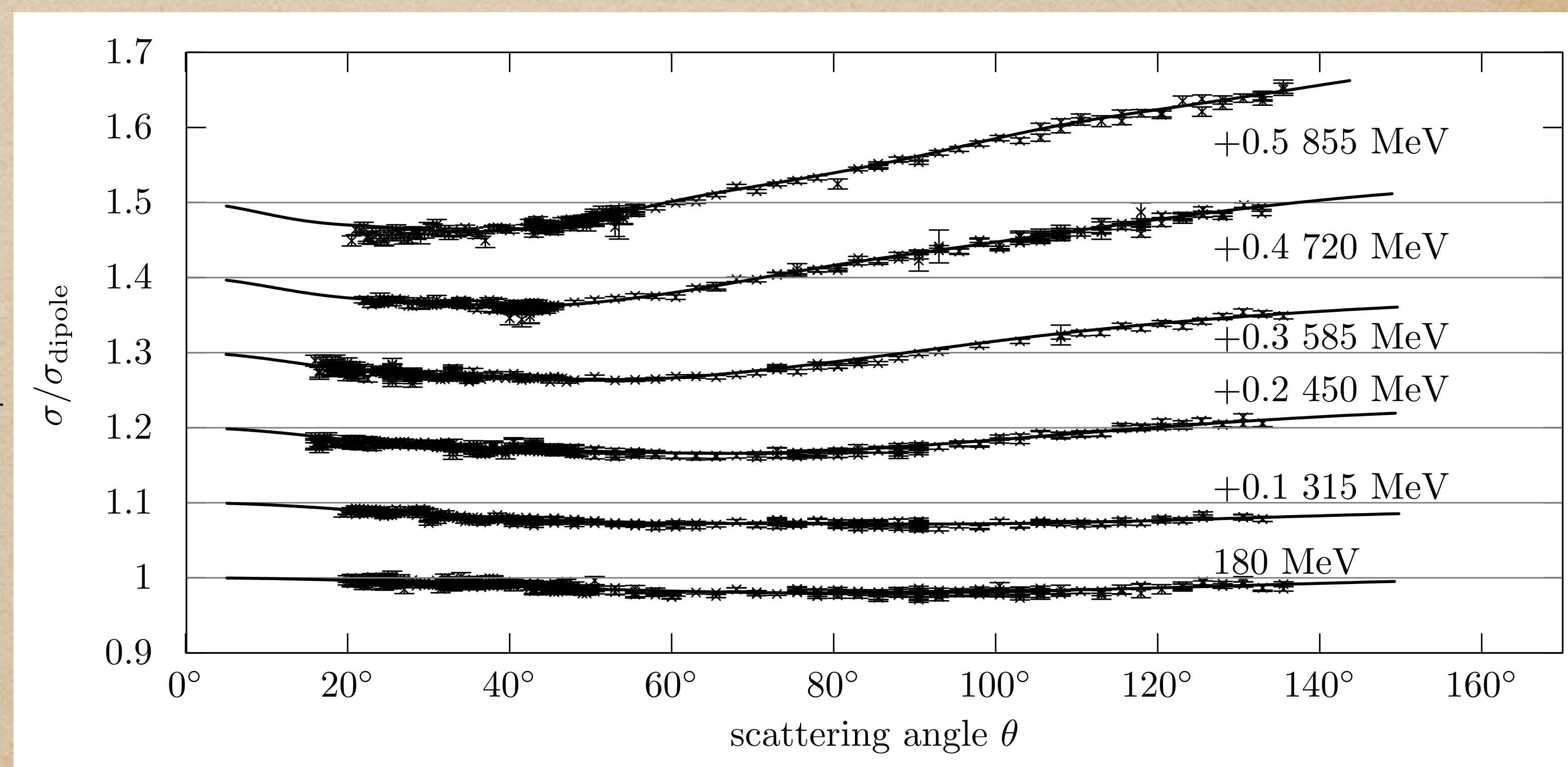
$$\left\langle r_{E/M}^2 \right\rangle = -\frac{6\hbar^2}{G_{E/M}\left(0\right)} \left.\frac{\mathrm{d}G_{E/M}\left(Q^2\right)}{\mathrm{d}Q^2}\right|_{Q^2=0}$$

Precise

measurement

at Mainz

$$r_E^p = 0.8791(79)\ \mathrm{fm}$$

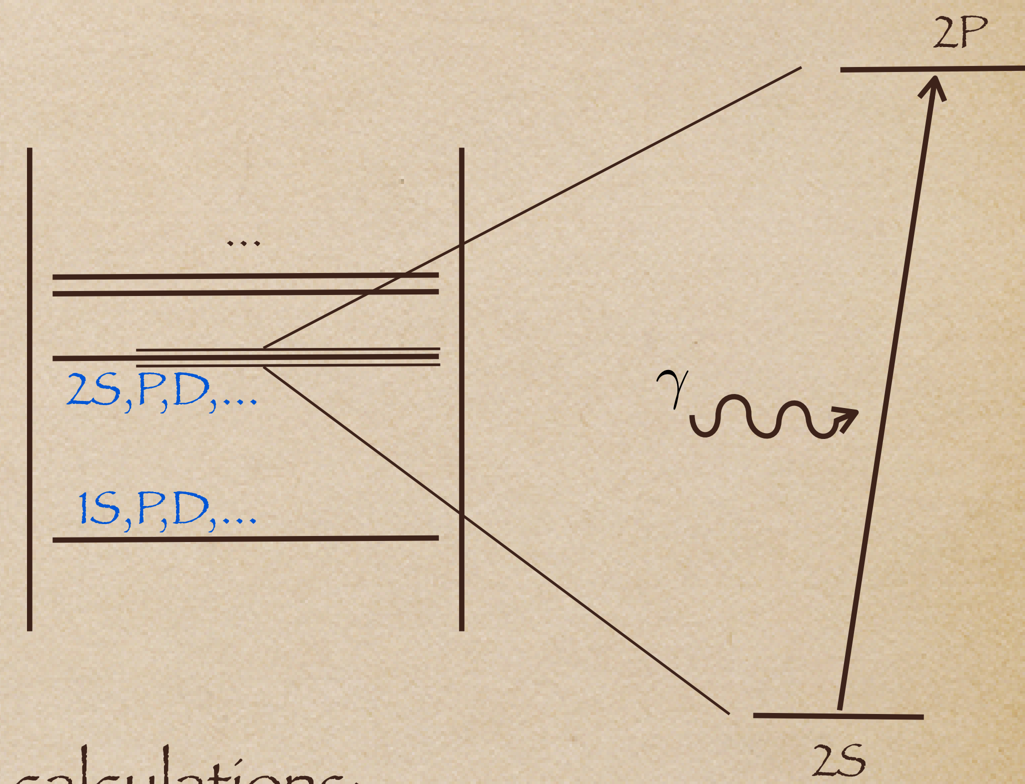


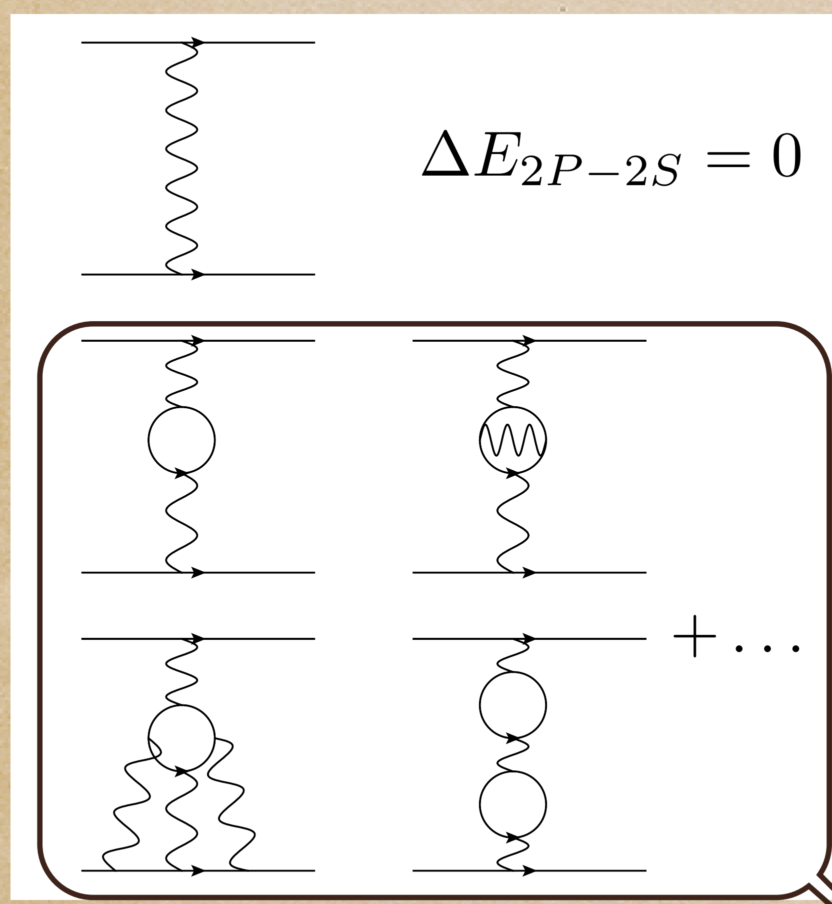
Bernauer et al [A1 Coll.], PRL 105 (2010) 242001

Proton Radius from Lamb Shift in Hydrogen Atom

Leading order: degenerate spectrum

Radiative corrections: fine structure





Modern QED calculations:

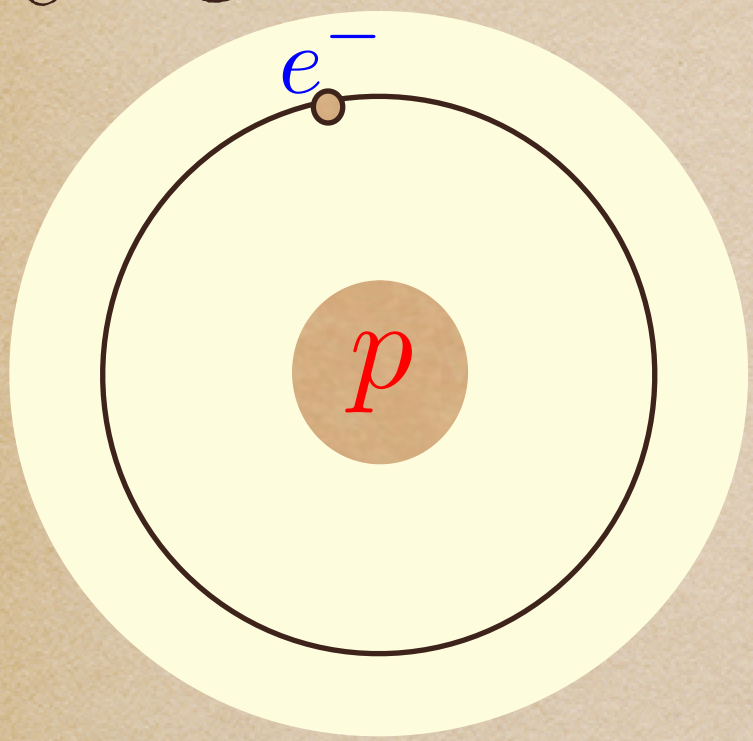
Borie, Annals Phys. 327 (2012) 733;
Eides et al., Phys.Rept. 342 (2001) 63;
Indelicato, arXiv:1210.5828, ...

Finite size correction

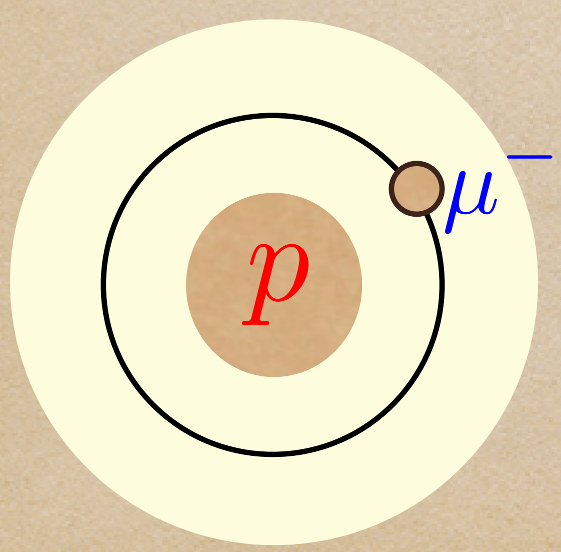
$$\Delta E_{nP-nS} = \Delta E^{QED}_{nP-nS} - \boxed{\frac{2(Z\alpha)^4}{3n^3} m_r^3\, r_E^2} + \mathcal{O}(\alpha_{em}^5)$$

Muonic vs Electronic Hydrogen

Hydrogen atom



muonic Hydrogen atom



SM: the only difference is the mass

Bohr radius $\frac{R_{\mu-H}}{R_{e-H}} = \frac{m_e}{m_\mu} \approx \frac{1}{200}$

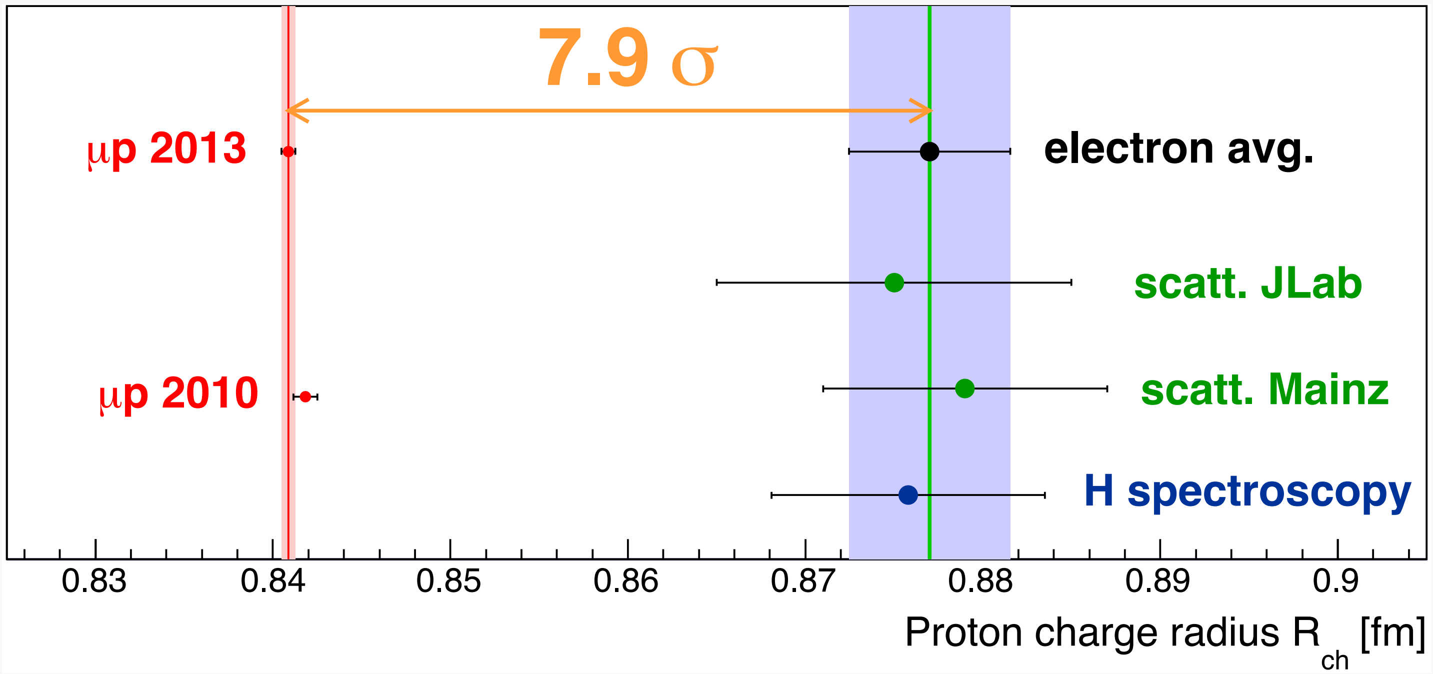
$$\Delta E^{FS,\, e-H}_{2P_{3/2}-2S_{1/2}} = -8.1 \times 10^{-7}\, r_E^2\, (\mathrm{meV})$$

$$\Delta E^{FS,\, \mu-H}_{2P_{3/2}-2S_{1/2}} = -5.2275(10)\, r_E^2\, (\mathrm{meV})$$

The proton radii puzzle

3 ways to the proton radius

e-p scattering
H precision laser spectroscopy
μp laser spectroscopy



Pohl *et al.*, Nature 466, 213 (2010)
Antognini *et al.*, Science 339, 417 (2013)

μ-H Lamb shift: pure QED corrections

#	Contribution	Pachucki [11, 12]	Nature [14]	Borie-v6 [77]	Indelicato [78]	Our choice	Ref.
1	NR one-loop electron VP (eVP)	205.0074					
2	Rel. corr. (Breit-Pauli)	0.0169[a]					
3	Rel. one-loop eVP		205.0282	205.0282	205.02821	205.02821	[78] Eq.(54)
19	Rel. RC to eVP, $\alpha(Z\alpha)^4$	(incl. in #2)[b]	-0.0041	-0.0041		-0.00208[c]	[75, 76]
4	Two-loop eVP (Källén-Sabry)	1.5079	1.5081	1.5081	1.50810	1.50810	[78] Eq.(57)
5	One-loop eVP in 2-Coulomb lines $\alpha^2(Z\alpha)^5$	0.1509	0.1509	0.1507	0.15102	0.15102	[78] Eq.(60)
7	eVP corr. to Källén-Sabry	0.0023	0.00223	0.00223	0.00215	0.00215	[78] Eq.(62), [85]
6	NR three-loop eVP	0.0053	0.00529	0.00529		0.00529	[85, 86]
9	Wichmann-Kroll, "1:3" LBL		-0.00103	-0.00102	-0.00102	-0.00102	[78] Eq.(64), [87]
10	Virtual Delbrück, "2:2" LBL		0.00135	0.00115		0.00115	[72, 87]
new	"3:1" LBL			-0.00102		-0.00102	[87]
20	μSE and μVP	-0.6677	-0.66770	-0.66788	-0.66761	-0.66761	[78] Eqs.(72)+(76)
11	Muon SE corr. to eVP $\alpha^2(Z\alpha)^4$	$-0.005(1)$	-0.00500	-0.004924 [d]		-0.00254	[83] Eq.(29a) [e]
12	eVP loop in self-energy $\alpha^2(Z\alpha)^4$	-0.001	-0.00150			[f]	[72, 88–90]
21	Higher-order corr. to μSE and μVP		-0.00169	-0.00171 [g]		-0.00171	[84] Eq.(177)
13	Mixed eVP+μVP		0.00007	0.00007		0.00007	[72]
new	eVP and μVP in two Coulomb lines				0.00005	0.00005	[78] Eq.(78)
14	Hadronic VP $\alpha(Z\alpha)^4 m_r$	0.0113(3)	0.01077(38)	0.011(1)		0.01121(44)	[91–93]
15	Hadronic VP $\alpha(Z\alpha)^5 m_r$		0.000047			0.000047	[92, 93]
16	Rad corr. to hadronic VP		-0.000015			-0.000015	[92, 93]
17	Recoil corr.	0.0575	0.05750	0.0575	0.05747	0.05747	[78] Eq.(88)
22	Rel. RC $(Z\alpha)^5$	-0.045	-0.04497	-0.04497	-0.04497	-0.04497	[78] Eq.(88), [72]
23	Rel. RC $(Z\alpha)^6$	0.0003	0.00030		0.0002475	0.0002475	[78] Eq.(86)+Tab.II
new	Rad. (only eVP) RC $\alpha(Z\alpha)^5$					0.000136	[83] Eq.(64a)
24	Rad. RC $\alpha(Z\alpha)^n$ (proton SE)	-0.0099	-0.00960	-0.0100		$-0.01080(100)$	[43][h] [72]
	Sum	206.0312	206.02915	206.02862		206.03339(109)	

μ-H Lamb shift: finite size + structure corrections

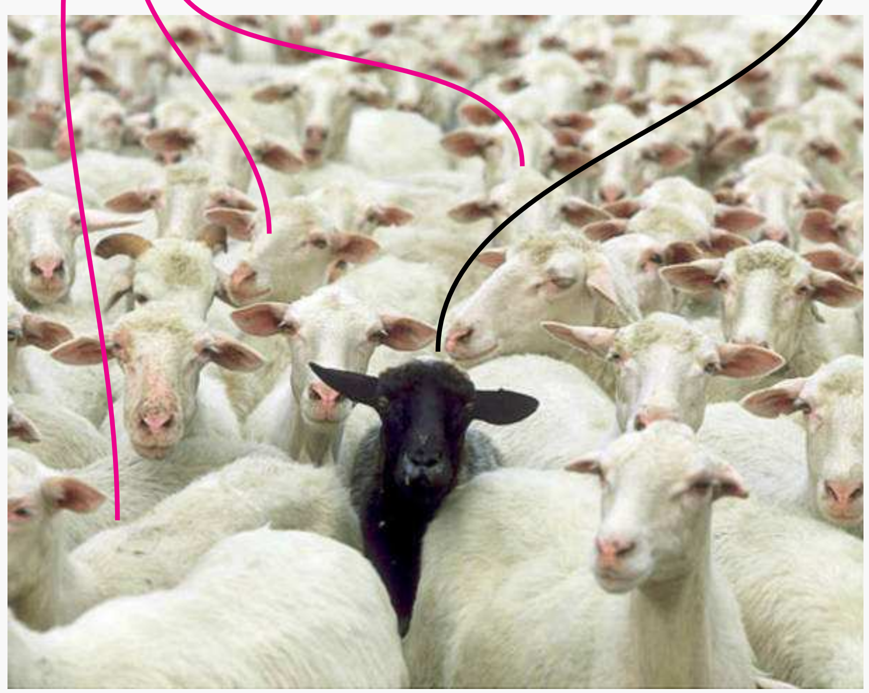
#	Contribution	Borie-v6 [77]	Karshenboim [76]	Pachucki [11, 12]	Indelicato [78]	Carroll [82]	Our choice
	Non-rel. finite-size	-5.1973$\langle r^2 \rangle$	-5.1975$\langle r^2 \rangle$	-5.1975$\langle r^2 \rangle$			
	Rel. corr. to non-rel. finite size	-0.0018$\langle r^2 \rangle$		-0.0009 meV[a]			
	Rel. finite-size						
	exponential				-5.1994$\langle r^2 \rangle$	-5.2001$\langle r^2 \rangle$	-5.1994$\langle r^2 \rangle$
	Yukawa					-5.2000$\langle r^2 \rangle$	
	Gaussian					-5.2001$\langle r^2 \rangle$	
	Finite size corr. to one-loop eVP	-0.0110$\langle r^2 \rangle$	-0.0110$\langle r^2 \rangle$	-0.010$\langle r^2 \rangle$	-0.0282$\langle r^2 \rangle$		-0.0282$\langle r^2 \rangle$
	Finite size to one-loop eVP-it.	-0.0165$\langle r^2 \rangle$	-0.0170$\langle r^2 \rangle$	-0.017$\langle r^2 \rangle$	(incl. in -0.0282)		
	Finite-size corr. to Källén-Sabry	[b]			-0.0002$\langle r^2 \rangle$		-0.0002$\langle r^2 \rangle$
new	Finite size corr. to μ self-energy	(0.00699) [c]			0.0008$\langle r^2 \rangle$		0.0009(3)$\langle r^2 \rangle$[d]
	ΔE_{TPE} [46]						0.0332(20) meV
	elastic (third Zemach)[e]						
	measured $R^3_{(2)}$	0.0365(18)$\langle r^2 \rangle^{3/2}$					(incl. above)
	exponential			0.0363$\langle r^2 \rangle^{3/2}$	0.0353$\langle r^2 \rangle^{3/2}$ [f]	0.0353$\langle r^2 \rangle^{3/2}$	
	Yukawa					0.0378$\langle r^2 \rangle^{3/2}$	
	Gaussian					0.0323$\langle r^2 \rangle^{3/2}$	
25	inelastic (polarizability)	0.0129(5) meV [99]		0.012(2) meV			(incl. above)
new	Rad. corr. to TPE	-0.00062$\langle r^2 \rangle$					-0.00062$\langle r^2 \rangle$
26	eVP corr. to polarizability						0.00019 meV [93]
27	SE corr. to polarizability						-0.00001 meV [93]
18	Finite-size to rel. recoil corr.	(0.013 meV) [g]		[h]			(incl. in ΔE_{TPE})
	Higher-order finite-size corr.	-0.000123 meV			0.00001(10) meV		0.00001(10) meV
	$2P_{1/2}$ finite-size corr.	-0.0000519$\langle r^2 \rangle$ [i]			(incl. above)	(incl. above)	(incl. above)

Proton radius from muonic hydrogen

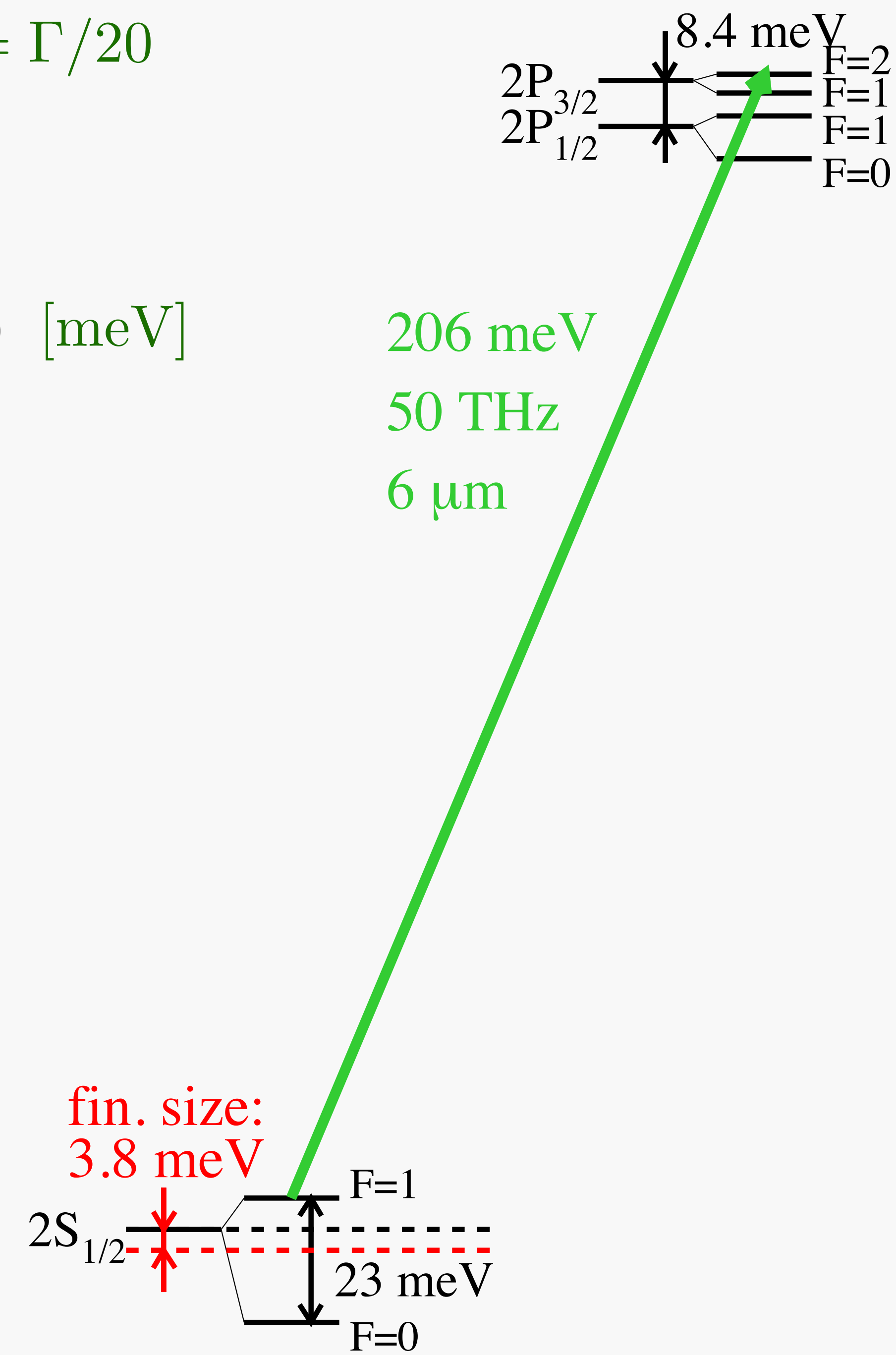
- Measure $\Delta E^{\text{exp}}_{2P-2S}$ in μp with $u_r = 10^{-5} \leftrightarrow 0.5$ GHz $= \Gamma/20$

- Compute theoertical prediction

$$\Delta E^{\text{th}}_{2P-2S} = 206.0336(15) - 5.2275(10)\, r_{\text{p}}^2 + 0.0332(20) \quad [\text{meV}]$$

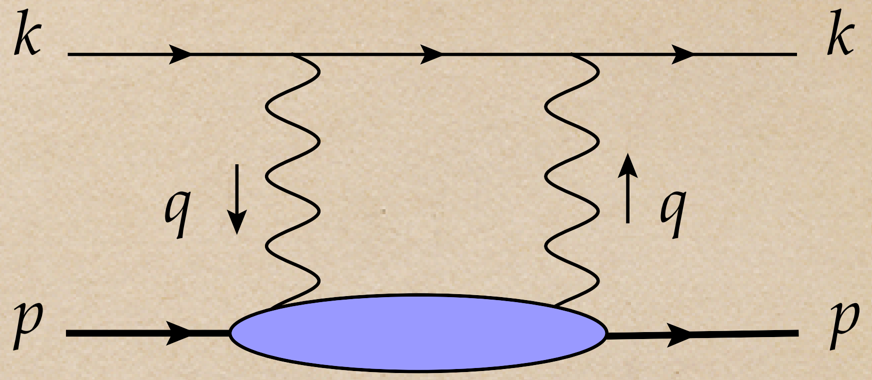


The Lamb shift contributions



Comparing theory with experiment $\Longrightarrow r_{\text{p}}$

Two photon exchange contribution to Lamb shift



Kinematics: 2 loop variables

q^2 and $\nu=(pq)/M$

$$\mathcal{M} = e^4 \int \frac{d^4q}{(2\pi)^4} \frac{1}{q^4} \bar{u}(k) \left[\gamma^\nu \frac{1}{\not{k} - \not{q} - m_l + i\epsilon} \gamma^\mu + \gamma^\mu \frac{1}{\not{k} + \not{q} - m_l + i\epsilon} \gamma^\nu \right] u(k) T_{\mu\nu}$$

Forward virtual Compton amplitude

$$T^{\mu\nu} = \frac{i}{8\pi M} \int d^4x e^{iqx} \langle p | T\, j^\mu(x) j^\nu(0) | p \rangle$$

$$= \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right) T_1(\nu, Q^2) + \frac{1}{M^2} (p - \frac{pq}{q^2} q)^\mu (p - \frac{pq}{q^2} q)^\nu T_2(\nu, Q^2)$$

Lamb shift (nS-nP)

$$\Delta E = -\frac{\alpha^2}{2\pi m_l M_d} \phi_n^2(0) \int d^4q \frac{(q^2 + 2\nu^2) T_1(\nu, q^2) - (q^2 - \nu^2) T_2(\nu, q^2)}{q^4 [(q^2/2m_l)^2 - \nu^2]}$$

Two photon exchange contribution to Lamb shift

T_1, T_2 - the imaginary parts known (Optical theorem)

$$\mathrm{Im}T_1(\nu, Q^2) = \frac{1}{4M} F_1(\nu, Q^2)$$

$$\mathrm{Im}T_2(\nu, Q^2) = \frac{1}{4\nu} F_1(\nu, Q^2)$$

Inelastic structure functions = data

Real parts - from forward dispersion relation

$F_1(\nu \to \infty, q^2) \sim \nu^{1+\epsilon}$ - subtraction needed

$F_2(\nu \to \infty, q^2) \sim \nu^{\epsilon}$ - no subtraction

$$\mathrm{Re}T_1(\nu, Q^2) = \bar{T}_1(0, Q^2) + T_1^{pole}(\nu, Q^2) + \frac{\nu^2}{2\pi M} \int_{\nu_0}^{\infty} \frac{d\nu'}{\nu(\nu'^2 - \nu^2)} F_1(\nu', Q^2)$$

$$\mathrm{Re}T_2(\nu, Q^2) = T_2^{pole}(\nu, Q^2) + \frac{1}{2\pi} \int_{\nu_0}^{\infty} \frac{d\nu'}{\nu'^2 - \nu^2} F_2(\nu', Q^2)$$

Proton Radius Puzzle vs Standard Model

3 contributions

$$\Delta E \;=\; \Delta E^{subt} + \Delta E^{el} + \Delta E^{inel}$$

2 out of 3 are directly fixed by data

"Elastic": elastic proton form factors

$$\Delta E^{el} = -\frac{\alpha^2 m_l}{M(M^2 - m_l^2)} \phi_n^2(0) \int_0^\infty \frac{dQ^2}{Q^2}$$
$$\times \left[\left(\frac{\gamma_2(\tau_p)}{\sqrt{\tau_p}} - \frac{\gamma_2(\tau_l)}{\sqrt{\tau_l}} \right) \frac{G_E^2 + \tau_p G_M^2}{\tau_p(1+\tau_p)} - \left(\frac{\gamma_1(\tau_p)}{\sqrt{\tau_p}} - \frac{\gamma_1(\tau_l)}{\sqrt{\tau_l}} \right) G_M^2 \right]$$

"Inelastic": real and virtual photoabsorption data

$$\Delta E^{inel} = -\frac{2\alpha^2}{m_l M} \phi_n^2(0) \int_0^\infty \frac{dQ^2}{Q^2} \int_{\nu_\pi}^\infty \frac{d\nu}{\nu}$$
$$\times \left[\tilde{\gamma}_1(\tau, \tau_l) F_1(\nu, Q^2) + \frac{M\nu}{Q^2} \tilde{\gamma}_2(\tau, \tau_l) F_2(\nu, Q^2) \right]$$

Proton Radius Puzzle vs Standard Model

3 groups evaluated the integrals - agreement

$$\begin{aligned} \text{Pachucki 1996}: &\quad \Delta E^{el} = -27.8\,\mu\text{eV},\ \Delta E^{inel} = -13.9\,\mu\text{eV} \\ \text{Vanderhaeghen\&Carlson 2011}: &\quad \Delta E^{el} = -29.5(1.3)\,\mu\text{eV},\ \Delta E^{inel} = -12.7(5)\,\mu\text{eV} \\ \text{MG, Llanes-Estrada, Szczepaniak 2013}: &\quad \Delta E^{el} = -30.1(1.2)\,\mu\text{eV},\ \Delta E^{inel} = -13.0(6)\,\mu\text{eV} \end{aligned}$$

Looking for 300 μeV

"Subtraction function" - generally unknown

$$\Delta E^{subt} = \frac{\alpha}{m_l}\phi_n^2(0)\int_0^\infty \frac{dQ^2}{Q^2}\frac{\gamma_1(\tau_l)}{\sqrt{\tau_l}}T_1(0,Q^2)$$

Usually, identify $\bar{T}_1(0,Q^2) = \frac{Q^2}{e^2}\beta_M F_\beta(Q^2)$

$\beta_M = 3.5(6)10^{-4}\ \text{fm}^3$

Not fully model-independent

Can this subtraction function resolve the discrepancy?

Finite Energy Sum Rule for subtraction constant

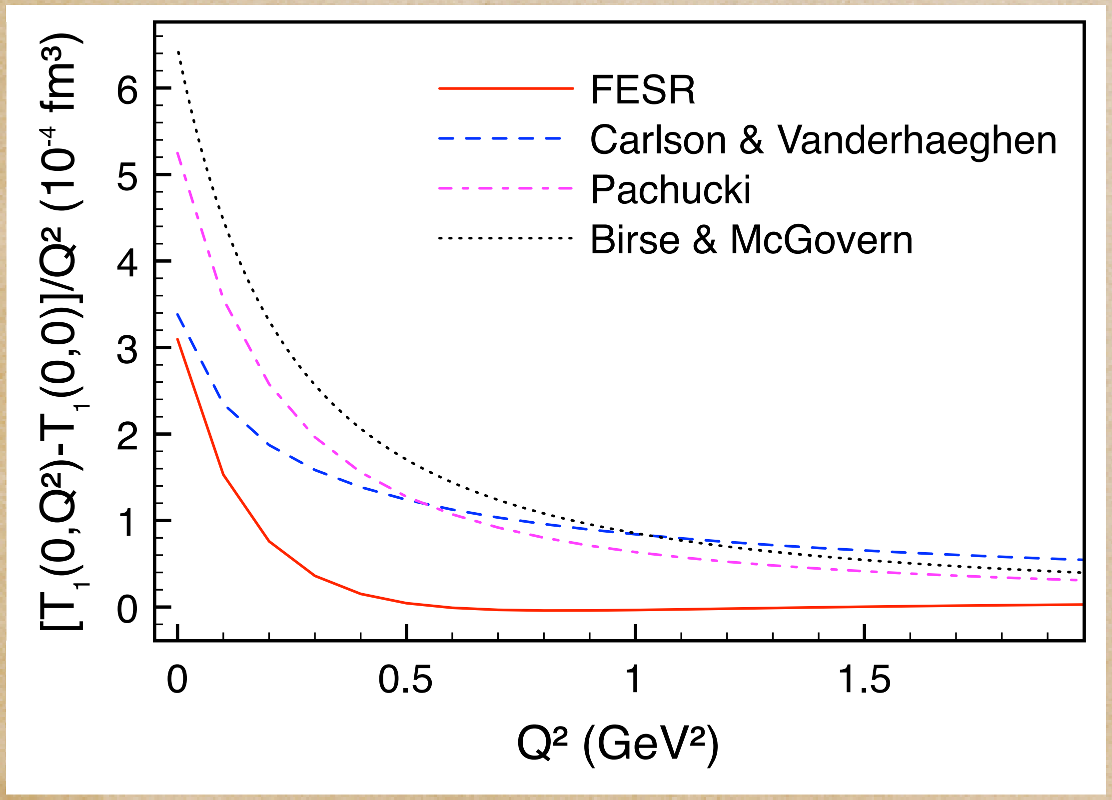
Evaluate the Lamb shift $$\Delta E^{subt} = \frac{\beta_M}{\pi}\phi^2(0)\int_0^\infty dQ\gamma_1(\tau_l)F_\beta(Q^2)$$

$$\text{Pachucki 1996}: \quad \Delta E^{subt} = 6.6\,\mu\text{eV}$$

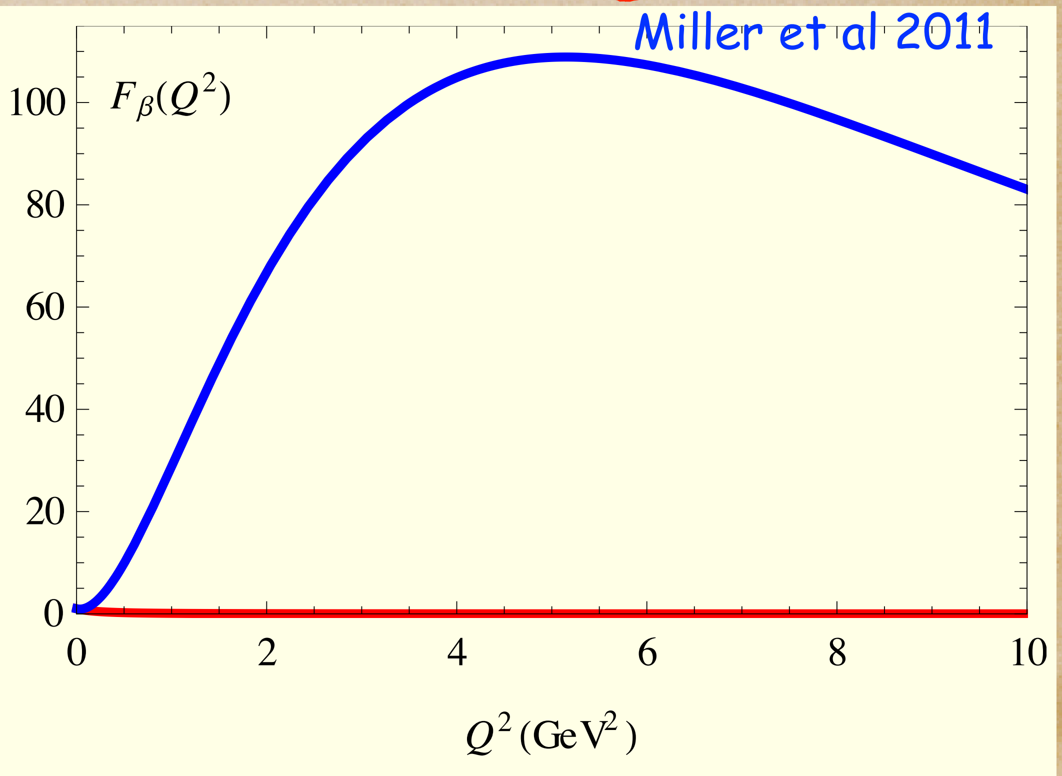
$$\text{Vanderhaeghen\&Carlson 2011}: \quad \Delta E^{subt} = 5.9(1.9)\,\mu\text{eV}$$

$$\text{MG, Llanes-Estrada, Szczepaniak 2013}: \quad \Delta E^{subt} = 3.3(4.6)\,\mu\text{eV}$$

Reasonable hadronic models



To get ~300 meV Lamb shift: need something like this



Proton Radius Puzzle vs New Physics

Possible Exotic explanations?

New heavy particles are excluded by the $(g\text{-}2)_\mu$

$$
\begin{aligned}
a_\mu(data) &= (116\,591\,785 \pm 51) \times 10^{-11} \\
a_\mu(thy.) &= (116\,592\,080 \pm 63) \times 10^{-11} \\
\delta a_\mu &= (295 \pm 81) \times 10^{-11}
\end{aligned}
$$

$$\Delta E_{Lamb}^{HEAVY} = 0.0044\, meV$$

$$\Delta E_{Lamb}^{missing} \approx 0.300\, meV$$

Viable explanation due to a light weakly-coupled boson

- finely tuned (mass ~ MeV, coupling ~ 10^{-4})
- breaks e- unversality (natural for scalar/tensor)

Should be seen in μ-D, μ-He$^+$ spectroscopy!

Batell, McKeen, Pospelov, PRL 2011

Rislow, Carlson, PRD 2012

Trucker-Smith, Yavin, PRD 2011

Deuteron Radius from μ-D Lamb Shift

Deuteron radius from μd (preliminary)

– Three transitions frequencies measured in μd
– 2P fine and hyperfine contributions from theory
Borie, Martynenko

$\Rightarrow$ Fit Lamb shift and 2S-HFS

μd:	$\Delta E_{\mathrm{LS}}^{\mathrm{exp}} = 202.8759(34)$ meV (prel.!)
μp:	$\Delta E_{\mathrm{LS}}^{\mathrm{exp}} = 202.3706(23)$ meV

Theory			QED		fin. size		TPE	
μp:	$\Delta E_{\mathrm{LS}}^{\mathrm{th}}$	=	206.0336(15)	-	5.2275(10) r_{p}^2	+	0.0332(20) meV	
μd:	$\Delta E_{\mathrm{LS}}^{\mathrm{th}}$	=	228.7972(15)	-	6.1094(10) r_{d}^2	+	1.6800(160) meV	Borie + Pachucki
		=	...	-	...	+	1.6980(???) meV	Borie + Ji (arXiv:1307.6577)
		=	228.7711(15)	-	6.1085(10) r_{d}^2	+	1.6800(160) meV	Martynenko+Pachucki

- Pachucki TPE term should be completed with:
  - finite-size of the nucleons (0.029 meV)?
  - neutron polarisabilities (0.040 meV?)?

Priv. Com. Friar

- Pachucki, Ji et al., and Friar agree on the 2% level
- Ongoing work of Carlson, Gorchtein and Vanderhaegen using inelastic data and dispersion relations.

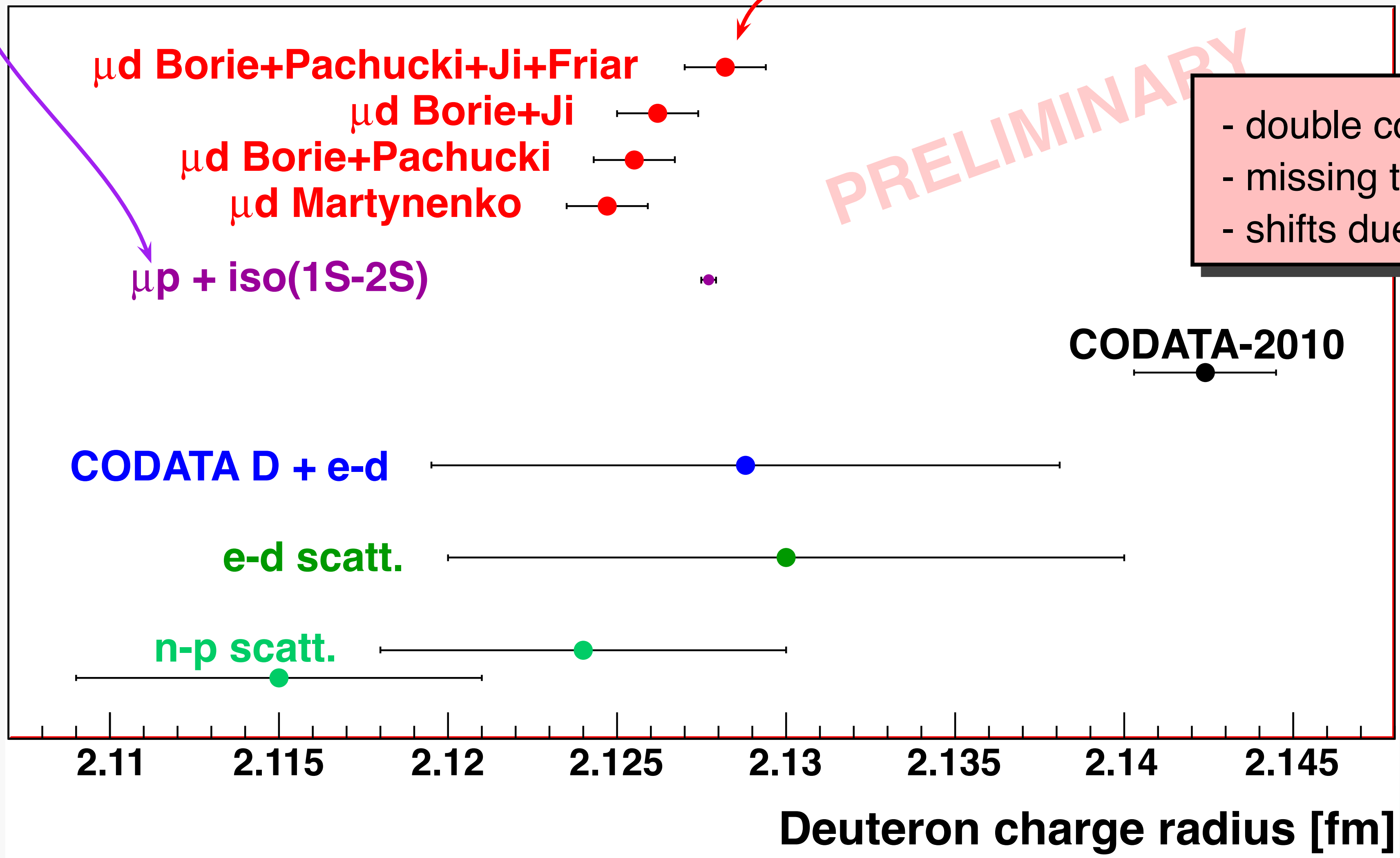
- NO $R_{(2)}^3$ term (third Zemach term) in μd. Pachucki, PRL 106, 193007 (2011)
  Exact cancellation for point-like nucleons between elastic (third Zemach) and part of the inelastic contributions

Deuteron radius from μd and μp (**preliminary**)

H-D iso-shift: $r_d^2 - r_p^2 = 3.820\,07(65)\ \mathrm{fm}^2$
μp : $r_p = 0.84087(39)\ \mathrm{fm}$
$\Rightarrow r_d = 2.12771(22)\ \mathrm{fm}$

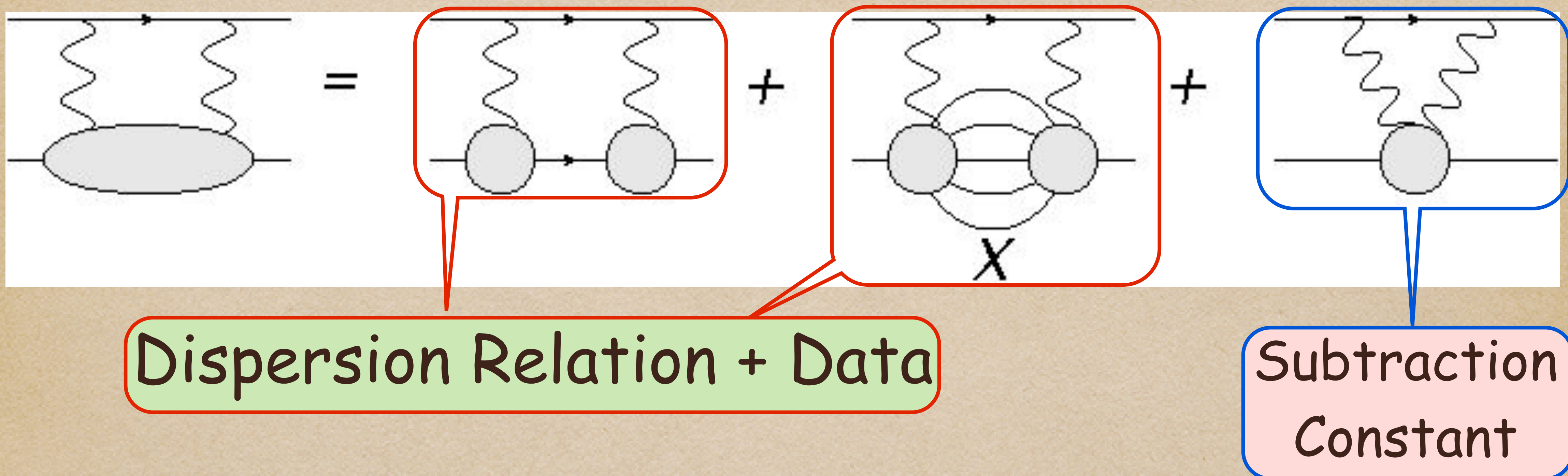
Directly from μd spectroscopy using μd polarizabiliy with ± 0.03 meV

- double counting (th)?
- missing terms (th)?
- shifts due to close levels (exp)?



Polarizability Correction to Lamb Shift

Order $\mathcal{O}(\alpha^5)$ correction



TPE: elastic contribution to Lamb shift

$$\Delta E^{el} = \alpha^2 \phi^2(0) \int_0^\infty \frac{dQ^2}{Q^2}$$
$$\left\{ \frac{16Mm}{(M+m)Q} G'_C(0) - \frac{m}{M(M^2-m^2)} \left(\frac{\gamma_2(\tau_d)}{\sqrt{\tau_d}} - \frac{\gamma_2(\tau_l)}{\sqrt{\tau_l}} \right) \left[\frac{G_C^2(Q^2)-1}{\tau_d} + \frac{8}{9}\tau_d \left(G_Q^2(Q^2) - G_{Q,pt}^2\right) \right] \right.$$
$$\left. + \frac{m}{M(M^2-m^2)} \frac{2}{3} \left(G_M^2(Q^2) - G_{M,pt}^2\right) \left[(1+\tau_d) \left(\frac{\gamma_1(\tau_d)}{\sqrt{\tau_d}} - \frac{\gamma_1(\tau_l)}{\sqrt{\tau_l}} \right) - \frac{\gamma_2(\tau_d)}{\sqrt{\tau_d}} + \frac{\gamma_2(\tau_l)}{\sqrt{\tau_l}} \right] \right\}$$

Deuteron form factors parametrization from:

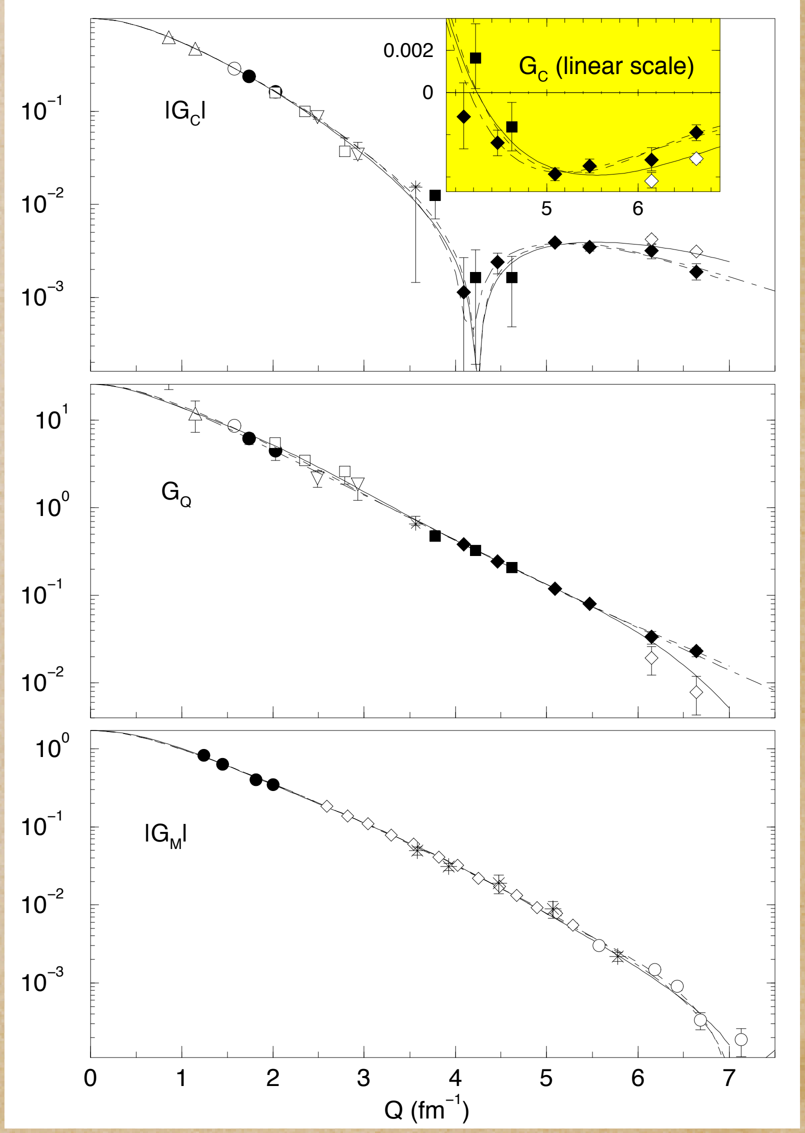
Abbott et al., EPJ A7 (2000) 421

$$\Delta E_C^{el} = -0.4162\,\mathrm{meV}$$
$$\Delta E_M^{el} = 0.00008\,\mathrm{meV}$$
$$\Delta E_Q^{el} = -0.0007\,\mathrm{meV}$$

Total elastic $\boxed{\Delta E_{tot}^{el} = -0.417(2)\,\mathrm{meV}}$

Compare to $\Delta E_{tot}^{el} = -0.37\,\mathrm{meV}$

Martynenko, Faustov,
Phys. Atom. Nucl. 67 (2004) 457

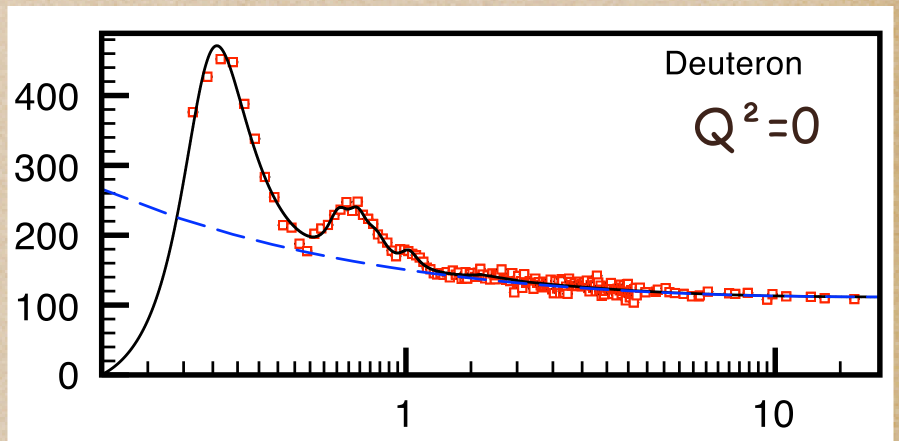


TPE: hadronic contribution to Lamb shift

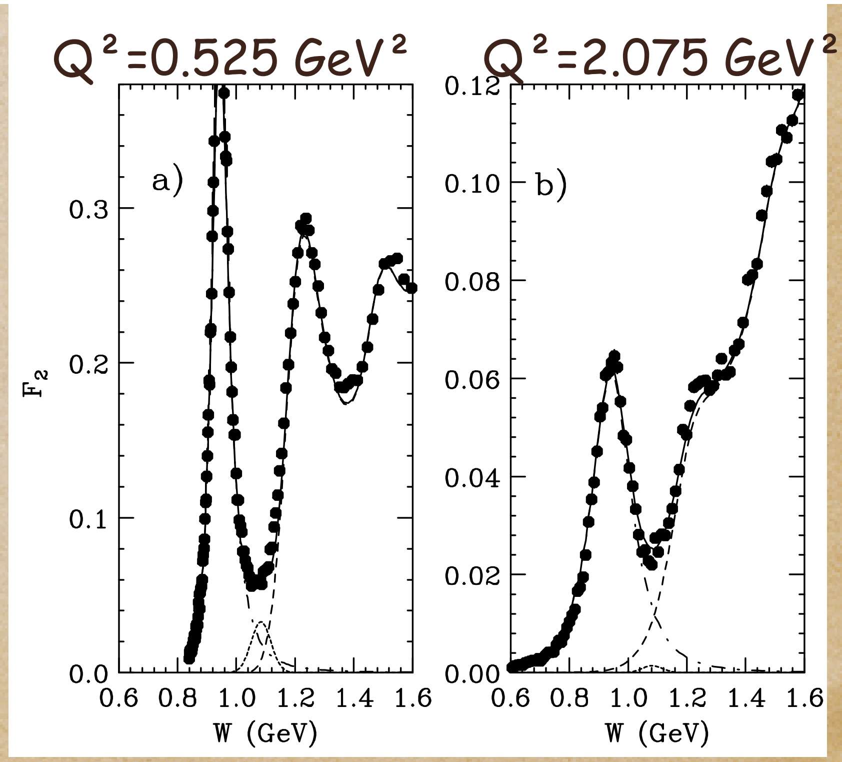
$$\Delta E^{inel} = \frac{2\alpha^2}{m_l M_d}\phi_n^2(0)\int_0^\infty \frac{dQ^2}{Q^2}\int_{\nu_{thr}}^\infty \frac{d\nu}{\nu}\left[\frac{\sqrt{\tau_l}\gamma_1(\tau_l)-\sqrt{\tau}\gamma_1(\tau)}{\tau_l-\tau}F_1(\nu,Q^2)+\frac{M_d\nu}{Q^2}\frac{\frac{\gamma_2(\tau)}{\sqrt{\tau}}-\frac{\gamma_2(\tau_l)}{\sqrt{\tau_l}}}{\tau_l-\tau}F_2(\nu,Q^2)\right]$$

Contains quasi-elastic and hadronic contributions $\quad \tau_l = \frac{Q^2}{4m_l^2}, \qquad \tau = \frac{\nu^2}{Q^2}$

Hadronic contribution:
fit to deuteron resonance data
matched to Regge-behaved background



Bosted and Christy, PR C77 (2008) 065206;
MG et al, PR C84 (2011) 065202



Contribution to the Lamb shift

$$\Delta E^{hadr} = -0.028\,\mathrm{meV}$$

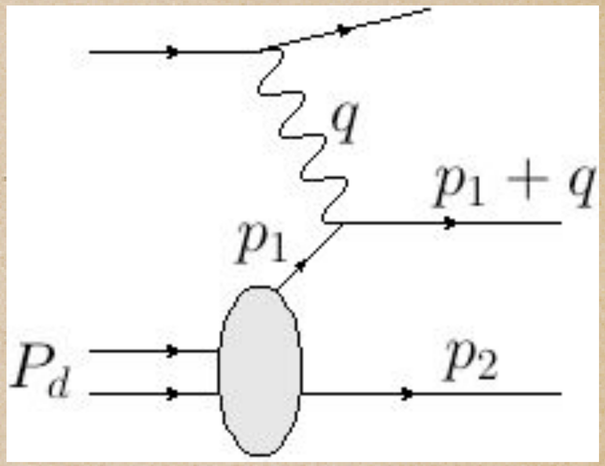
Compare to $\quad \Delta E^{hadr} = -0.043\,\mathrm{meV}$

Pachucki, PRL '13

TPE: quasielastic contributions to Lamb shift

QE in the Plane-Wave Born Approximation

$$F_{1,2}^{d,QE}(\nu, Q^2) = \frac{1}{4\pi}\int d^3\vec{k}\phi^2(\vec{k})\left[F_{1,2}^{p}(\nu', Q^2) + F_{1,2}^{n}(\nu', Q^2)\right]$$



Deuteron momentum distribution

$$S(\nu, Q^2) = \frac{1}{2}\int_{k_{min}}^{k_{max}} kdk\phi^2(k)$$

Analytic parametrization of the Paris NN potential

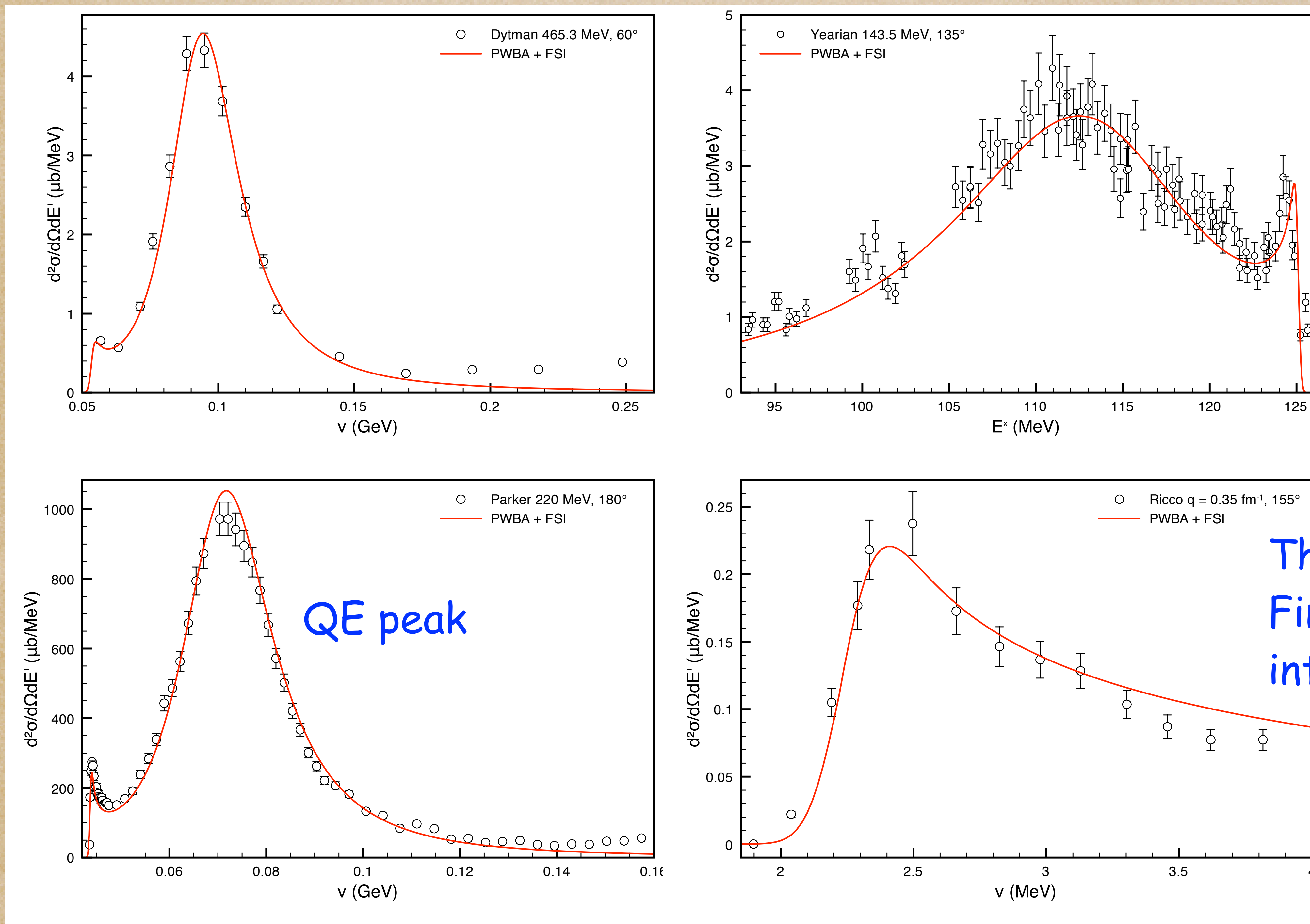
Lacombe et al. PL B101 (1981) 139

Works fine at substantial photon virtualities, not so fine at low Q^2.

But we just need to parametrize data - rescale by a function of Q^2 that will be obtained from a fit to all available data.

Quasielastic data

PWBA normalized to data   0.005 GeV² < Q² < 3 GeV²

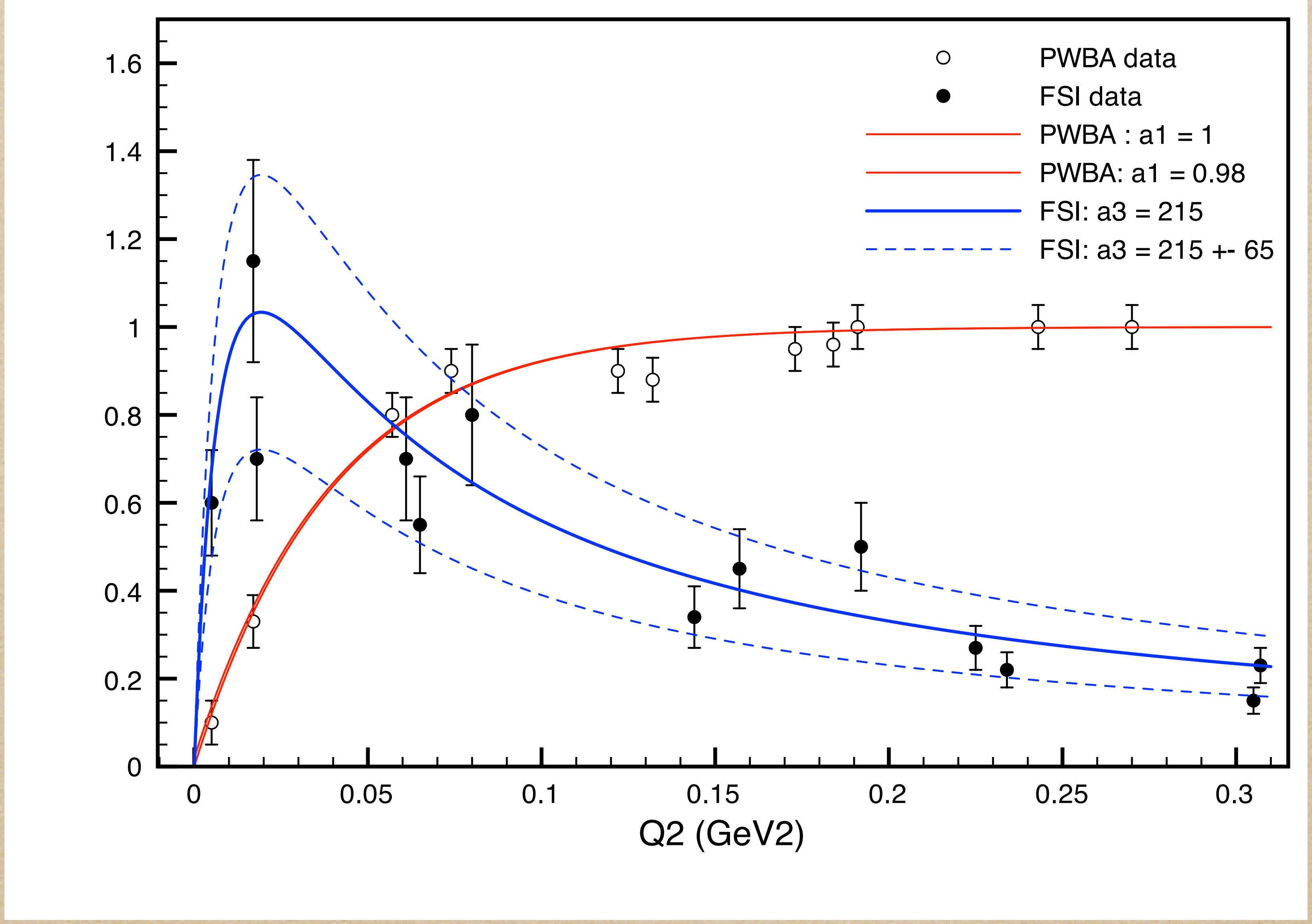


Threshold peak:
Final state
interaction

Quasielastic data

$$f^{PWBA}(Q^2) \;=\; [1 - a_1 e^{-b_1 Q^2}], \qquad a_1 = 0.995(5),\, b_1 = 25.4 GeV^{-2}$$

$$f^{FSI}(Q^2) \;=\; \frac{a_3 Q^2}{(1+b_3 Q^2)^2}, \qquad a_3 = 215(65) GeV^{-3},\, b_3 = 52(8) GeV^{-2}$$



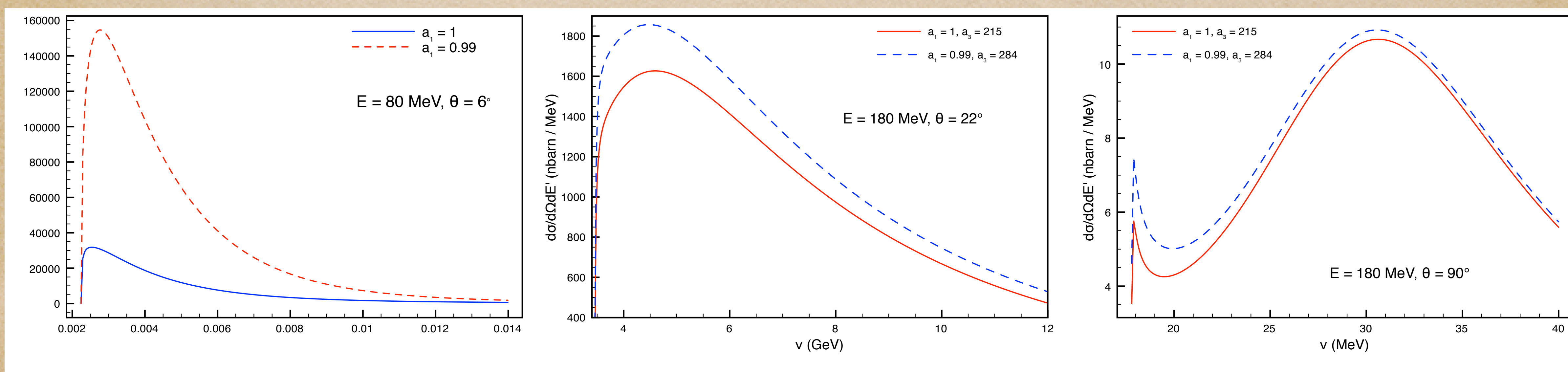
The uncertainty to Lamb shift: from the error bands on this plot

TPE: quasielastic contribution to Lamb shift

$$\Delta E_{F_1}^{PWBA} = -52(1)\,\mu eV, \quad \boxed{\Delta E_{F_2}^{PWBA} = -1568(740)\,\mu eV}$$
$$\Delta E_{F_1}^{FSI} = -228(69)\,\mu eV, \quad \Delta E_{F_2}^{FSI} = -172(48)\,\mu eV$$

1% uncertainty in one parameter ≈ 50% uncertainty in Lamb shift

At lowest Q^2 only backward data available: F_2 unconstrained



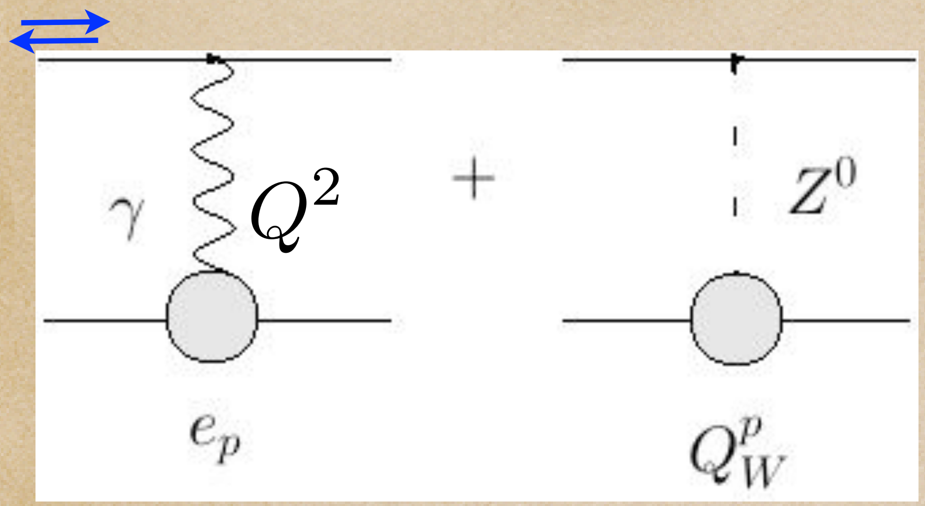
Sample plots in MAMI and MESA/P2 kinematics
show how sensitivity changes from 6° to 22° and 90°

No wonder: forward Compton amplitude is sensitive to forward data

PROTON's WEAK CHARGE

Dispersive γZ-Box correction

Weak Charge of the Proton



Elastic e-p scattering with polarized e⁻ beam

$$A^{PV} = \frac{\sigma_\uparrow - \sigma_\downarrow}{\sigma_\uparrow + \sigma_\downarrow} = -\frac{G_F Q^2}{4\sqrt{2}\pi\alpha_{em}} Q_W^p + \mathcal{O}(Q^4)$$

Effective e-q interaction

$$-\mathscr{L}^{eh} = -\frac{G_F}{\sqrt{2}} \sum_i \left[C_{1i}\, \bar{e}\, \gamma_\mu \gamma^5 e\, \bar{q}_i\, \gamma^\mu q_i + C_{2i}\, \bar{e}\, \gamma_\mu e\, \bar{q}_i\, \gamma^\mu \gamma^5 q_i \right]$$

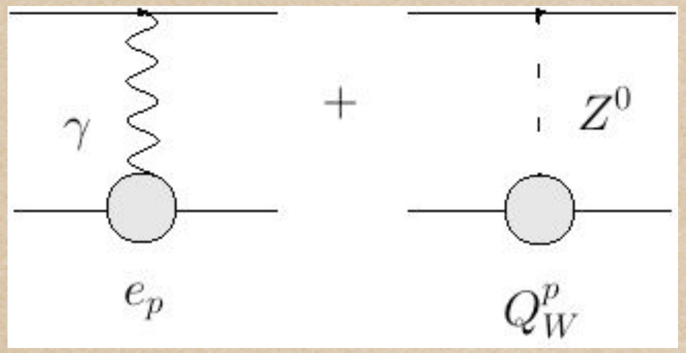
Standard Model (tree-level)

$$Q_W^{p,\,tree} = -2(2C_{1u} + C_{1d}) = 1 - 4\sin^2\theta_W \approx 0.05$$

Weak Charge of the Proton: EW corrections

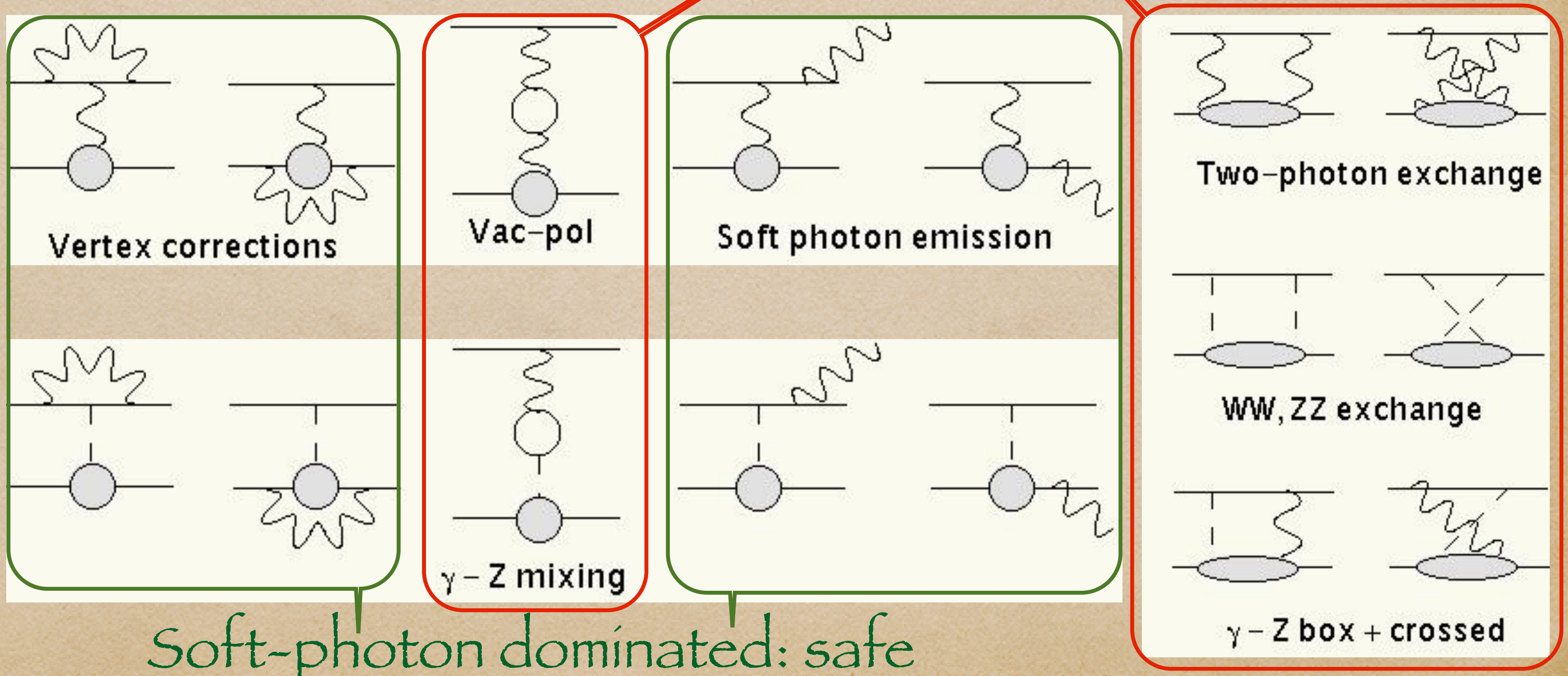
To match the experimental precision - include radiative corrections

$\mathcal{O}(1)$



Hadronic structure-dependent

$\mathcal{O}(\alpha_{em})$ $\alpha_{em} \approx 1/137$



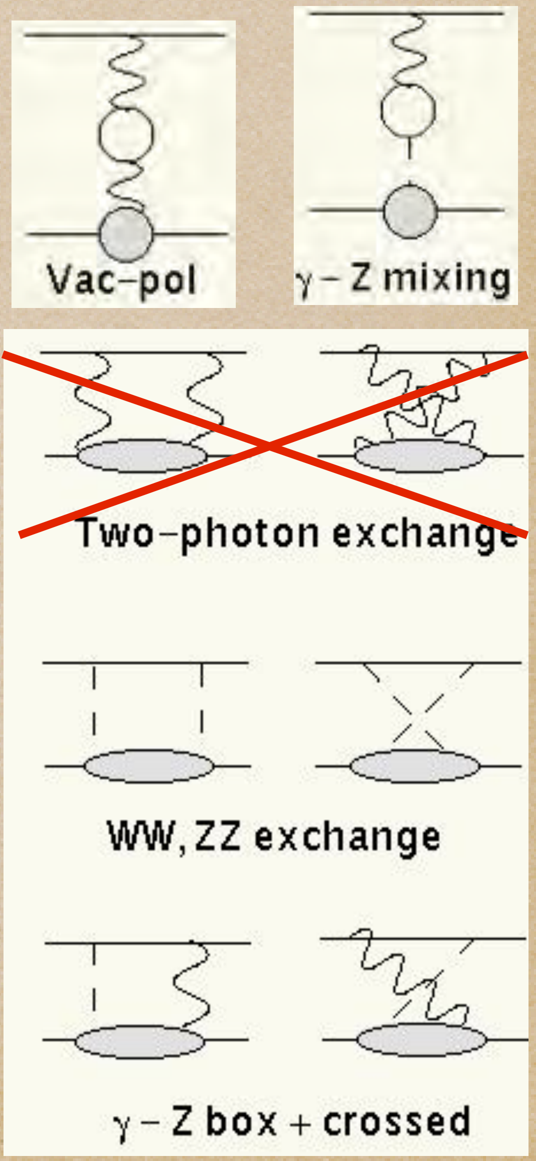
Soft-photon dominated: safe

Weak Charge of the Proton: EW corrections

Hadronic structure effects are under control

$$Q_W^p = \boxed{(1+\Delta_\rho+\Delta_e)(1-4\sin^2\hat{\theta}_W+\Delta_e')+\Box_{WW}+\Box_{ZZ}}+\boxed{\Box_{\gamma Z}}$$

W. J. Marciano and A. Sirlin, PRD **27**, 552 (1983); **29**,75 (1984); **31**, 213 (1985).
M.J. Ramsey-Musolf, PRC 60, 015501 (1999).



Vacuum polarization: reconstructed from $e^+e^- \to$ hadrons with dispersion relations

2γ-Box: kinematically suppressed

WW,ZZ-Box: perturbative- calculable reliably

γZ: for low energies (atomic PV experiments) cancellation between box and crossed
- not true for ~1 GeV energy any more!

Energy dependence of the γZ-Correction

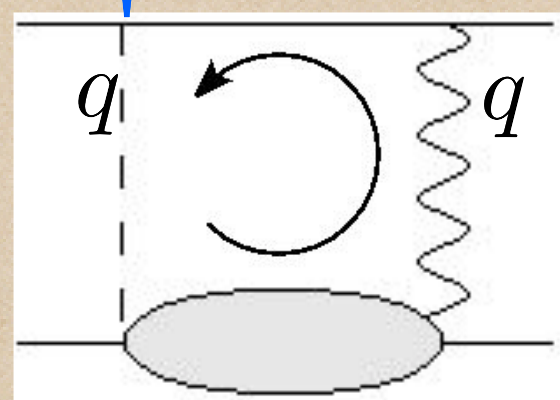
MG & C.J. Horowitz, PRL102, 091806 (2009)

$g_V^e,\ g_A^e$

$k^\mu = (E, \vec{k})$

$Q^2 = -q_\mu q^\mu \geq 0$

$p^\mu = (M, \vec{0})$



$W^2 = (p+q)^2$

Forward dispersion relation for

$$\Box_{\gamma Z} = g_V^e \Box_{\gamma Z_A} + g_A^e \Box_{\gamma Z_V}$$

Possess different symmetry between box and crossed terms:

$$\mathrm{Re}\Box_{\gamma Z_A}(E) = \frac{2}{\pi}\int\limits_{\nu_0}^{\infty} \frac{E' dE'}{E'^2 - E^2} \mathrm{Im}\Box_{\gamma Z_A}(E')$$

$$\mathrm{Re}\Box_{\gamma Z_V}(E) = \frac{2E}{\pi}\int\limits_{\nu_0}^{\infty} \frac{dE'}{E'^2 - E^2} \mathrm{Im}\Box_{\gamma Z_V}(E')$$

$$\mathrm{Re}\Box_{\gamma Z_A}(0) \neq 0$$

$$\mathrm{Re}\Box_{\gamma Z_V}(0) = 0$$

APV result

Can quantify the energy dependence

Energy dependence of the γZ-Correction

$$\mathrm{Re}\Box_{\gamma Z_V}(E) = \frac{2E}{\pi}\int_0^\infty dQ^2 \int_{W_\pi^2}^\infty dW^2 \left[A F_1^{\gamma Z}(W^2,Q^2) + B F_2^{\gamma Z}(W^2,Q^2)\right]$$

PV DIS data
- not (YET!) available

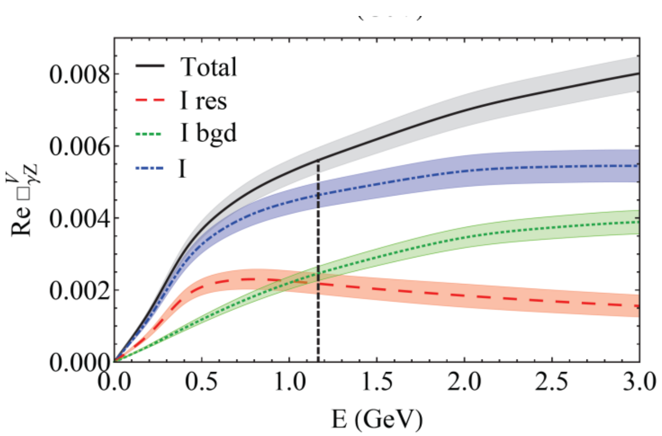
Isospin-rotate the e.-m. data

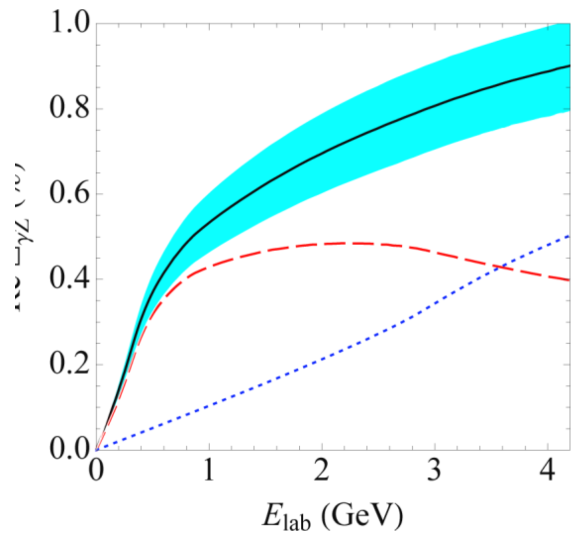
Evaluate at E = 1.165 GeV (QWEAK)

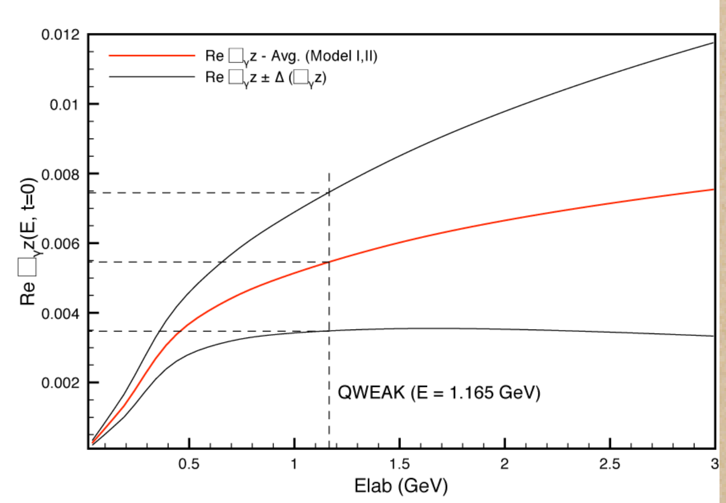
Hall *et al.*
PRD 88, 013011 (2013)

Carlson and Rislow
PRD 83, 113007 (2011)

Gorchtein *et al.*
PRC 84, 015502 (2011)







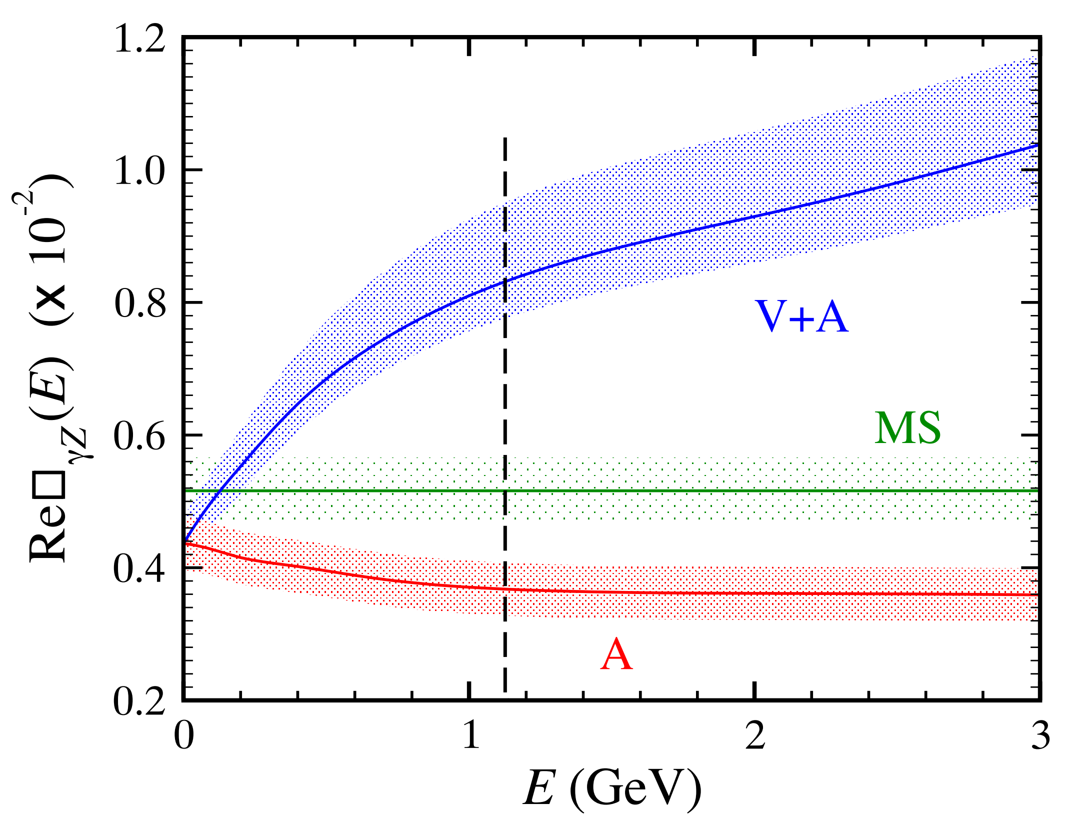
$$\mathrm{Re}\Box^V_{\gamma Z}(E = 1.165\ \mathrm{GeV})$$

$(5.6 \pm 0.36)\times 10^{-3}$ $(5.7 \pm 0.9)\times 10^{-3}$ $(5.4 \pm 2.0)\times 10^{-3}$

Energy dependence of the γZ-Correction

Also the axial part can be evaluated through a DR - check the old Marciano & Sirlin's calculation

Blunden et al., PRL 107 (2011) 081801



New SM prediction for the proton's weak charge

$$Q_W^p + \mathrm{Re}\Box_{\gamma Z}(E = 1.165\,\mathrm{GeV}) = 0.0767 \pm 0.0008 \pm 0.0020_{\gamma Z}$$

To be compared to the previous prediction $Q_W^p = 0.0713 \pm 0.0008$

4σ(theory) effect was missed in the original QWEAK analysis;
Theory uncertainty needs to be further reduced

γZ-Correction to the QWEAK Measurement

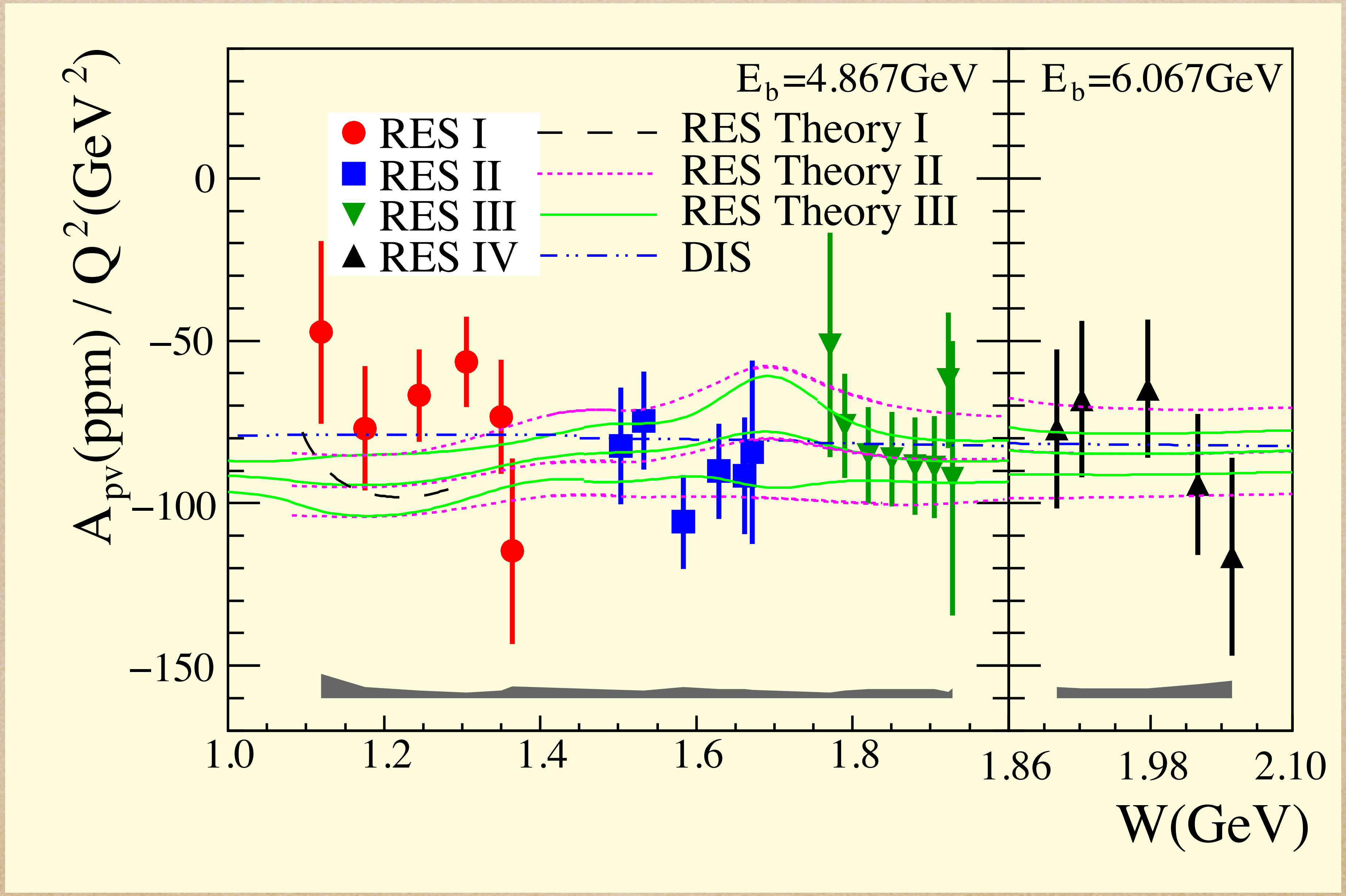
γZv -correction for QWEAK

Uncertainty thereof

Reducing Theory Uncertainty

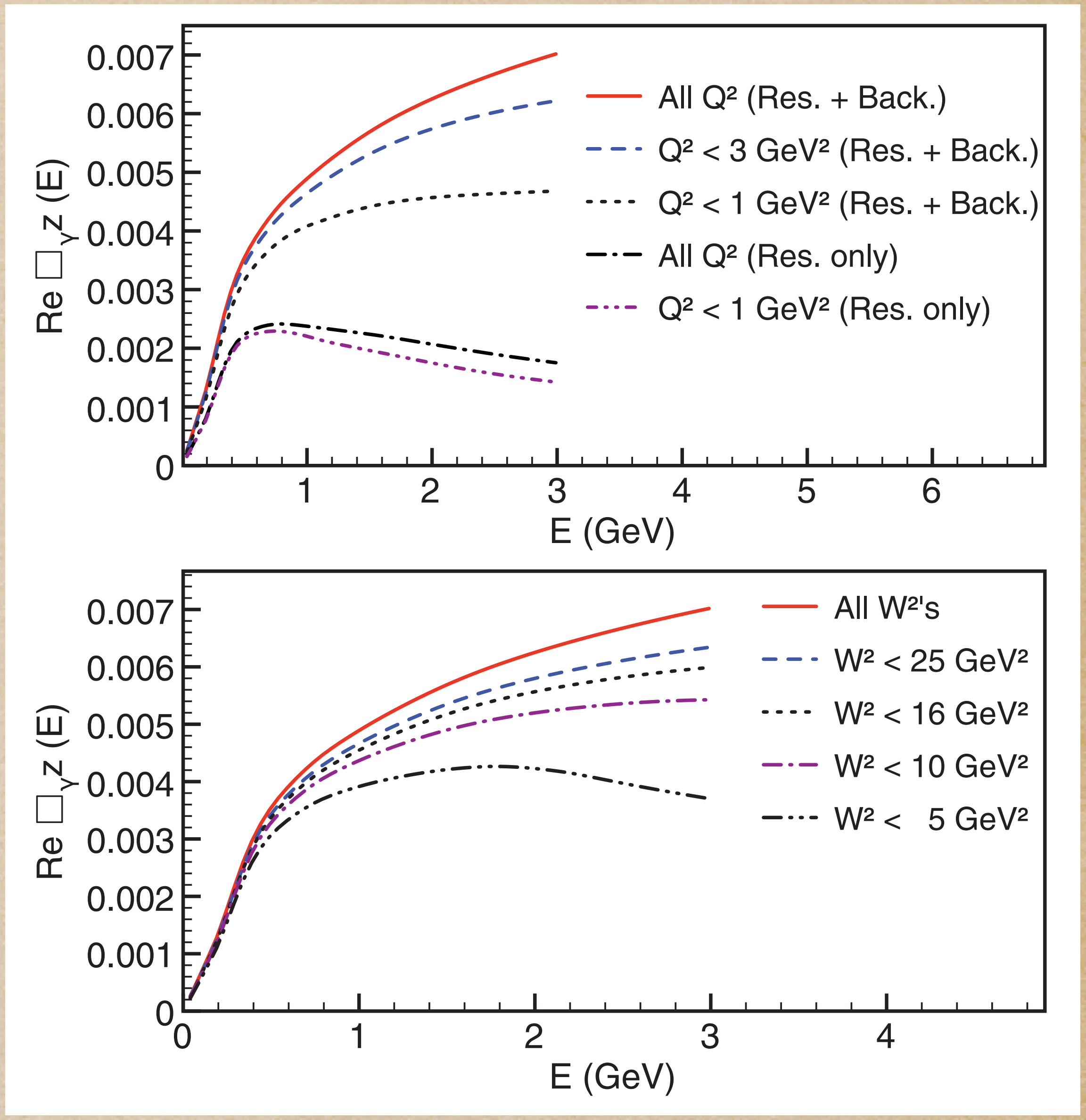
New data on PV DIS structure functions coming

- PV DIS, SOLID @ JLab
- will help constraining the theory uncertainty



Theory uncertainty for MESA/P2 kinematics

Energy dependence;
Saturation of the box;



Steep energy dependence
of the dispersion correction

A measurement at Mainz with 180MeV beam is planned - MESA/P2

$$Q_W^p + \mathrm{Re}\Box_{\gamma Z}(E = 0.180\,\mathrm{GeV}) = 0.0726 \pm 0.0008 \pm 0.0003_{\gamma Z}$$

SUMMARY

Proton radius puzzle:

- TPE correction unlikely to resolve the puzzle
- Check μ-He spectroscopy (PSI, Munich); μ-p scattering (MUSE);
- Radiative corrections to e-p scattering?

TPE correction to Lamb shift in μ-D:

- lack of forward data at low Q^2
- extend A1@Mainz measurement of elastic e-D scattering to quasielastic
- may need to take some further data at lower energy with P2
- further input from theory - EFT calculation?

γZ-box:

- at low energy - smaller effect, big advantage for MESA/P2;
- to further reduce uncertainty - include πN states explicitly
- at 1 GeV - need some more input from exp. (PVDIS, VDM sum rule)
- MOLLER@JLab: Q^p_w could be measured as a by-product

Спасибо за внимание!

Isospin rotation of e.-m. data: resonances

Isospin 1/2, 3/2 resonances

$$\langle X|J^\mu_{NC,V}|p\rangle \;=\; (1-4s^2\theta_W)\langle X|J^\mu_{em}|p\rangle - \langle X|J^\mu_{em}|n\rangle$$

$$\langle p|J^\mu_{em}|R\rangle\langle R|J^\mu_{NC,V}|p\rangle \;=\; (1-4s^2\theta_W)|\langle R|J^\mu_{em}|p\rangle|^2 - \langle p|J^\mu_{em}|R\rangle\langle R|J^\mu_{em}|n\rangle$$

Rescale each I=1/2 resonance from e.-m. to γZ cross section as

$$\boxed{\xi^R_{Z/\gamma} \equiv \frac{\sigma^{\gamma Z,p}_{T,R}}{\sigma^{\gamma\gamma p}_{T,R}} \;=\; (1-4s^2\theta_W) - \underbrace{\frac{A^p_{R,1/2}A^{n*}_{R,1/2} + A^p_{R,3/2}A^{n*}_{R,3/2}}{|A^p_{R,1/2}|^2 + |A^p_{R,3/2}|^2}}_{y_R}}$$

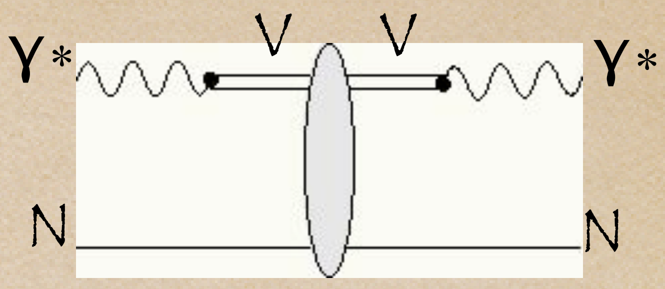
	$P_{33}(1232)$	$S_{11}(1535)$	$D_{13}(1520)$	$S_{11}(1665)$	$F_{15}(1680)$	$P_{11}(1440)$	$F_{37}(1950)$
y_R	$-1.0^{-0.1}_{+0.1}$	$-0.51^{-0.71}_{+0.35}$	$-0.77^{-0.125}_{+0.125}$	$-0.28^{-0.86}_{+0.45}$	$-0.27^{-0.12}_{+0.1}$	$-0.62^{-0.2}_{+0.19}$	-1^{-1}_{+1}

Uncertainties - from PDG helicity amplitudes values

Isospin rotation of e.-m. data: background

Vector Dominance Model (VDM)

$$|\gamma^*\rangle = \sum_V C_{\gamma^* V}|V\rangle \qquad V = \rho,\ \omega,\ \phi$$



$$\sigma_{tot}^{\gamma p} = \sum_{V=\rho,\omega,\phi} \frac{4\pi\alpha}{f_V^2}\sigma_{Vp}$$

Elastic Vp cross section - independent of V

VM decay constants $\frac{4\pi}{f_V^2} = 0.4545, 0.04237, 0.05435 \quad (\rho,\omega,\phi)$

VDM sum rule: $$\sigma_{tot}(\gamma p) = \sum_{V=\rho,\omega,\phi}\sqrt{16\pi\frac{4\pi\alpha}{f_V^2}\frac{d\sigma^{\gamma p\to Vp}}{dt}(t=0)}$$

Measured experimentally

HERA: NPB' 02 ZEUS: Z.Phys.'95,'96, PLB'96

$139 \pm 4\ (\mu b) \quad \leftrightarrow \quad 111 \pm 13\ (\mu b)$ at $W = 70$ GeV

Generalized VDM - continuum contribution $$\sigma_{tot}^{\gamma p} = \sum_{V=\rho,\omega,\phi}\frac{4\pi\alpha}{f_V^2}\sigma_{Vp} + \sigma_{Cp}$$

Isospin rotation of e.-m. data: background

Rescale the background according to

$$\boxed{\frac{\sigma^{\gamma^* p\to Zp}}{\sigma^{\gamma^* p\to\gamma^* p}} = \frac{\frac{g_V^{I=1}}{e_{I=1}} + \frac{g_V^{I=0}}{e_{I=0}}\frac{\sigma^{\gamma^* p\to\omega p}}{\sigma^{\gamma^* p\to\rho p}} + \frac{g_V^{s}}{e_{s}}\frac{\sigma^{\gamma^* p\to\phi p}}{\sigma^{\gamma^* p\to\rho p}} + \frac{X'}{\sigma^{\gamma^* p\to\rho p}}}{1 + \frac{\sigma^{\gamma^* p\to\omega p}}{\sigma^{\gamma^* p\to\rho p}} + \frac{\sigma^{\gamma^* p\to\phi p}}{\sigma^{\gamma^* p\to\rho p}} + \frac{X}{\sigma^{\gamma^* p\to\rho p}}}}$$

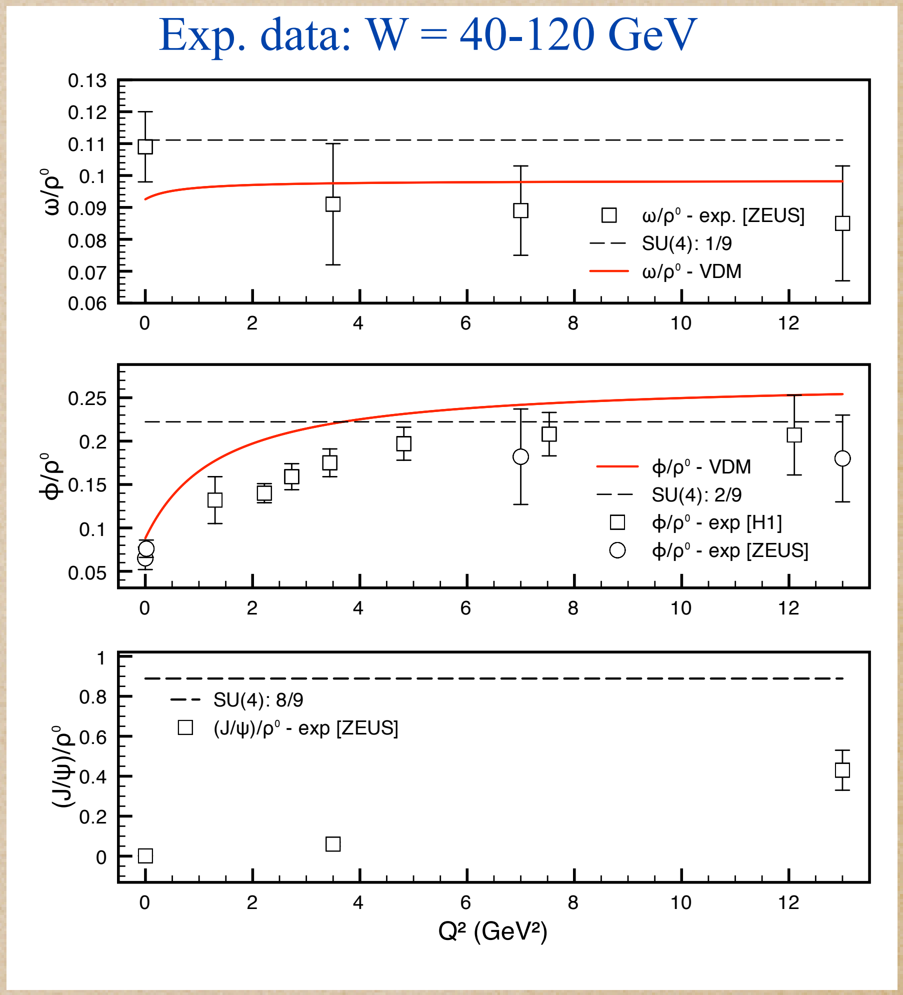
VDM: identify X(X') with continuum

$$\frac{\sigma^{\gamma^* p\to Vp}}{\sigma^{\gamma^* p\to\rho p}} = \frac{r_V}{r_\rho}\frac{m_V^4}{m_\rho^4}\frac{(m_\rho^2+Q^2)^2}{(m_V^2+Q^2)^2}$$

Uncertainty estimate - from data!

$$\Delta\xi_{Z/\gamma}^{V,Model\ A} = \left[\left(\frac{\sigma^{\gamma^*\to V}}{\sigma^{\gamma^*\to\rho}}\right)^{exp} - \left(\frac{\sigma^{\gamma^*\to V}}{\sigma^{\gamma^*\to\rho}}\right)^{Model\ A}\right]\sigma^{\gamma^*\to V\to Z}$$

Continuum - 100% uncertainty



Correcting input in γZ-calculation

Strong indication that S11 and D13 are misidentified in Bosted's fit!

PDG:

$$D_{12}(1520): \quad A^p_{3/2} = 150(15)\,\mathrm{GeV}^{-1/2}, \quad A^p_{1/2} = -24(9)\,\mathrm{GeV}^{-1/2}$$

$$S_{11}(1535): \quad A^p_{3/2} = 0, \quad A^p_{1/2} = 90(30)\,\mathrm{GeV}^{-1/2}$$

Bosted:

$$D_{12}(1520): \quad \sqrt{(A^p_{3/2})^2 + (A^p_{1/2})^2} \sim 17\,\mathrm{GeV}^{-1/2}$$

$$S_{11}(1535): \quad A^p_{1/2} \sim 170\,\mathrm{GeV}^{-1/2}$$

Correcting the fit for more realistic strengths of these resonances would reduce the uncertainty due to isospin rotation of the resonances