Determination of Planck constant and the new SI

Savely G Karshenboim

Pulkovo observatory (ГАО) (St. Petersburg)
and Max-Planck-Institut für Quantenoptik (Garching)

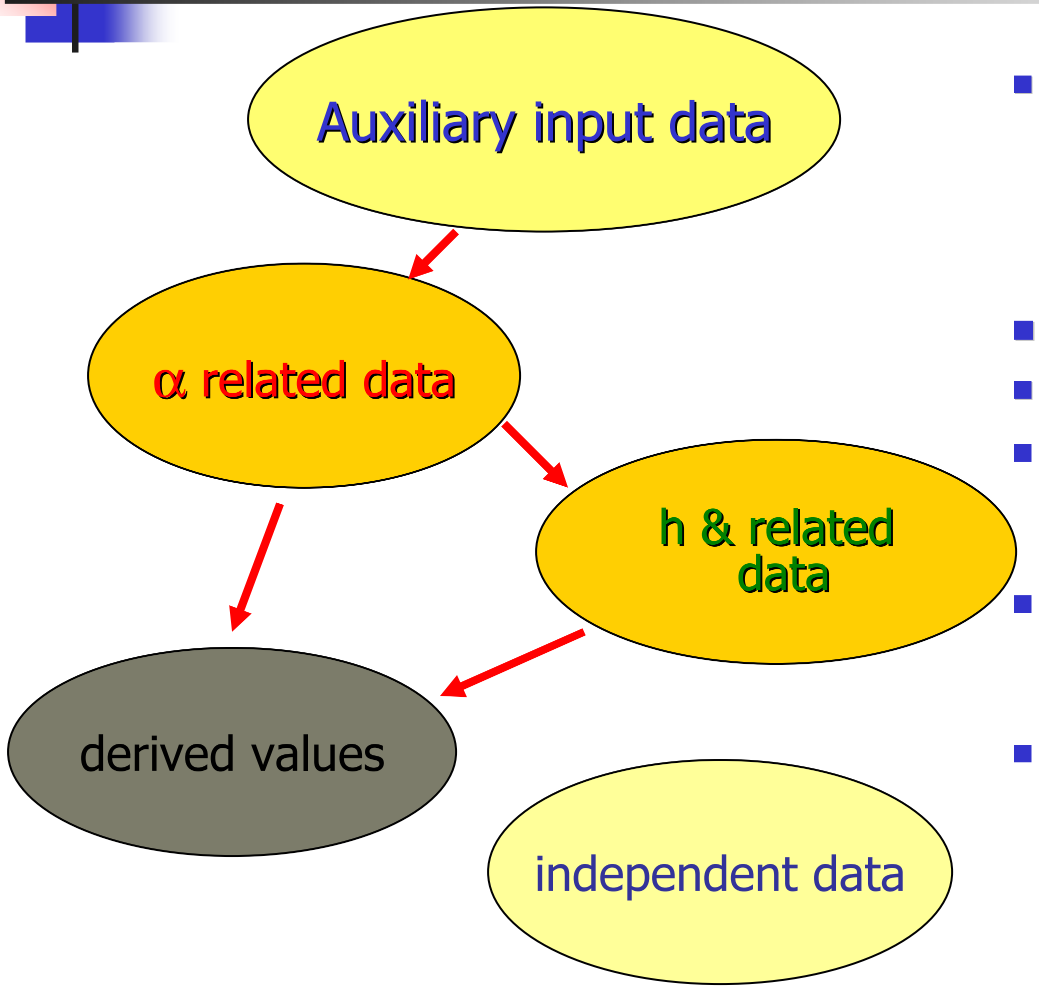
MAX-PLANCK-INSTITUTE
OF QUANTUM OPTICS
GARCHING
MPQ

Pulkovo Observatory

Outline

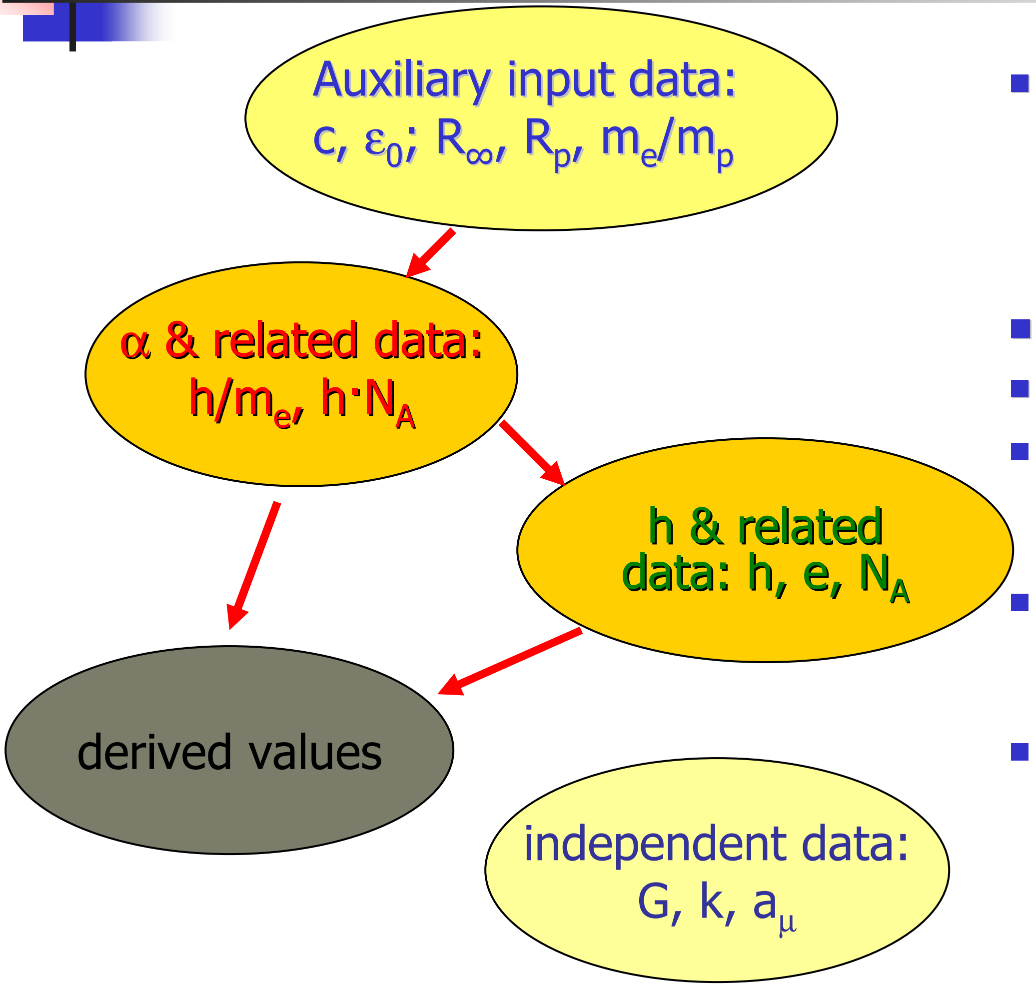
- *structure of input and output of the adjustment*
- *Rydberg constant*
- α
- h
- *quantum Hall standards*
- *quantum Josephson standards*
- *Watt-balance*
- *silicon crystale*

Structure of the input data and output values



- Auxiliary data = **exact** + the most accurate data which are to be evaluated prior the adjustment: R_∞, m_e/m_p, atomic masses.
- α related data: h/m, hN_A ...
- h related data: e, e/h, ...
- The lines (→) are equations: e.g., theoretical expressions for h/M, the Lamb shift, ...
- Some data are measured, a lot are derived: m_p [kg], m_e [Mev/c²], ...
- G is uncorrelated,...

Structure of the input data and output values



- Auxiliary data = **exact** + the most accurate data which are to be evaluated prior the adjustment: R_∞, m_e/m_p, atomic masses.
- α related data: h/m, hN_A ...
- h related data: e, e/h, N_A ...
- The lines ($\rightarrow$) are equations: e.g., theoretical expressions for h/M, the Lamb shift, ...
- Some data are measured, a lot are derived: m_p [kg], m_e [Mev/c^2], ...
- G is uncorrelated; k, a_μ, ...

Example: multiplicative vs. additive: R_∞ vs. α

- equations:

$$1/2\ \alpha^2 = R_\infty \frac{h}{m_e c}$$

$$c_1 R_\infty c + c_2 \alpha^2 R_\infty c = \nu$$

- uncertainties:
  - $R_\infty \sim 10^{-11}$
  - $\alpha \sim 10^{-9} - 10^{-10}$
  - $\alpha^2 \rightarrow 10^{-4} \times 10^{-9}$

Example: multiplicative vs. additive: R_∞ vs. α

- equations:

$$1/2\ \alpha^2 = R_\infty \frac{h}{m_e c}$$

$$c_1 R_\infty c + c_2 \alpha^2 R_\infty c = \nu$$

- uncertainties:
  - $R_\infty \sim 10^{-11}$
  - $\alpha \sim 10^{-9} - 10^{-10}$
  - $\alpha^2 \to 10^{-4} \times 10^{-9}$

`almost´

exact

Rydberg constant

- hydrogen & deuterium spectroscopy
- electron-proton elastic scattering
- Lamb shift in muonic hydrogen

few parts in 10^{12}

α block

- equations:

$$R_\infty = \frac{\alpha^2 m_e c}{2h}$$

- input data
  - α
  - h/m_e

α block

- equations:

$$R_\infty = \frac{\alpha^2 m_e c}{2h}$$

- m_e/m_p

- input data
  - α
  - h/m_e
  - h/m_p

α block

- equations:

$$R_\infty = \frac{\alpha^2 m_e c}{2h}$$

- m_e/m_p
- m_p in u
- m_{at} in u

- input data
  - α
  - h/m_e
  - h/m_p
  - h/m_{at}

α block

- equations:

$$m(^{12}\mathrm{C})/12 \cdot N_A = 1\,\mathrm{g\,mol}^{-1}$$

- m_e/m_p
- m_p in u
- m_{at} in u

- input data
  - α
  - h/m_e
  - h/m_p
  - h/m_{at}
- output
  - $h \cdot N_A$

$$\frac{mc^2}{h} = \frac{1}{(h \cdot N_A)} \times \frac{m}{m(^{12}\mathrm{C})/12} \times c^2 \times \left(m(^{12}\mathrm{C})/12 \cdot N_A\right)$$

α block

- equations:

$$m(^{12}\mathrm{C})/12 \cdot N_A = 1\,\mathrm{g\,mol}^{-1}$$

- m_e/m_p
- m_p in u
- m_{at} in u

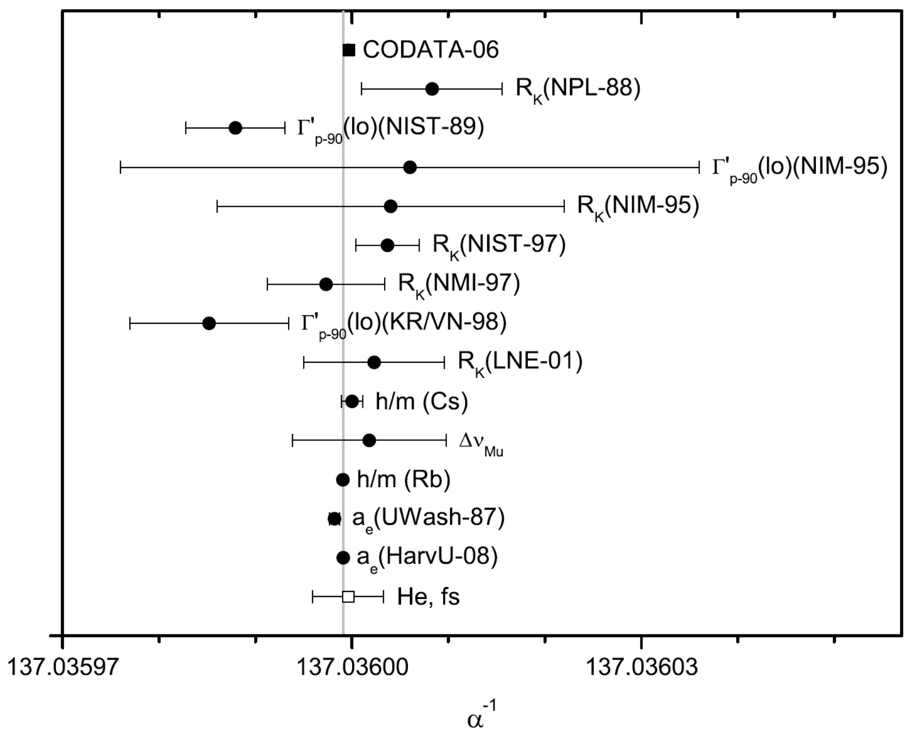
- input data
  - α
  - h/m_e
  - h/m_p
  - h/m_{at}
- output
  - $h \cdot N_A$

$$\frac{mc^2}{h} = \frac{1}{(h \cdot N_A)} \times \frac{m}{m(^{12}\mathrm{C})/12} \times c^2 \times \left(m(^{12}\mathrm{C})/12 \cdot N_A\right)$$

α block

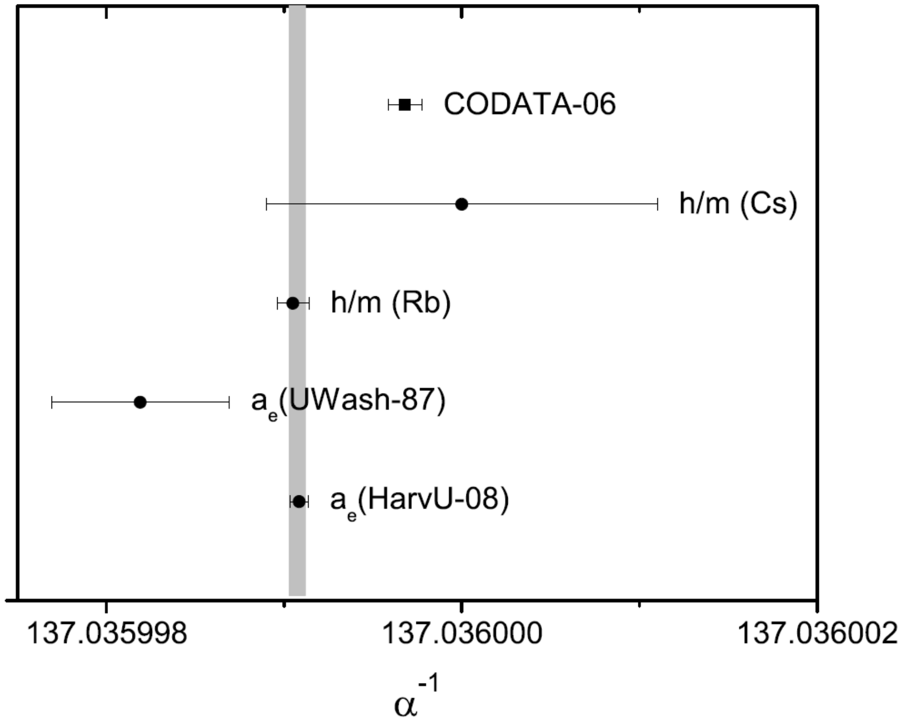
Quantity	Symbol	Value	u_r
inverse fine structure constant	α^{-1}	$137.035\,999\,074(44)$	$[3.2 \times 10^{-10}]$
molar Planck constant	$h \cdot N_A$	$3.990\,312\,7176(28) \times 10^{-10}$ J s mol^{-1}	$[7.0 \times 10^{-10}]$
quantum of circulation	$h/(2m_e)$	$3.636\,947\,5520(24) \times 10^{-4}$ m^2 s^{-1}	$[6.5 \times 10^{-10}]$
Compton wavelength	$\lambda_C = h/(m_e c)$	$2.426\,310\,2389(16) \times 10^{-12}$ m	$[6.5 \times 10^{-10}]$
von Klitzing constant	$R_K = h/e^2$	$25\,812.807\,4434(84)$ Ω	$[3.2 \times 10^{-10}]$
muon-electron mass ratio	m_μ/m_e	$206.768\,2843(52)$	$[2.5 \times 10^{-8}]$

α block



- QED vs. Penning trap: a_e
- recoil spectroscopy
  - h/m_{Rb}
  - h/m_{Cs}
- quantum Hall standard vs calculable capacitor: R_K

α block

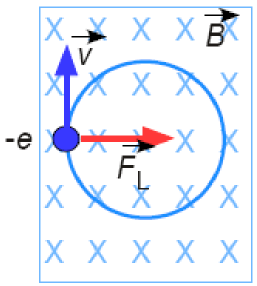


below 1 part in 10^9

- QED vs Penning trap: a_e
- recoil spectroscopy
  - h/m_{Rb}
  - h/m_{Cs}

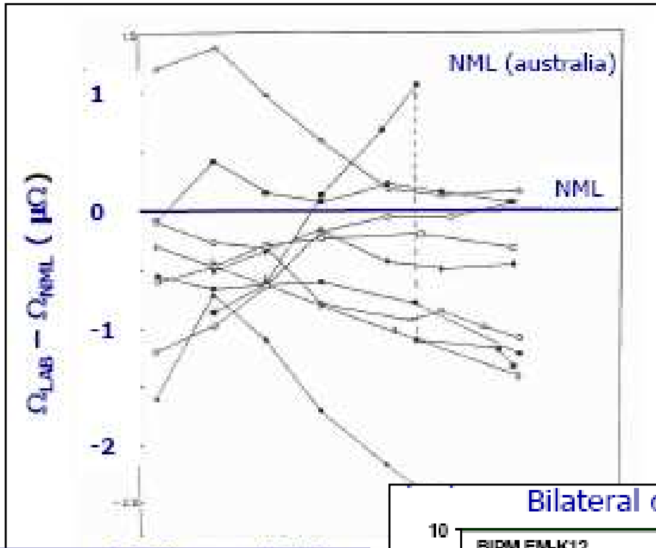
Quantum Hall effect and a standard of resistance

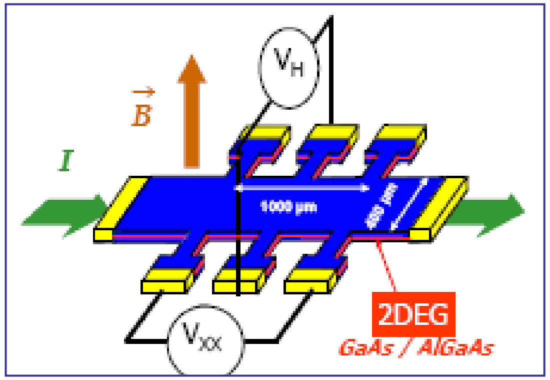
Classical Hall effect (1879)

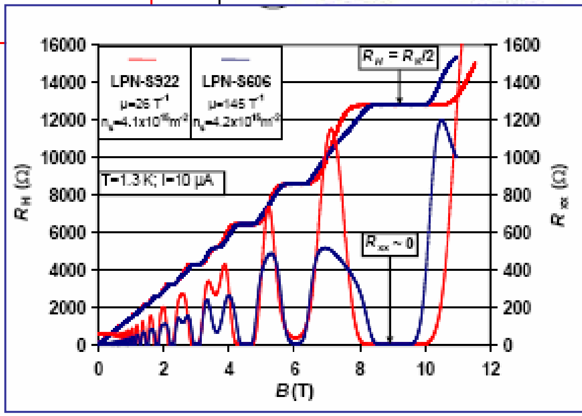


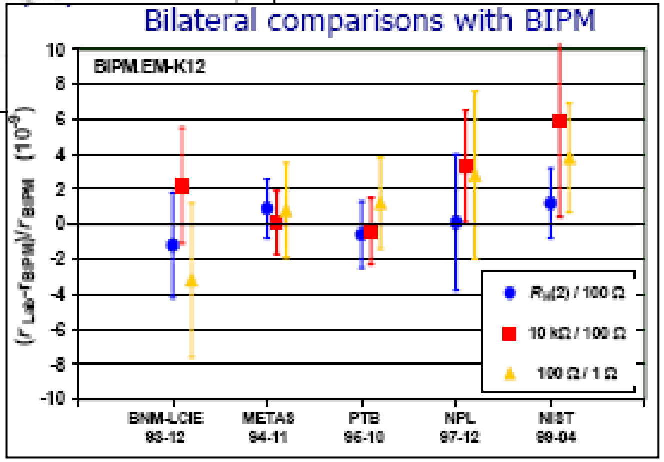
Lorentz Force

$$\vec{F}_L = q\left(\vec{v} \times \vec{B}\right)$$



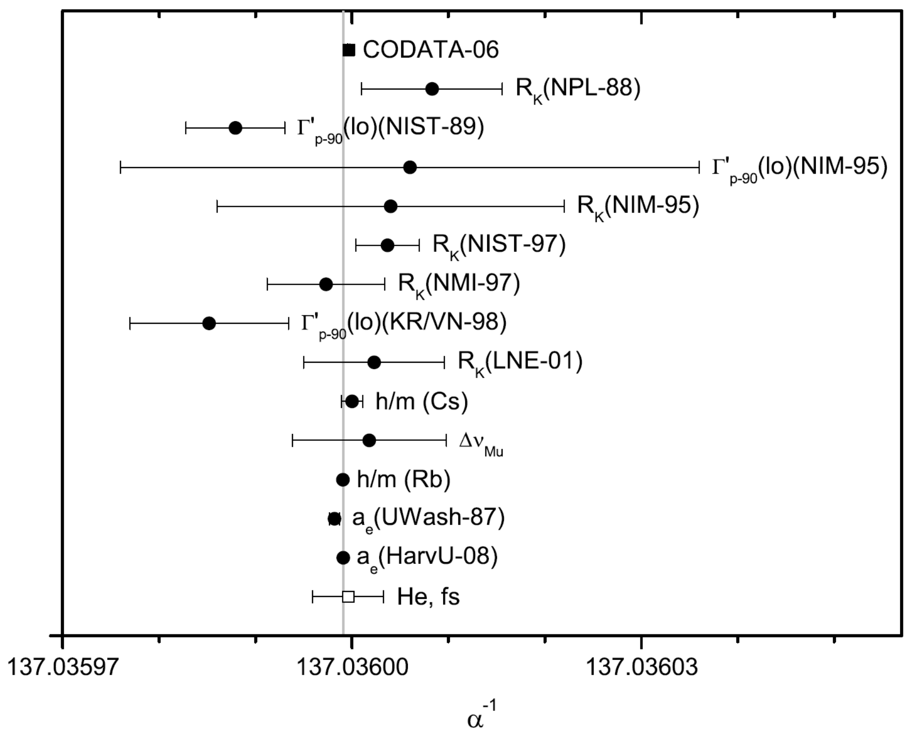






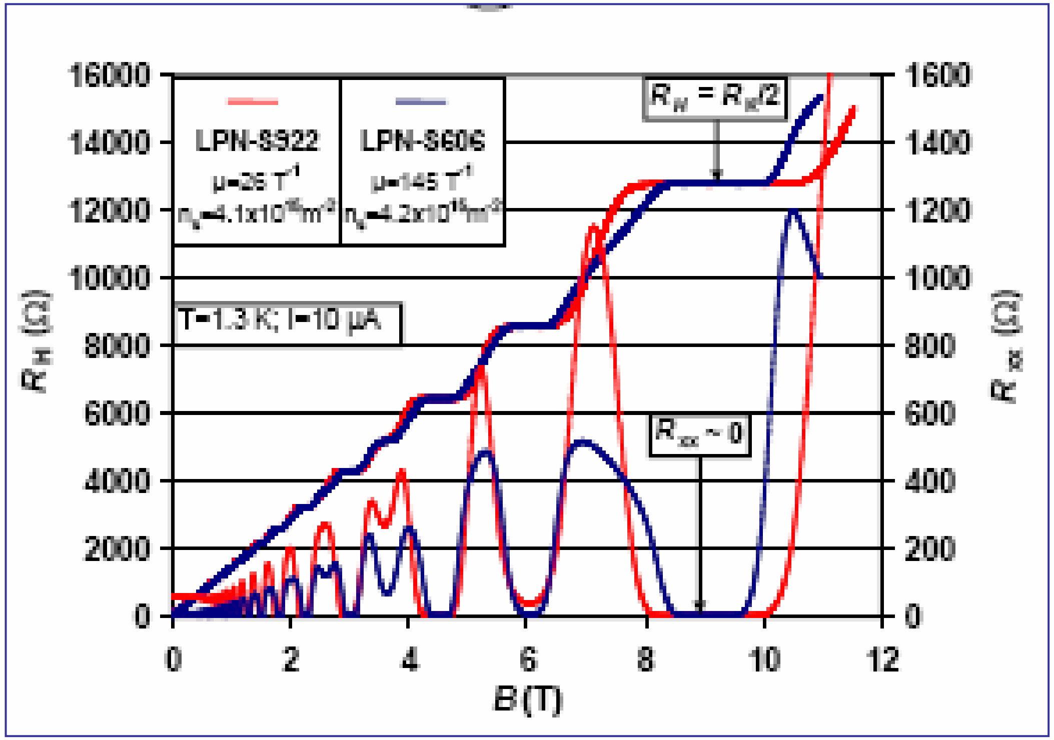
W. Poirier, Les Houches, 2007

α block



- QED vs. Penning trap: a_e
- recoil spectroscopy
  - h/m_{Rb}
  - h/m_{Cs}
- quantum Hall standard vs calculable capacitor: R_K

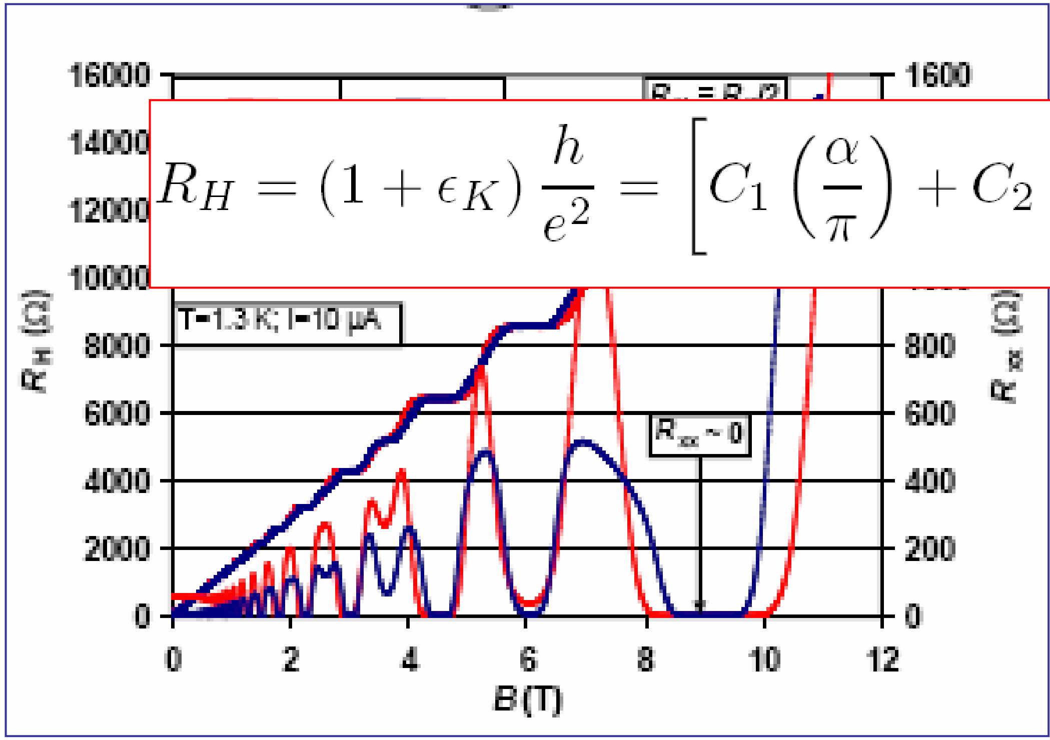
Needs for a `theory´ for QHE



- steps R_n
- rational $R_n = R_1/n$
- universal $R_1 = R_H$
- relation to α

$$R_H = R_K \equiv h/e^2$$

Needs for a `theory´ for QHE



- steps R_n

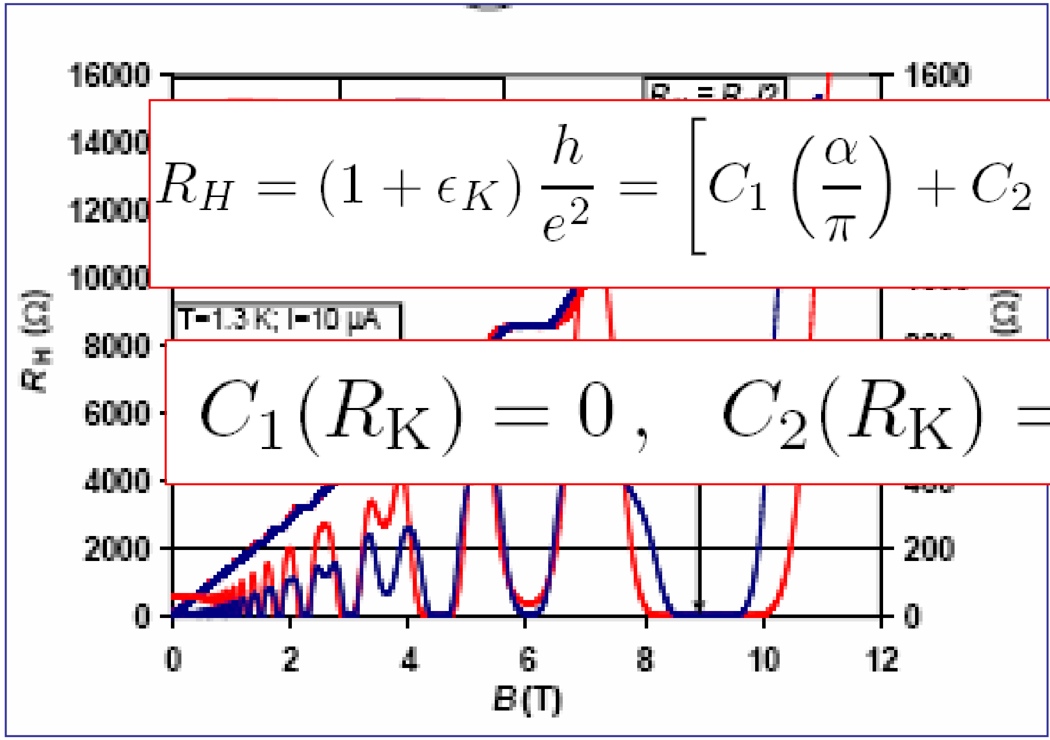
$$R_H = (1+\epsilon_K)\frac{h}{e^2} = \left[C_1\left(\frac{\alpha}{\pi}\right) + C_2\left(\frac{\alpha}{\pi}\right)^2 + C_3\left(\frac{\alpha}{\pi}\right)^3 + \dots\right]\frac{h}{e^2}\,1/n$$

- universal $R_1 = R_H$
- relation to α

$$R_H = R_K \equiv h/e^2$$

Needs for a `theory´ for QHE

- steps R_n



$$R_H = (1+\epsilon_K)\,\frac{h}{e^2} = \left[C_1 \left(\frac{\alpha}{\pi}\right) + C_2 \left(\frac{\alpha}{\pi}\right)^2 + C_3 \left(\frac{\alpha}{\pi}\right)^3 + \dots \right] \frac{h}{e^2}$$

$1/n$

$$C_1(R_\mathrm{K}) = 0\,, \quad C_2(R_\mathrm{K}) = 0\,, \quad C_3(R_\mathrm{K}) \simeq 2(2)$$

R_H

- relation to α

$$R_H = R_K \equiv h/e^2$$

h block

known from α block

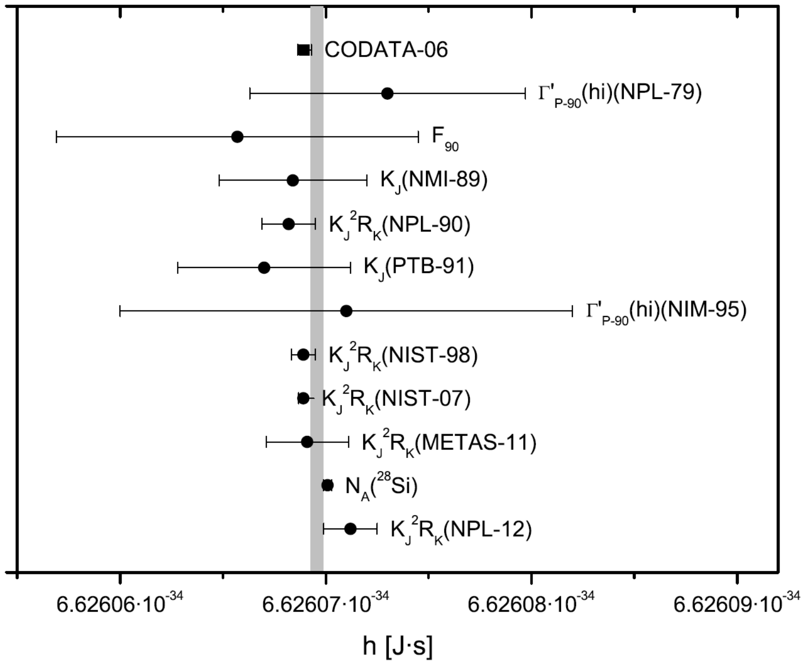
- $$\alpha = \frac{e^2}{4\pi\epsilon_0 hc}$$
- $h \cdot N_A$
- h/m_e

- input:
  - h
  - e
  - N_A
- output
  - m_e
  - μ_B

h block

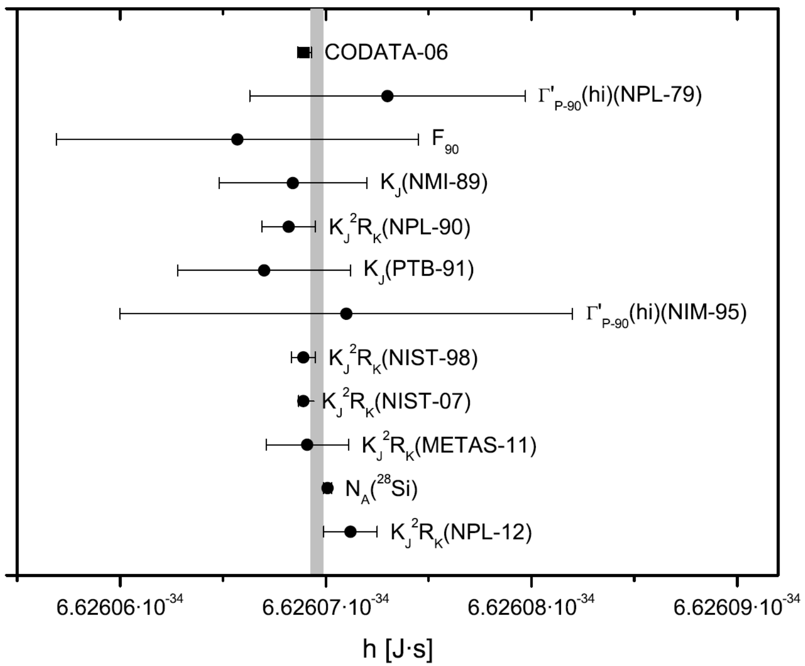
Quantity	Symbol	Value	u_r
Planck constant	h	$6.626\,069\,57(29) \times 10^{-34}$ J s	$[4.4 \times 10^{-8}]$
elementary charge	e	$1.602\,176\,565(35) \times 10^{-19}$ C	$[2.2 \times 10^{-8}]$
Avogadro constant	N_A	$6.022\,141\,29(27) \times 10^{23}$ mol^{-1}	$[4.4 \times 10^{-8}]$
Faraday constant	$F = e \cdot N_A$	$96\,485.3365(21)$ C mol^{-1}	$[2.2 \times 10^{-8}]$
electron charge to mass quotient	e/m_e	$1.758\,820\,088(39) \times 10^{11}$ C kg^{-1}	$[2.2 \times 10^{-8}]$
electron gyromagnetic ratio	$\gamma_e = 2\mu_e/\hbar$	$1.760\,859\,708(39) \times 10^{11}$ s^{-1} T^{-1}	$[2.2 \times 10^{-8}]$
electron mass	m_e	$9.109\,382\,91(40) \times 10^{-31}$ kg	$[4.4 \times 10^{-8}]$
		$0.510\,998\,928(11)$ MeV$/c^2$	$[2.2 \times 10^{-8}]$
proton mass	m_p	$1.672\,621\,777(74) \times 10^{-27}$ kg	$[4.4 \times 10^{-8}]$
		$938.272\,046(21)$ MeV$/c^2$	$[2.2 \times 10^{-8}]$
Bohr magneton	$\mu_B = e\hbar/2m_e$	$927.400\,968(20) \times 10^{-26}$ J T^{-1}	$[2.2 \times 10^{-8}]$
nuclear magneton	$\mu_N = e\hbar/2m_p$	$5.050\,783\,53(11) \times 10^{-27}$ J T^{-1}	$[2.2 \times 10^{-8}]$
Josephson constant	$K_J = 2e/h$	$483\,597.870(11) \times 10^{9}$ Hz V^{-1}	$[2.2 \times 10^{-8}]$

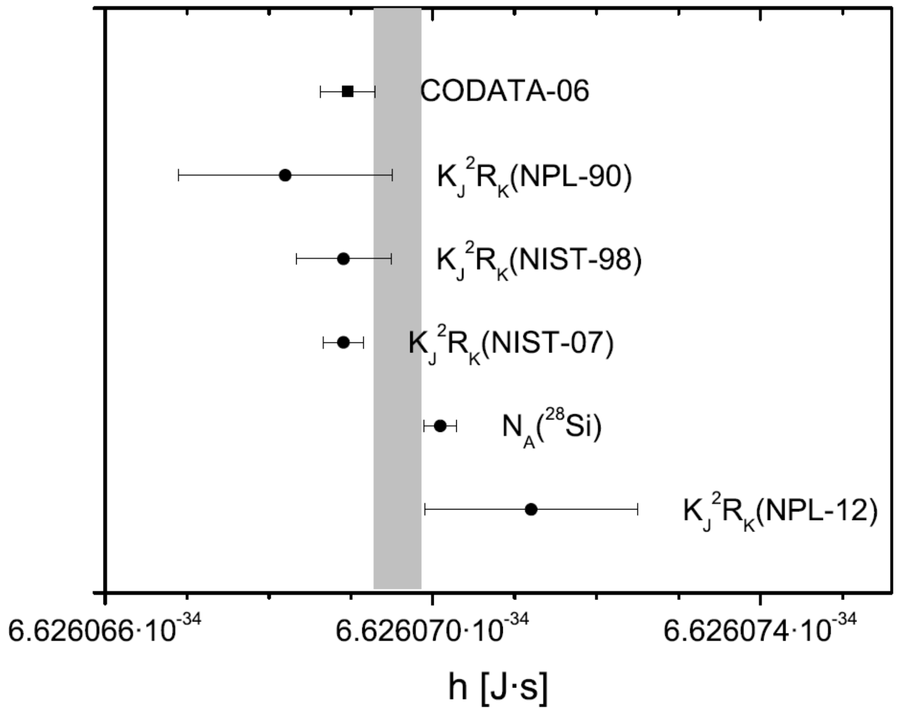
h block



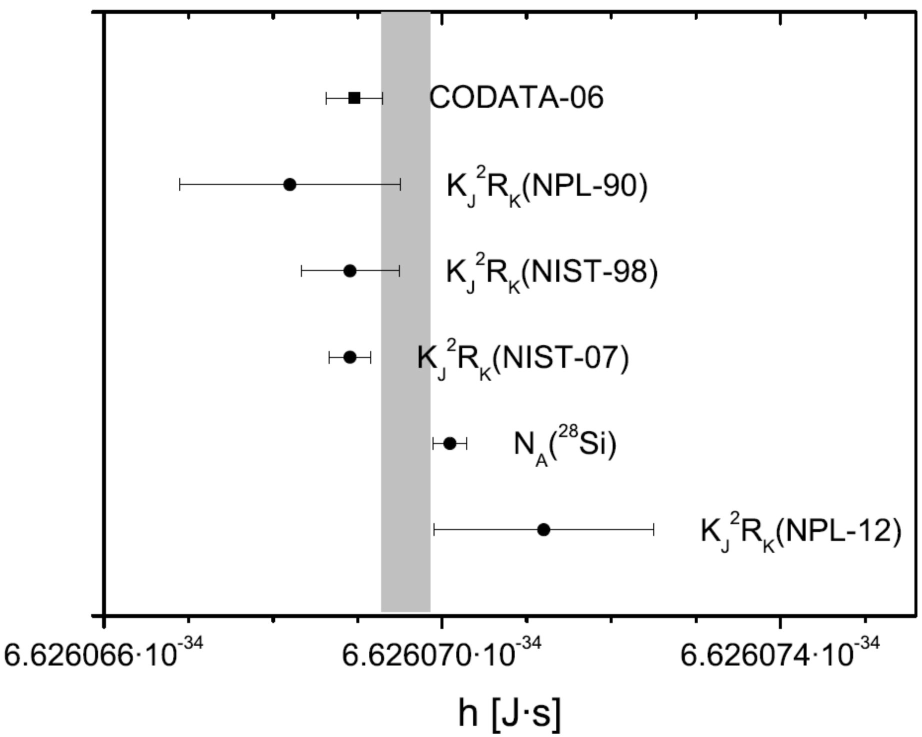
CODATA-06
Γ'P-90(hi)(NPL-79)
F90
KJ(NMI-89)
KJ2RK(NPL-90)
KJ(PTB-91)
Γ'P-90(hi)(NIM-95)
KJ2RK(NIST-98)
KJ2RK(NIST-07)
KJ2RK(METAS-11)
NA(28Si)
KJ2RK(NPL-12)
6.62606·10-34
6.62607·10-34
6.62608·10-34
6.62609·10-34
h [J·s]

h block: the most important data



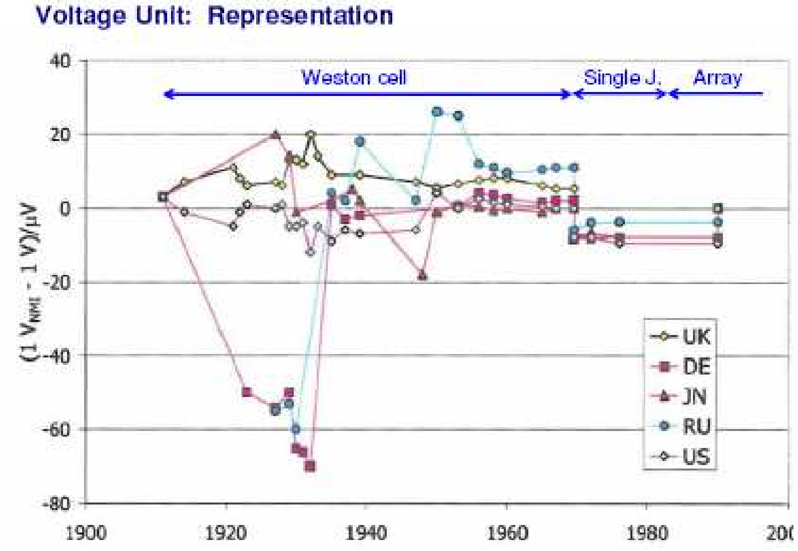


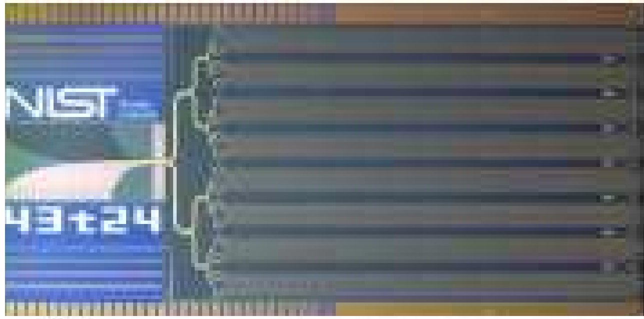
h block: the most important data

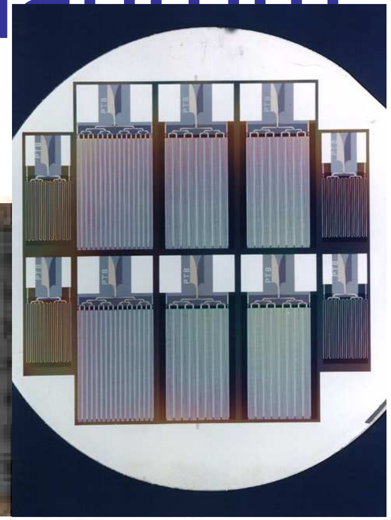


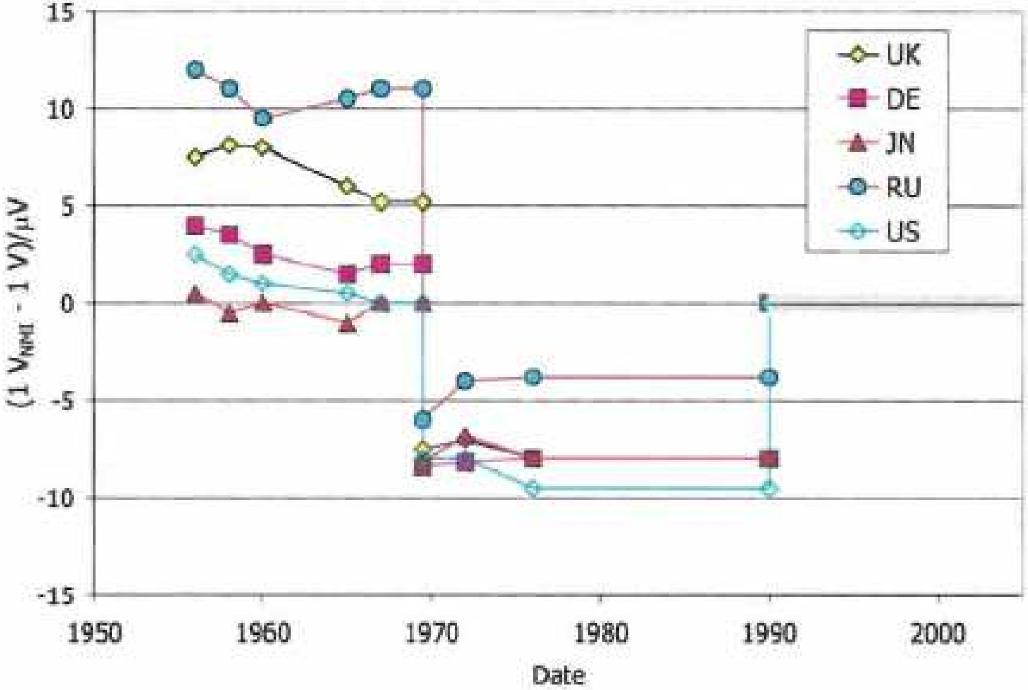
- watt ballance
- Avogadro constant from ehrhiched Si

Josephson effect and quantum volt stardard









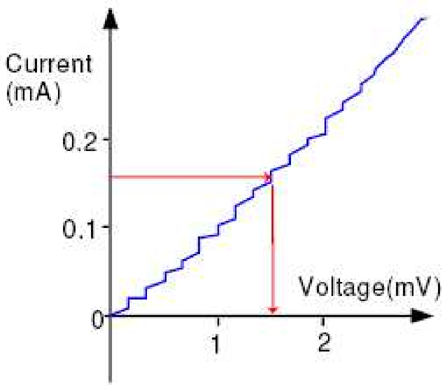
Irradiation with microwave:

- Cooper pairs synchronize with radiation
- Voltage steps appear

$$V_n = n\frac{h}{2e}f$$

Shapiro step, 1963

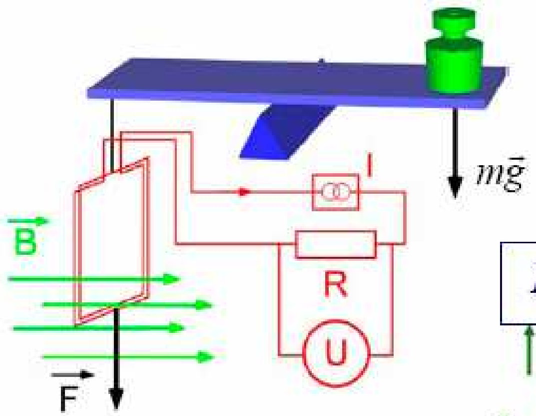
V_1 ~ 145 µV @ 70 GHz



B. Jeanneret, Les Houches, 2007

watt-ballance

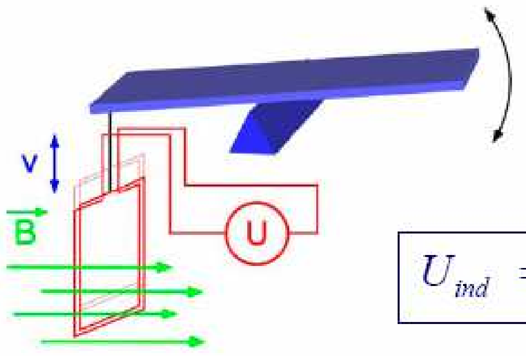
WB Principle (1): static phase / weighing mode



$$I \cdot \int d\ell \times B = mg$$

$$I = \frac{U_j}{R}$$ (QHE & JVS)

WB Principle (2): dynamic phase / velocity mode



$$U_{ind} = v \cdot \int d\ell \times B$$

WB Principle (3): combination of modes

Only if $G_m = G_e$

$$G(B,\ell) = \underbrace{\frac{mg}{I}}_{\text{static}} = \underbrace{\frac{U}{v}}_{\text{dynamic}} \Rightarrow \boxed{UI = mgv}$$

electrical power — mechanical power

B. Jeanneret, Les Houches, 2007

watt-ballance

WB Principle (1): static phase / weighing mode

$$K_J = \frac{2e}{h}$$

…phase / velocity mode

$$R_K = \frac{h}{e^2}$$

WB Pri… …ination of …

$$U_{ind} = v \cdot \int d\ell \times B$$

Only if $G_m = G_e$

mg U

$$\Rightarrow UI = mgv$$

$$\frac{1}{R_K K_J^2} = \frac{h}{4}$$

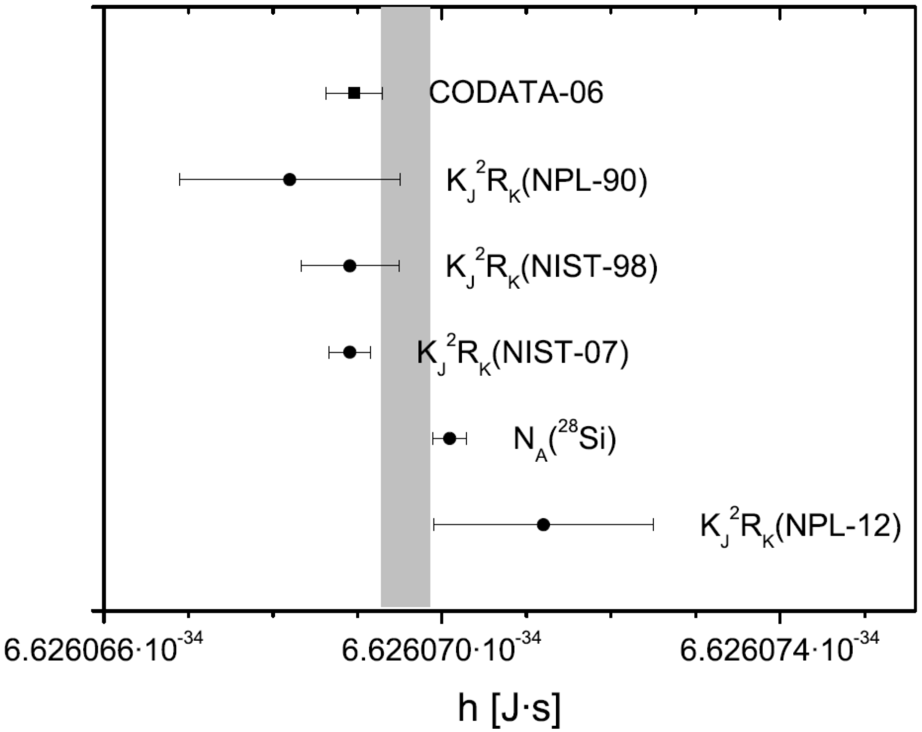
…amic

electrical power

mechanical power

B. Jeanneret, Les Houches, 2007

h block: the most important data



- watt ballance
- Avogadro constant from ehrhiched Si

Monocrystale of ^{28}Si

monocrystale ~ 1 kg

isotopic composition

- ^{28}Si: 92%
- ^{29}Si: 5%
- ^{30}Si: 3%

Monocrystale of ^{28}Si

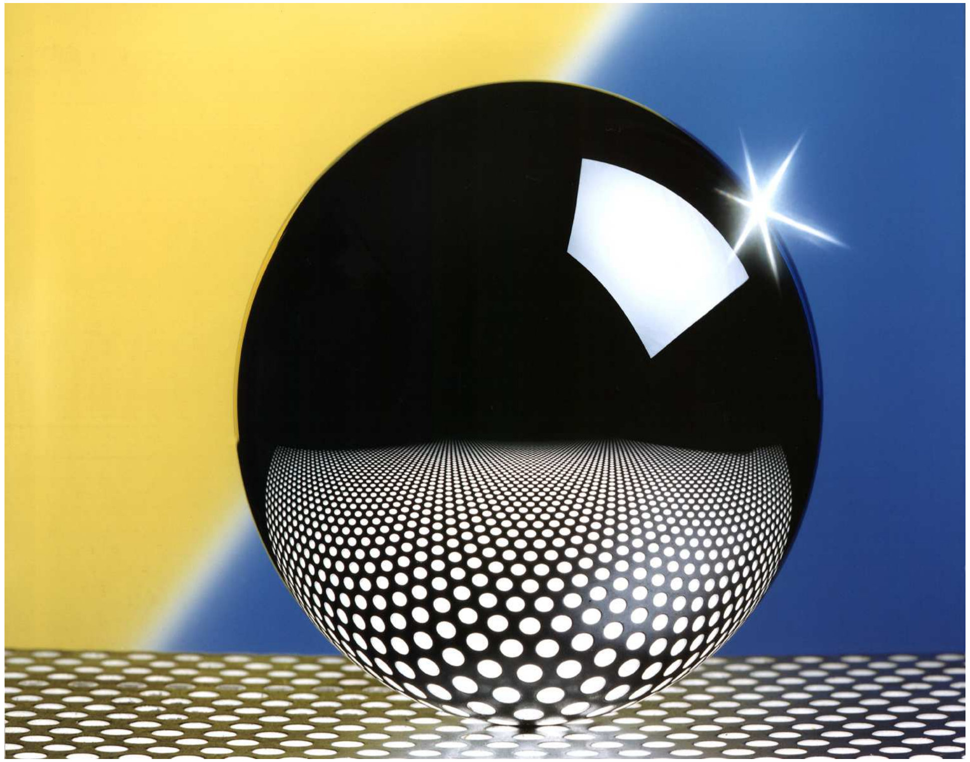
monocrystale ~ 1 kg

isotopic composition

- ^{28}Si: ~~92%~~ 99.985%
- ^{29}Si: ~~5%~~
- ^{30}Si: ~~3%~~

Monocrystale of ^{28}Si

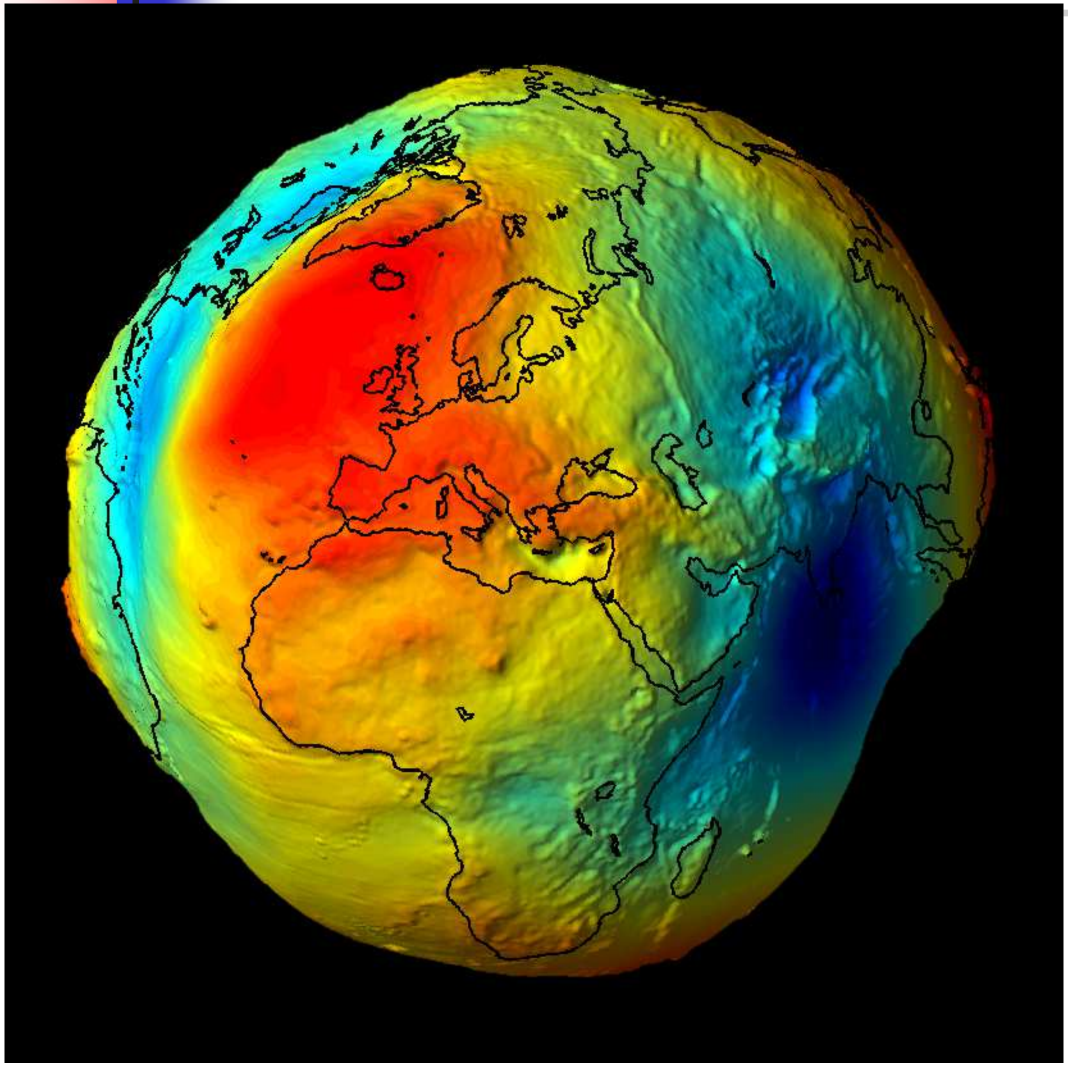
monocrystale ~ 1 kg



isotopic composition

- ^{28}Si: ~~92%~~ 99.985%
- ^{29}Si: ~~5%~~
- ^{30}Si: ~~3%~~

Monocrystale of ^{28}Si



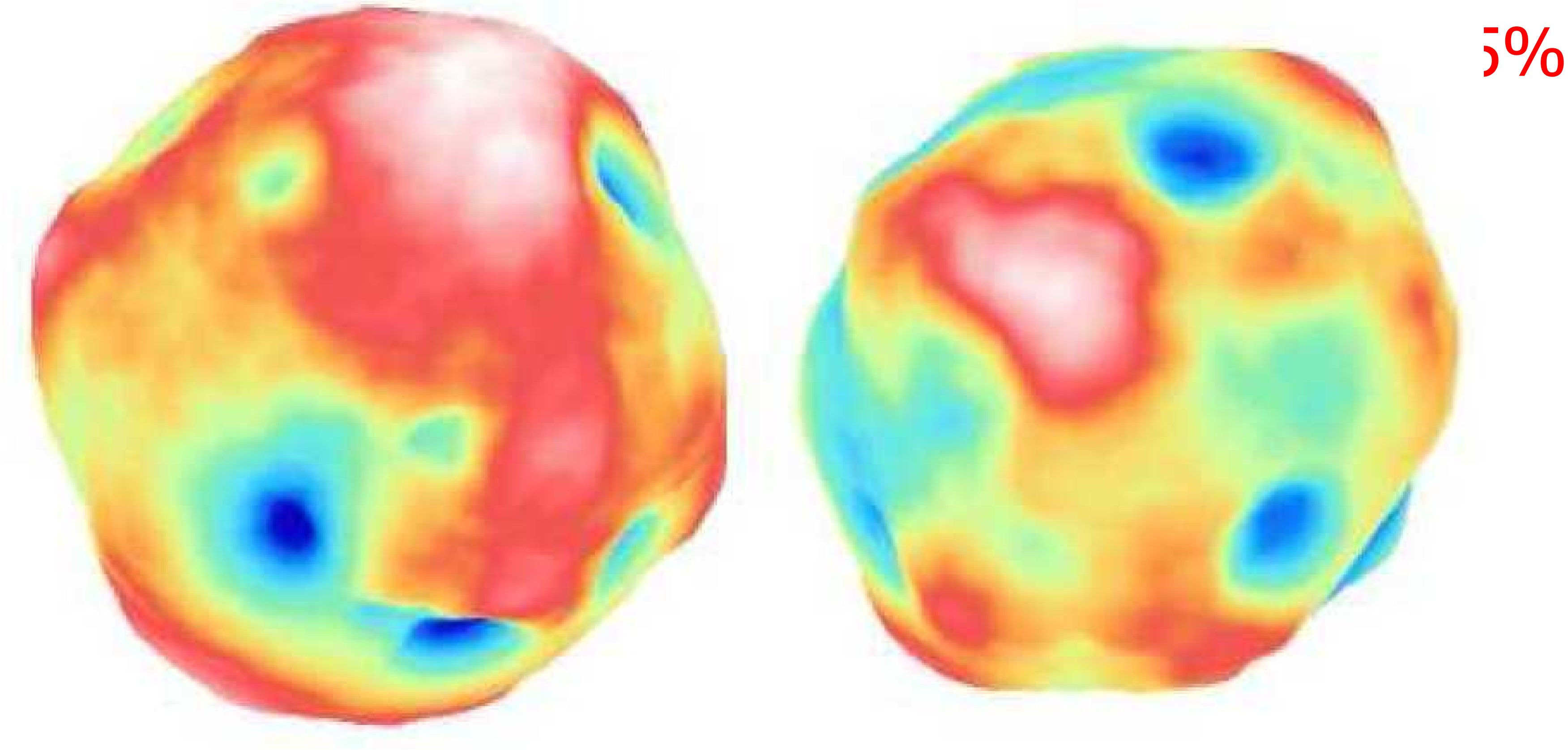
isotopic composition

- ^{28}Si: ~~92%~~ 99.985%
- ^{29}Si: ~~5%~~
- ^{30}Si: ~~3%~~

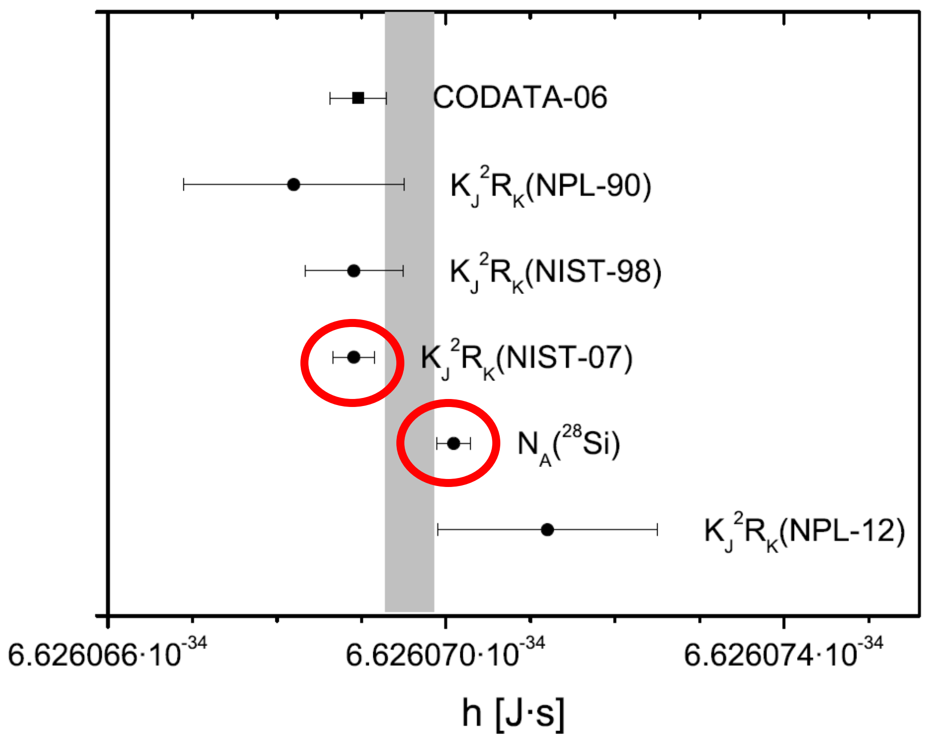
Monocrystale of ^{28}Si

monocrystale ~ 1 kg

isotopic composition

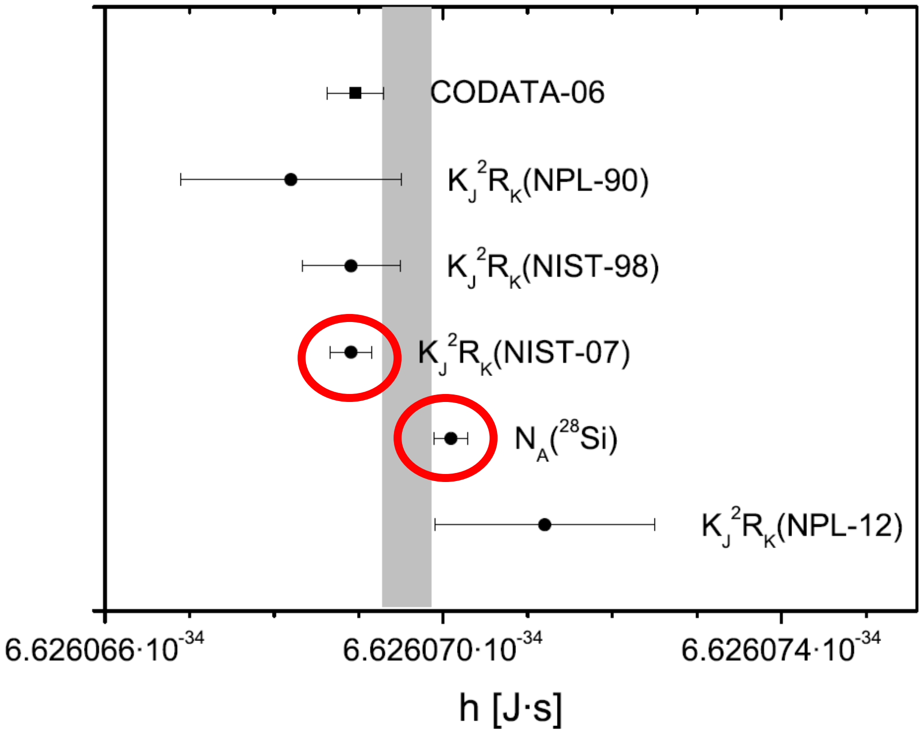


h block: the most important data

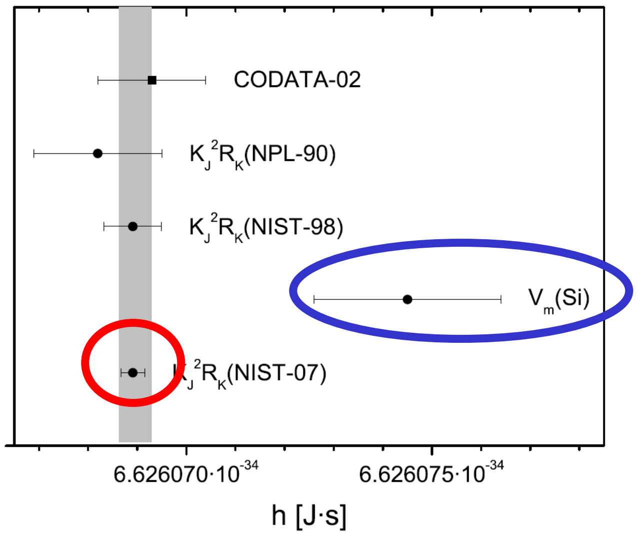


- watt ballance
- Avogadro constant from ehrhiched Si
- problem remains

h block: the most important data

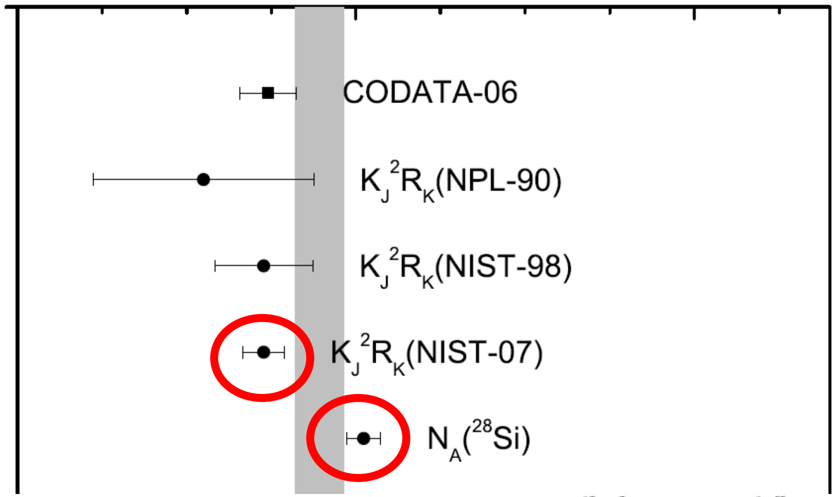


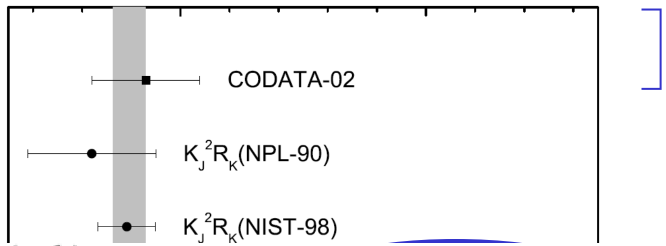
- watt ballance
- Avogadro constant from ehrhiched Si

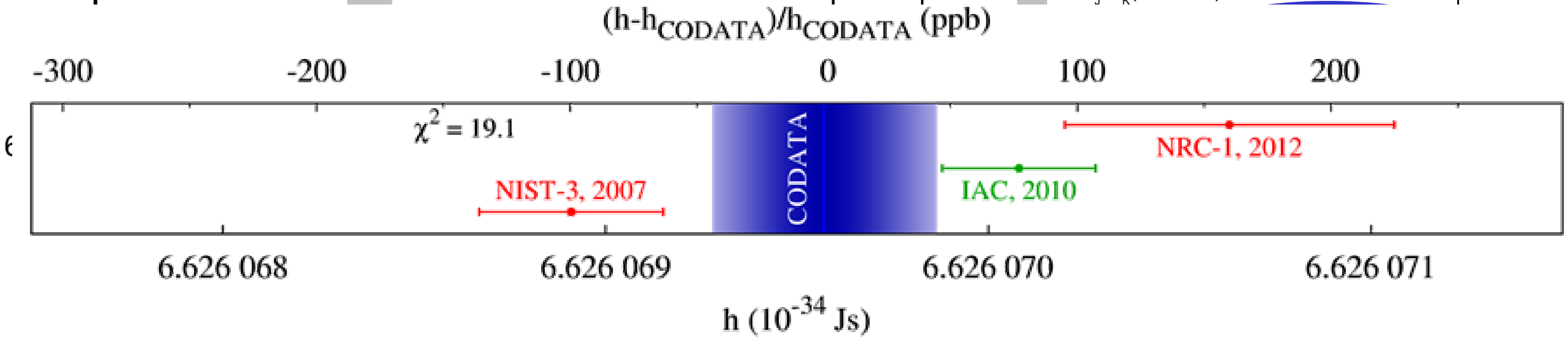


h block: the most important data

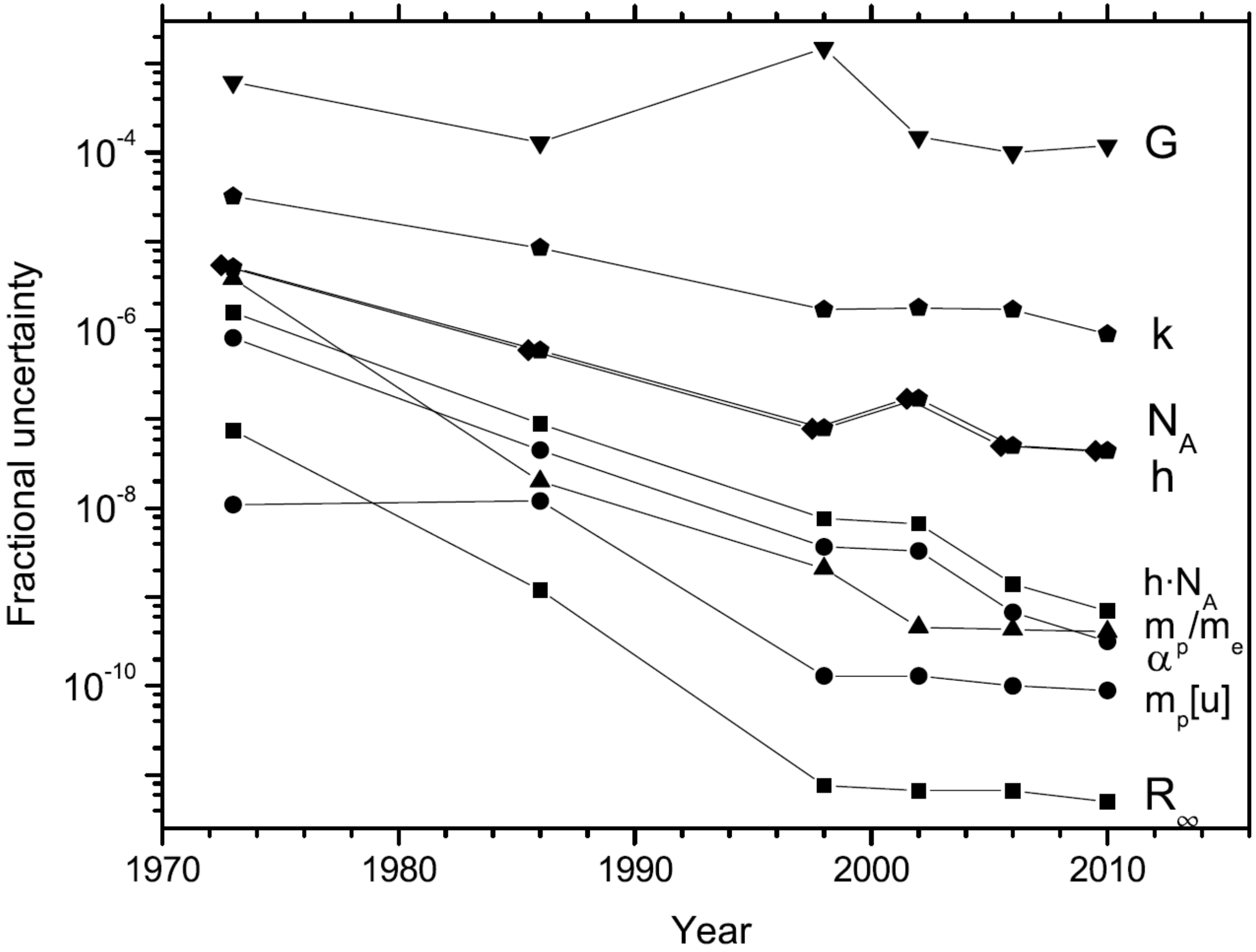
- watt ballance
- Avogadro constant from ehrhiched Si





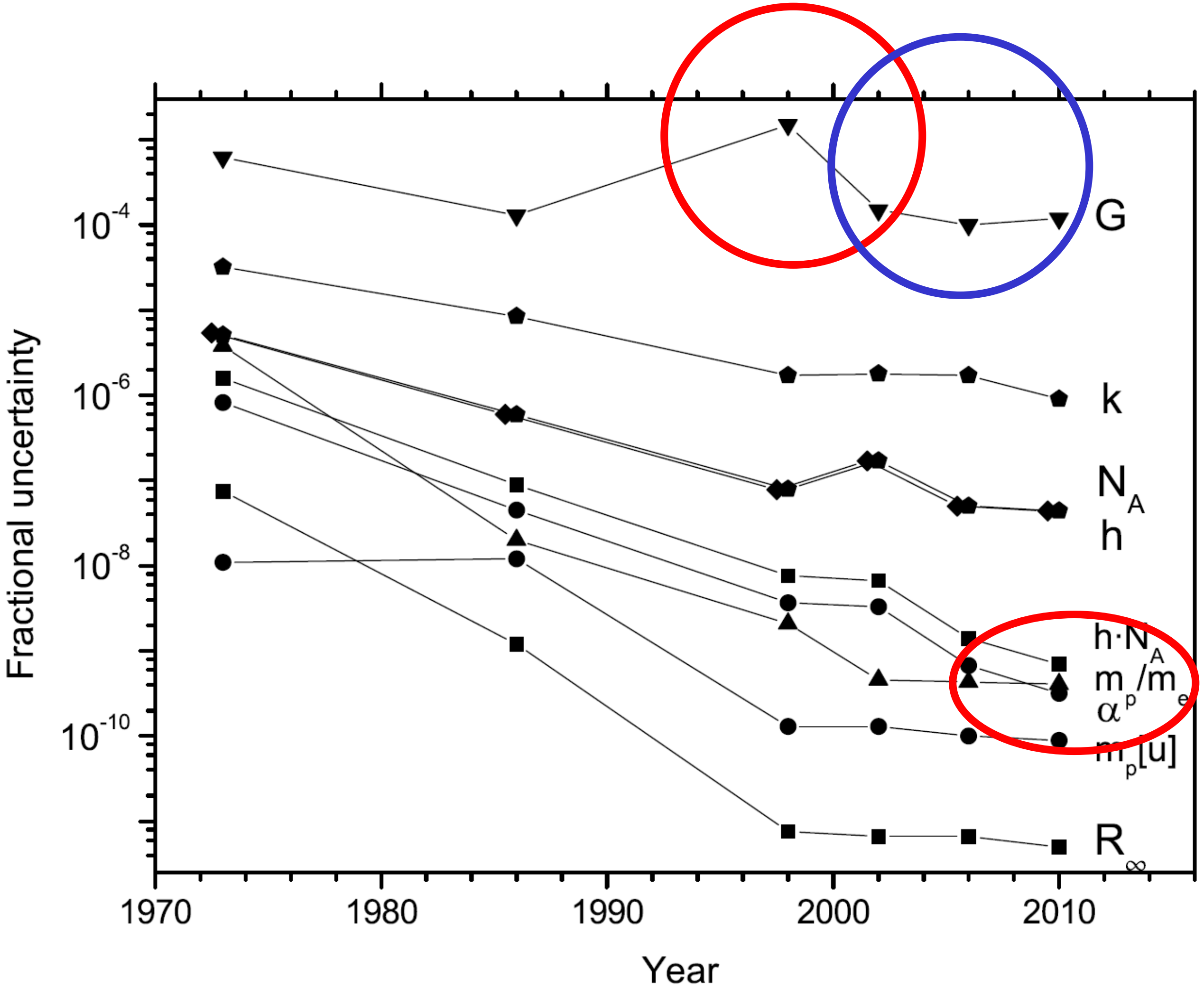


Progress



Fractional uncertainty
10^{-4}
10^{-6}
10^{-8}
10^{-10}
1970
1980
1990
2000
2010
Year
G
k
N_A
h
$h \cdot N_A$
m_p/m_e
α
$m_p[u]$
R_∞

Progress



G
k
N_A
h
$h \cdot N_A$
m_p/m_e
α
m_p[u]
R_∞
Fractional uncertainty
10^{-4}
10^{-6}
10^{-8}
10^{-10}
1970
1980
1990
2000
2010
Year

Progress

Quantity	$u_r(2006)$	Δ	$\Delta/u_r(2006)$	$u_r(2010)$	$u_r(2010)/u_r(2006)$
R_∞	6.6×10^{-12}	1.1×10^{-12}	0.17	5.0×10^{-12}	0.76
m_e/m_p	4.3×10^{-10}	0.1×10^{-10}	0.03	4.1×10^{-10}	0.95
α	6.8×10^{-10}	44.2×10^{-10}	6.50	3.2×10^{-10}	0.47
h	5.0×10^{-8}	9.2×10^{-8}	1.84	4.4×10^{-8}	0.88
k	1.7×10^{-6}	-1.2×10^{-6}	-0.68	9.1×10^{-7}	0.53
G	1.0×10^{-4}	-0.7×10^{-4}	-0.66	1.2×10^{-4}	1.2

Progress

Quantity	$u_r(2006)$	Δ	$\Delta/u_r(2006)$	$u_r(2010)$	$u_r(2010)/u_r(2006)$
R_∞	6.6×10^{-12}	1.1×10^{-12}	0.17	5.0×10^{-12}	0.76
m_e/m_p	4.3×10^{-10}	0.1×10^{-10}	0.03	4.1×10^{-10}	0.95
α	6.8×10^{-10}	44.2×10^{-10}	6.50	3.2×10^{-10}	0.47
h	5.0×10^{-8}	9.2×10^{-8}	1.84	4.4×10^{-8}	0.88
k	1.7×10^{-6}	-1.2×10^{-6}	-0.68	9.1×10^{-7}	0.53
G	1.0×10^{-4}	-0.7×10^{-4}	-0.66	1.2×10^{-4}	1.2

Problems

- R_∞ & R_p
- m_e/m_p
- α
- h
- G
- k
- a_μ

+ better accuracy

+ two methods

+ sensitivity to 5 loops

– 6-sigma jump

Problems

- R_∞ & R_p
- m_e/m_p
- α
- h
- G
- k
- a_μ

+ natural-silicon discrepacy resolved

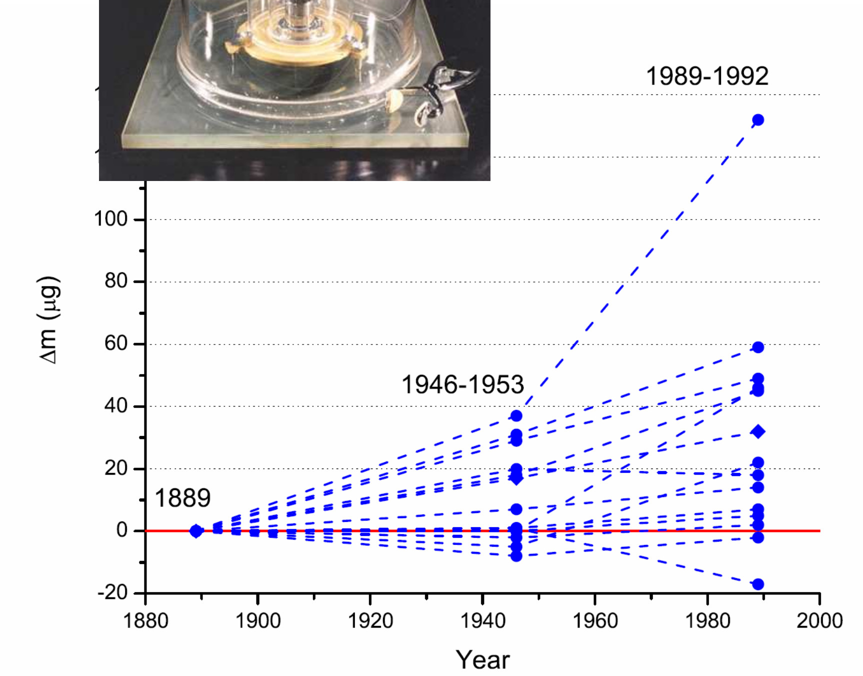
+ better accuracy for Avodagro

- new discrepancy

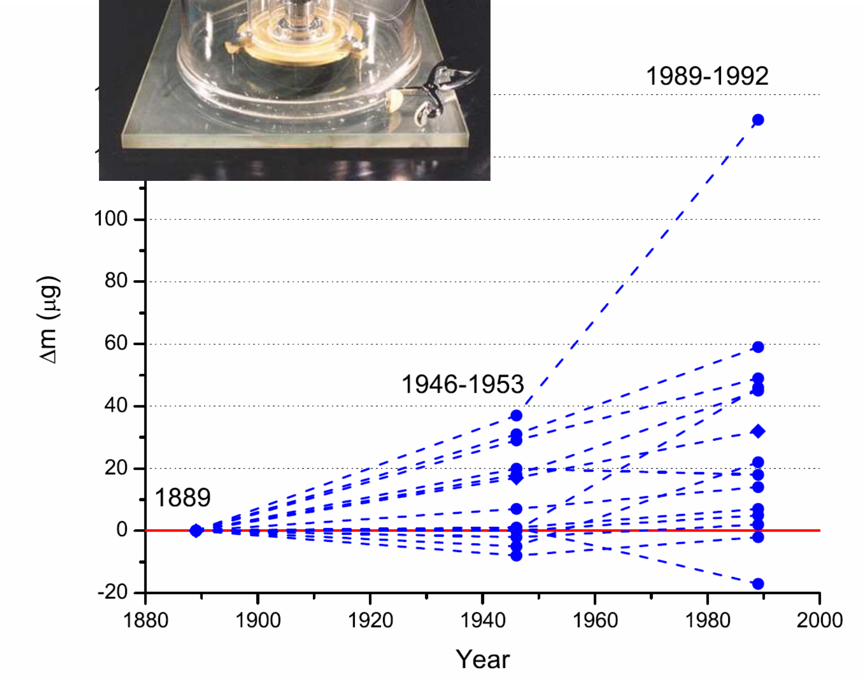
NPL $\rightarrow$ NRC

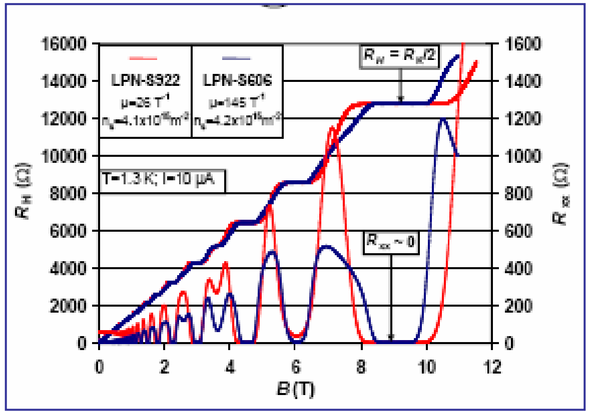
NIST-3

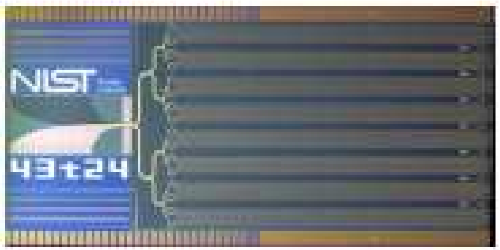
Do we need a new definition?
The most probably, yes, we do!

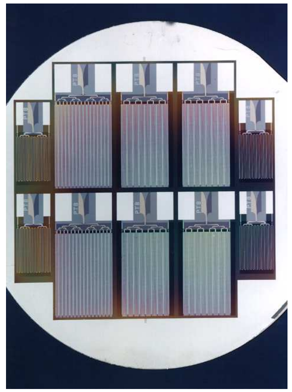


Do we need a new definition?
The most probably, yes, we do!

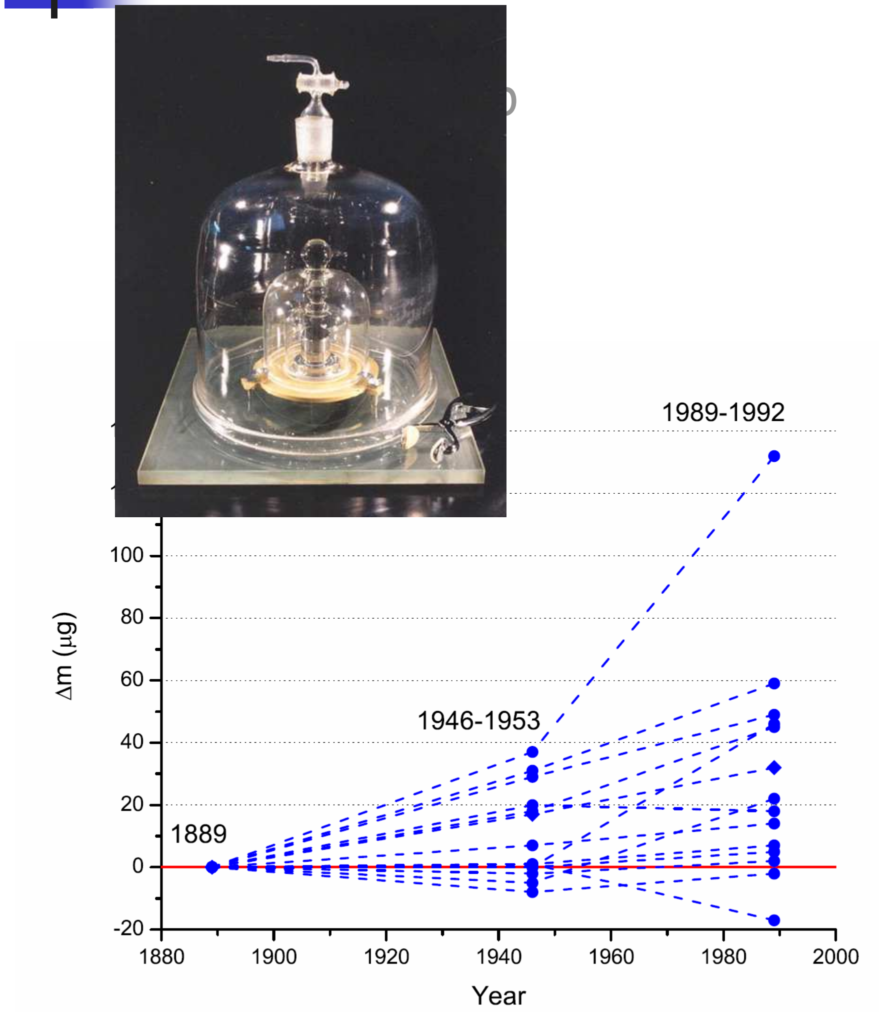








Are we ready for the new definition?



+ natural-silicon discrepancy resolved
+ better accuracy for Avodagro

- new discrepancy