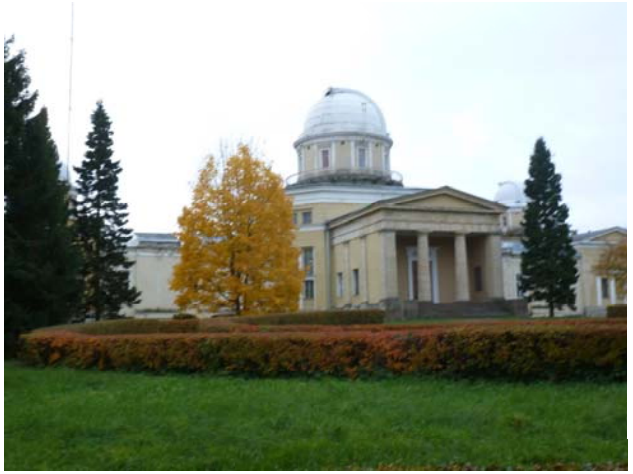

Workshop on Precision Physics and Fundamental Physical Constants

(FFK 2013, Pulkovo, 7-11 October)

Fundamental constants and transition to unity of system of physical quantities and units in electrodynamics

K. Tomilin

S.I.Vavilov Institute for the History of Science and Technology RAS

tomilin@ihst.ru

FFK2013, CPO 11 October 2013

Be careful! Check the system of units
when you open a new book about electricity!

Ja.A. Smorodinsky

Outline

1. Systems of Natural Units

2. Problems caused by Gaussian system of units

3. Fundamental constants in electrodynamics

4. Systems of units in main textbooks (Landau-Lifshits and others)

5. Electrodynamics: physical quantities and laws

6. Relations between physical quantities in CGS and SI.

7. CGS systems should be replaced to the system which is equivalent to new (quantum) SI for c=1, h=1, e=1.

8. Laws of electrodynamics in CGS(Gaussian) and CGS(Quantum)

Conclusion

1. Systems of Natural Units (Natürliche Einheiten)

in theoretical physics:

Maxwell J.C. (c and G), (e)	1873	
Stoney G. (c, e, G)	1874/81	
Planck M. (c, h, k and G)	1899/06, 1950s	→ GR, Quantum gravity
Hartree D. ($\hbar, e$ and m_e)	1928	→ Atomic physics
Ruark A. ($c, \hbar$ and m_e)	1931	→ ($c, \hbar$ and eV) High energy physics
Stille U. (c, h, e, k and m_p, μ_B)	1949	→ Modern quantum metrology

in metrology:

Z.Bay et al.: c	exact from 20 October 1983
(c, h, e, k)	exact ~2014

motivation:

1) creating of universal units for any civilizations,

2) simplification of physical formulas

reviews: Dolinsky E., Pilipchuk B. *(in Russian, 1965)*

↓

Tomilin K. *(1999)*

↓

wikipaedia *(~2004)*

2. Problems caused by Gaussian system of units

CGS(Gaussian): $\alpha = \frac{e^2}{\hbar c}$

(*Sommerfeld*, 1915)

CGS(Heaviside): $\alpha = \frac{e^2}{4\pi\hbar c}$

Applying of these systems of units is the source a number of groundless speculations:

1. one from three constants – c, $\hbar$ or e is not fundamental constant (*J.Jeans, M.Born, P.Dirac et al.*)

2. some physical principles can be non-fundamental (as uncertainty principle – *P.Dirac*)

3. applying of system of units where c=1, $\hbar$=1 and e=1 simultaneously is impossible (*P.Bridgeman, D.Hartree, F.Wilczek et al.*)

4. physical theories with variable constants (as cosmology with variable speed of light – *Moffat et al.*)

. . . .

But really this is not laws, but **definitions** of elementary charge e in mechanical units in systems:

CGS(Gaussian): $e^2 = \alpha\hbar c$

CGS(Heaviside): $e^2 = 4\pi\alpha\hbar c$

In SI (nowadays): $\alpha = \frac{e^2}{4\pi\varepsilon_0\hbar c}$ (*Sommerfeld*, 1935)

SI (nowadays): $e^2 \equiv 4\pi\varepsilon_0\alpha\hbar c$

new SI (quantum): $\varepsilon_0^{-1} \equiv 4\pi\alpha\hbar c / e^2$

3. Fundamental constants in electrodynamics

one dimensionless constant:

fine-structure constant α – *dimensionless coupling constant*

dimensional constants:

velocity of light c
elementary charge e
Planck constant $\hbar$
} *basic constants*

electric constant ε_0
magnetic constant μ_0
impedance of free space Z_0
} *secondary constants defined by combinations of basic constants, fine-structure constant and mathematical constants*

dimensional constants have different physical meanings in different systems of units:

CGS(Gaussian): $e^2 \equiv \alpha\hbar c$
CGS(Heaviside): $e^2 \equiv 4\pi\alpha\hbar c$
SI (nowadays): $e^2 \equiv 4\pi\varepsilon_0\alpha\hbar c$
} *dimensional coupling constants*

new SI (quantum): $c,\ \hbar,\ e$ – *basic constants*

$\varepsilon_0^{-1} \equiv 4\pi\alpha\hbar c / e^2$
$Z_0 \equiv \varepsilon_0^{-1}c^{-1} \equiv \mu_0 c \equiv 4\pi\alpha\hbar / e^2$
$\mu_0 \equiv \varepsilon_0^{-1}c^{-2} \equiv 4\pi\alpha\hbar / ce^2$
} *dimensional coupling constants*

in new SI for c=1, $\hbar$=1, e=1: $\varepsilon_0^{-1} = \mu_0 = Z_0 = 4\pi\alpha$

4. Systems of units in main textbooks

Textbooks	System of units	Fields	Dimension	Constants
Landau L.D., Lifshits E.M. Theory of field. QED Electrodynamics of Continuous Media	CGS(Gaussian)	(E, H) F_{ik} (E, H) E, B, D, H	3D 4D 3D	
Sommerfeld A. Elektrodynamik. Leipzig, 1949.	LTMQ	(E, B), (D, H) F_{ik}, f_{ik}	3D 4D	ε_0 and μ_0
Tonnelat M.-A. Les principés de la théorie électromagnétique et de la relativité, 1959	SI (but without rationanization)	E, D, B, H $\varphi_{\mu\nu}$, $f_{\mu\nu}$	3D 4D	ε_0 and μ_0
Jackson J.D. Classical electodynamics. N.Y.-L.: J. Wiley&sons. 1962.	CGS(Gaussian)	E, B, D, H	3D	

Feynman, R.P., R.B. Leighton, and M. Sands, *The Feynman Lectures on Physics, Vol. II: the Electromagnetic Field,* Addison-Wesley, Reading, Mass. 1965.	SI (but only with ε_0)	E, B	3D	ε_0
Purcell E. Electricity and Magnetism (Berkeley Physics Course, Vol. 2). 1965.	CGS(Gaussian)	(E, B)	3D	
Sivukhin D. Electricity. 2 ed. 1977	CGS(Gaussian) SI (one §)	(E, B), (D, H)	3D	ε_0 and μ_0
Tamm I.E. Principles of theory of electricity.	CGS(Gaussian)	(E, H), (E, B, D, H)	3D 3D	
Ugarov V.A. The theory of relativity. 2 ed. 1977	SI	(E, B), (D, H) F_{ik}, f_{ik}	3D 4D	ε_0 and μ_0
Akhiezer A.I., Berestetsky V.B., QED. 1981.	CGS(Heaviside)	(E, H)		

5. Electrodynamics: physical quantities and laws

$$E = F / q \qquad \mathrm{div} D = \rho$$

$$[E] = \frac{force}{\mathrm{charge}} \qquad [\varepsilon_0^{-1}] = \frac{\mathrm{force} \cdot \mathrm{length}^2}{charge^2} = \left[\frac{\hbar c}{e^2}\right] \qquad [D] = \frac{\mathrm{charge}}{length^2}$$

$$\mathrm{F} = (\mathrm{cB}, -\mathrm{iE})$$

"intensive" quantities (Intensitätsgrößen)

$$\mathrm{f} = (\mathrm{H}, -\mathrm{icD})$$

"extensive" quantities (Quantitätsgrößen)

$$\mathrm{F_{ik}} = \sqrt{\frac{\mu_0}{\varepsilon_0}}\mathrm{f_{ik}} = \mathrm{Z_0 f_{ik}}$$

(Sommerfeld A.)

$$F_{ik} = \varepsilon_0^{-1} f_{ik}$$

(Ugarov V.A.)

$$F_{ik} = 4\pi\alpha f_{ik}$$

(for c=1, $\hbar$=1, e=1)

6. Relations between physical quantities in CGS and SI.

CGS(Gaussian)	SI
E	$\sqrt{4\pi\varepsilon_0}\,$E
D	$\sqrt{\frac{4\pi}{\varepsilon_0}}\,$D
B	$\sqrt{\frac{4\pi}{\mu_0}}\,$B
H	$\sqrt{4\pi\mu_0}\,$H
q, ρ	$\frac{1}{\sqrt{4\pi\varepsilon_0}}(q, \rho)$ (Table from *Jackson J.D.*, 1962)

Thus electric charge q (СИ)= $(4\pi\varepsilon_0)^{-1/2}$ q (Gaussian), field E (SI)= $(4\pi\varepsilon_0)^{1/2}$ E (Gaussian),

where $\varepsilon_0^{-1} = 4\pi\alpha\frac{\hbar c}{e^2}$. However, between six-vectors and tensors in these systems there is no-one correspondence (this is is possible only when c=1):

$$F = (E, B) \text{ (Gaussian)} \rightarrow F = \sqrt{4\pi\varepsilon_0}\,(E, cB) \quad \text{(SI)}$$

$$f = (D, H) \text{ (Gaussian)} \rightarrow f = \sqrt{4\pi\mu_0}\,(cD, H) \quad \text{(SI)}$$

7. CGS system should be replaced to system which equivalent to quantum SI for *c*=1, *ħ*=1, *e*=1.

Gaussian system 1870s

MKS, *Giorgi* 1903

MKSQ, *Sommerfeld* 1933/34

MKSA 1950s

Heaviside system 1894

SI 1960

Classical electrodynamics
QED

Classical electrodynamics

new SI (quantum SI) 2014

CGS(Quantum) ≡ quantum SI for c=1, $\hbar$=1, e=1

Should be recommended to go over in new textbooks from Gaussian or Heaviside systems to QSI (quantum SI) or system which is equivalent QSI for c=1, $\hbar$=1, e=1. It requires transition to other physical quantities:

charges $q \to \alpha^{1/2} q$ and field $(E, B) \to \alpha^{-1/2}(E, B)$ and $(D, H) \to 4\pi\alpha^{1/2}(D, H)$.

8. Laws of electrodynamics in CGS(Gaussian) and CGS(Quantum)

	CGS(Gaussian):	
	$L = -\frac{1}{4}\mathbf{F}_{ik}\,\mathbf{F}^{ik}$ →	$L = -\frac{1}{4}\mathbf{F}_{ik}\mathbf{f}^{ik} = -\frac{1}{16\pi\alpha}\mathbf{F}_{ik}\mathbf{F}^{ik}$
	$L = \frac{1}{8\pi}(\mathbf{E}^2 - \mathbf{H}^2)$ →	$L = \frac{1}{2}(\mathbf{ED} - \mathbf{BH})$
Maxwell equations:	$\mathrm{Div}\ F^* = 0$ $\mathrm{Div}\ f = 4\pi J$ $F = f$ →	$\mathrm{Div}\ F^* = 0$ $\mathrm{Div}\ f = J$ $F = 4\pi\alpha f$, где $F^* = (-iE, cB)$ and $f = (H, -icD)$
Coulomb law:	$F = \frac{q_1 q_2}{r^2}$ →	$F = \alpha\frac{q_1 q_2}{r^2}$

Conclusion

1. A variety of systems of units in electromagnetism is unacceptable, because it is a source of errors and produces a variety of unfounded speculations.

2. The basis for the unity of the system of units in electromagnetism is provided by new (quantum) SI.

3. What should be changed in the textbooks (which uses SI)

(1) providing definitions of the secondary dimensional constant ε_0, μ_0, Z_0 through more fundamental constants c, $\hbar$ and e.

(2) it is desirable to transit from three dimensional constants ε_0, μ_0 and Z_0 to one (also taking into account the speed of light)

4 . What should be changed in the textbooks (which uses the Gaussian system and a Heaviside)

(1) In those books and articles that use the fields $\boldsymbol{E}$, $\boldsymbol{H}$ (as in the Landau-Lifshitz textbook), use $\boldsymbol{E}$ and $\boldsymbol{B}$.

(2) to introduce the fine-structure constant α explicitly in the laws of electromagnetism, so that such system of units was completely equivalent to new (quantum) system SI with the choice of c=1, $\hbar$=1, e=1. This is due to the transition to a different system physical quantities.

(3) two tensor (six-vectors) $\boldsymbol{F}_{ik}$ = (***E***, ***B***), $\boldsymbol{f}_{ik}$ = (***D***, ***H***) should be distinguished even in vacuum, since the fine-structure constant α is in relation between them. Thus, the "material equation" as it is considered in Gaussian system becomes the fundamental equation of electromagnetism. It is no accidentally Sommerfeld wrote it even in the preface of his book.
This will ensure the unity of the system quantities and units in electrodynamics, both classical and quantum electrodynamics.

5. Should be used and 4-dimensional quantities (tensors), and 3-dimensional, but not only 3-dimensional.

6. It is need to change the terminology in accordance with the physical meaning, since the terminology is formed in the XIX century, and does not reflect modern electrodynamics.

Thank you for attention!